10.3 Arcs and Chords- Notes

Part 1. In a circle or in congruent circles, two minor arcs are congruent iff their corresponding chords are ___≅___

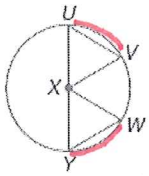
EXAMPLE Prove Theorem 10.2

1 Theorem 10.2 (part 1)

Given: $\odot X$, $\widehat{UV} \cong \widehat{YW}$

Prove: $\overline{UV} \cong \overline{YW}$

Proof:



Statements	Reasons
1. $\widehat{UV} \cong \widehat{YW}$	1. given
2. $\angle UXV \cong \angle WXY$	2. ≅ arcs ⇒ ≅ central ∠s
3. $\overline{XU} \cong \overline{XY} \cong \overline{XW} \cong \overline{XV}$	3. all radii in same circle are ≅
4. $\triangle XVU \cong \triangle XWY$	4. SAS
5. $UV \cong YW$	5. cpctc

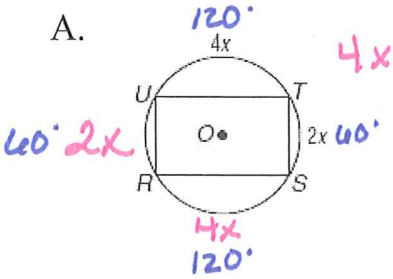
PROOF (Part 2 of Theorem 10.2) Given $\odot X$ and $\overline{UV} \cong \overline{YW}$, prove $\widehat{UV} \cong \widehat{YW}$. (Use the figure from part 1 of Theorem 10.2.)

1. $UV \cong YW$	1. given
2. $\overline{XU} \cong \overline{XY} \cong \overline{XW} \cong \overline{XV}$	2. all radii in same circle are ≅
3. $\triangle XVU \cong \triangle XWY$	3. SSS
4. $\angle UXV \cong \angle YXW$	4. cpctc
5. $\widehat{UV} \cong \widehat{YW}$	5. ≅ central ∠s make ≅ arcs.

Examples for part 1:

Determine the measure of each arc of the circle circumscribed about the polygon.

A.

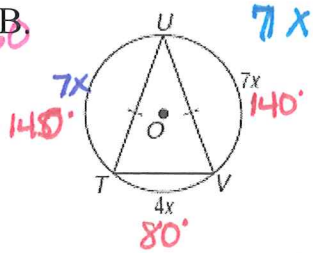


$4x + 2x + 4x + 2x = 360$

$12x = 360$

$x = 30$

B.



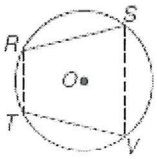
$7x + 7x + 4x = 360$

$18x = 360$

$x = 20$

----------**Part 2**----------

If all vertices of a polygon lie on a circle, the polygon is said to be ___inscribed___ in a circle and the circle is ___Circumscribed___ about the polygon.



$\overline{RS} \cong \overline{TV}$ if and only if $\widehat{RS} \cong \widehat{TV}$.

$RSVT$ is inscribed in $\odot O$.

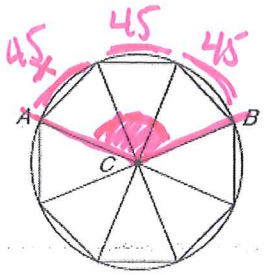
$\odot O$ is circumscribed about $RSVT$.

Examples from part 2:

A regular octagon is inscribed in a circle as part of a stained glass art piece. If opposite vertices are connected by line segments, what is the measure of angle ACB?

A 108 C 135

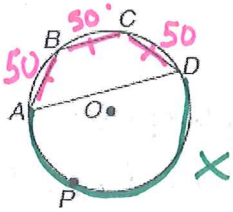
B 120 D 150



$360 \div 8$
$= 45°$

Example Trapezoid $ABCD$ is inscribed in $\odot O$. If $\overline{AB} \cong \overline{BC} \cong \overline{CD}$ and $m\widehat{BC} = 50$, what is $m\widehat{APD}$?

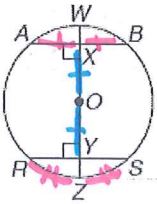
$\boxed{\widehat{APD} = 210°}$



Part 3

Diameters and Chords

- In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.
- In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

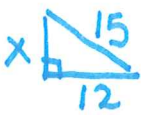


If $\overline{WZ} \perp \overline{AB}$, then $\overline{AX} \cong \overline{XB}$ and $\widehat{AW} \cong \widehat{WB}$.

If $OX = OY$, then $\overline{AB} \cong \overline{RS}$.

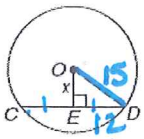
If $\overline{AB} \cong \overline{RS}$, then $\overline{AB}$ and $\overline{RS}$ are equidistant from point O.

Example In $\odot O$, $\overline{CD} \perp \overline{OE}$, $OD = 15$, and $CD = 24$. Find x.



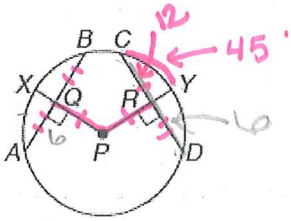
$x^2 + 12^2 = 15^2$

$\boxed{x = 9}$



Exercises

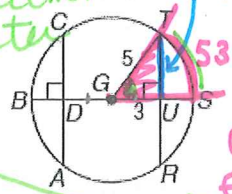
In $\odot P$, $CD = 24$ and $m\widehat{CY} = 45$. Find each measure.



1. $AQ = 6$
2. $RC = 6$
3. $QB = 6$
4. $AB = 12$
5. $m\widehat{DY} = 45°$
6. $m\widehat{AB} = 90°$
7. $m\widehat{AX} = 45°$
8. $m\widehat{XB} = 45°$
9. $m\widehat{CD} = 90°$

Radii are ⊥ to chords (bisectors)
chords equidistant to the center are ≅

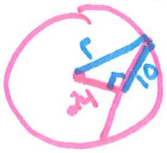
In $\odot G$, $DG = GU$ and $AC = RT$. Find each measure.



$\cos^{-1}\left(\frac{3}{5}\right)$
$\theta = 53°$

10. $TU = 4$
11. $TR = 4$
12. $m\widehat{TS}$ 53°
13. $CD = 4$
14. $GD = 3$
15. $m\widehat{AB}$ 53°

16. A chord of a circle 20 inches long is 24 inches from the center of a circle. Find the length of the radius.



$24^2 + 10^2 = r^2$
$r^2 = 676$
$\boxed{r = 26 \text{ inches}}$