NAME _____ DATE_____ PERIOD ____

12-2 Lesson Reading Guide

Permutations and Combinations

Get Ready for the Lesson

Read the introduction to Lesson 12-2 in your textbook.

Suppose that 20 students enter a math contest. In how many ways can first, second, and third places be awarded? (Write your answer as a product. Do not calculate the product.)
20 · 19 · 18

Read the Lesson

1. Indicate whether each situation involves a *permutation* or a *combination*.
   a. choosing five students from a class to work on a special project combination
   b. arranging five pictures in a row on a wall permutation
   c. drawing a hand of 13 cards from a 52-card deck combination
   d. arranging the letters of the word *algebra* permutation
2. Write an expression that can be used to calculate each of the following.
   a. number of combinations of n distinct objects taken r at a time $\frac{n!}{(n-r)!r!}$
   b. number of permutations of n objects of which p are alike and q are alike $\frac{n!}{p!q!}$
   c. number of permutations of n distinct objects taken r at a time $\frac{n!}{(n-r)!}$
3. Five cards are drawn from a standard deck of cards. Suppose you are asked to determine how many possible hands consist of one heart, two diamonds, and two spades.
   a. Which of the following would you use to solve this problem: *Fundamental Counting Principle*, *permutations*, or *combinations*? (More than one of these may apply.)
   Fundamental Counting Principle, combinations
   b. Write an expression that involves the notation $P(n, r)$ and/or $C(n, r)$ that you would use to solve this problem. (Do not do any calculations.)
   $C(13, 1) \cdot C(13, 2) \cdot C(13, 2)$

Remember What You Learned

4. Many students have trouble knowing when to use permutations and when to use combinations to solve counting problems. How can the idea of *order* help you to remember the difference between permutations and combinations?
   Sample answer: A permutation is an arrangement of objects in which order is important. A combination is a selection of objects in which order is *not* important.

NAME _____ DATE_____ PERIOD ____

12-2 Study Guide and Intervention

Permutations and Combinations

Permutations When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**.

Permutations	The number of permutations of n distinct objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$.
Permutations with Repetitions	The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

The rule for permutations with repetitions can be extended to any number of objects that are repeated.

Example **From a list of 20 books, each student must choose 4 books for book reports. The first report is a traditional book report, the second a poster, the third a newspaper interview with one of the characters, and the fourth a timeline of the plot. How many different orderings of books can be chosen?**

Since each book report has a different format, order is important. You must find the number of permutations of 20 objects taken 4 at a time.

$P(n, r) = \frac{n!}{(n-r)!}$ Permutation formula

$P(20, 4) = \frac{20!}{(20-4)!}$ $n = 20$, $r = 4$

$= \frac{20!}{16!}$ Simplify.

$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot \cancel{16} \cdot \cancel{15} \cdot \ldots \cdot \cancel{1}}{\cancel{16} \cdot \cancel{15} \cdot \ldots \cdot \cancel{1}}$ Divide by common factors.

$= 116{,}280$

Books for the book reports can be chosen 116,280 ways.

Exercises

Evaluate each expression.

1. $P(6, 3)$ 120
2. $P(8, 5)$ 6720
3. $P(9, 4)$ 3024
4. $P(11, 6)$ 332,640

How many different ways can the letters of each word be arranged?

5. MOM 3
6. MONDAY 720
7. STEREO 360

8. **SCHOOL** The high school chorus has been practicing 12 songs, but there is time for only 5 of them at the spring concert. How may different orderings of 5 songs are possible?
95,040

Answers

NAME _____ DATE _____ PERIOD ____

12-2 Study Guide and Intervention *(continued)*

Permutations and Combinations

Combinations An arrangement or selection of objects in which order is *not* important is called a combination.

Combinations	The number of combinations of n distinct objects taken r at a time is given by $C(n, r) = \frac{n!}{(n-r)!r!}$.

Example 1 **SCHOOL How many groups of 4 students can be selected from a class of 20?**

Since the order of choosing the students is not important, you must find the number of combinations of 20 students taken 4 at a time.

$C(n, r) = \frac{n!}{(n-r)!r!}$ Combination formula

$C(20, 4) = \frac{20!}{(20-4)!4!}$ n 20, r 4

$= \frac{20!}{16!4!}$ or 4845

There are 4845 possible ways to choose 4 students.

Example 2 **In how many ways can you choose 1 vowel and 2 consonants from a set of 26 letter tiles? (Assume there are 5 vowels and 21 consonants.)**

By the Fundamental Counting Principle, you can multiply the number of ways to select one vowel and the number of ways to select 2 consonants. Only the letters chosen matter, not the order in which they were chosen, so use combinations.

$C(5, 1)$ One of 5 vowels are drawn.

$C(21, 2)$ Two of 21 consonants are drawn.

$C(5, 1) \cdot C(21, 2) = \frac{5!}{(5-1)!1!} \cdot \frac{21!}{(21-2)!2!}$ Combination formula

$= \frac{5!}{4!} \cdot \frac{21!}{19!2!}$ Subtract.

$= 5 \cdot 210$ or 1050 Simplify.

There are 1050 combinations of 1 vowel and 2 consonants.

Exercises

Evaluate each expression.

1. $C(5, 3)$ 10
2. $C(7, 4)$ 35
3. $C(15, 7)$ 6435
4. $C(10, 5)$ 252

5. **PLAYING CARDS** From a standard deck of 52 cards, in how many ways can 5 cards be drawn? 2,598,960

6. **HOCKEY** How many hockey teams of 6 players can be formed from 14 players without regard to position played? 3003

7. **COMMITTEES** From a group of 10 men and 12 women, how many committees of 5 men and 6 women can be formed? 232,848

NAME _____ DATE _____ PERIOD ____

12-2 Skills Practice

Permutations and Combinations

Evaluate each expression.

1. $P(6, 3)$ 120
2. $P(8, 2)$ 56
3. $P(2, 1)$ 2
4. $P(3, 2)$ 6
5. $P(10, 4)$ 5040
6. $P(5, 5)$ 120
7. $C(2, 2)$ 1
8. $C(5, 3)$ 10
9. $C(4, 1)$ 4
10. $C(8, 7)$ 8
11. $C(3, 2)$ 3
12. $C(7, 4)$ 35

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. seating 8 students in 8 seats in the front row of the school auditorium
permutation; 40,320

14. introducing the 5 starting players on the Woodsville High School basketball team at the beginning of the next basketball game
permutation; 120

15. checking out 3 library books from a list of 8 books for a research paper
combination; 56

16. choosing 2 movies to rent from 5 movies
combination; 10

17. the first-, second-, and third-place finishers in a race with 10 contestants
permutation; 720

18. electing 4 candidates to a municipal planning board from a field of 7 candidates
combination; 35

19. choosing 2 vegetables from a menu that offers 6 vegetable choices
combination; 15

20. an arrangement of the letters in the word *rhombus*
permutation; 5040

21. selecting 2 of 8 choices of orange juice at a store
combination; 28

22. placing a red rose bush, a yellow rose bush, a white rose bush, and a pink rose bush in a row in a planter permutation; 24

23. selecting 2 of 9 kittens at an animal rescue shelter
combination; 36

24. an arrangement of the letters in the word *isosceles*
permutation; 30,240

NAME _____ DATE_____ PERIOD ____

12-2 Practice

Permutations and Combinations

Evaluate each expression.

1. $P(8, 6)$ 20,160
2. $P(9, 7)$ 181,440
3. $P(3, 3)$ 6
4. $P(4, 3)$ 24
5. $P(4, 1)$ 4
6. $P(7, 2)$ 42
7. $C(8, 2)$ 28
8. $C(11, 3)$ 165
9. $C(20, 18)$ 190
10. $C(9, 9)$ 1
11. $C(3, 1)$ 3
12. $C(9, 3) \cdot C(6, 2)$ 1260

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. selecting a 4-person bobsled team from a group of 9 athletes
combination; 126

14. an arrangement of the letters in the word *Canada*
permutation; 120

15. arranging 4 charms on a bracelet that has a clasp, a front, and a back
permutation; 24

16. selecting 3 desserts from 10 desserts that are displayed on a dessert cart in a restaurant
combination; 120

17. an arrangement of the letters in the word *annually*
permutation; 5040

18. forming a 2-person sales team from a group of 12 salespeople
combination; 66

19. making 5-sided polygons by choosing any 5 of 11 points located on a circle to be the vertices
combination; 462

20. seating 5 men and 5 women alternately in a row, beginning with a woman
permutation; 14,400

21. **STUDENT GROUPS** Farmington High is planning its academic festival. All math classes will send 2 representatives to compete in the math bowl. How many different groups of students can be chosen from a class of 16 students? 120

22. **PHOTOGRAPHY** A photographer is taking pictures of a bride and groom and their 6 attendants. If she takes photographs of 3 people in a group, how many different groups can she photograph? 56

23. **AIRLINES** An airline is hiring 5 flight attendants. If 8 people apply for the job, how many different groups of 5 attendants can the airline hire? 56

24. **SUBSCRIPTIONS** A school librarian would like to buy subscriptions to 7 new magazines. Her budget, however, will allow her to buy only 4 new subscriptions. How many different groups of 4 magazines can she choose from the 7 magazines? 35

NAME _____ DATE_____ PERIOD ____

12-2 Word Problem Practice

Permutations and Combinations

1. **WAITING IN LINE** When the 12 students in Mr. Jaybird's class go to lunch, they form a single file line. Does forming a line involve a permutation or a combination of the students?
A permutation

2. **ART** Isabel needs to select three different colors of construction paper to make a flag for a school project. She can choose from a selection of 15 different colors. In how many ways can she pick her colors?
455

3. **SUDOKU** A popular game called "Sudoku" involves square arrays of numbers. In a game of Sudoku, every entry is an integer between 1 and 9, inclusive. No number appears twice in any row or column.

7	1	8	6	9	4	2	3	5
9	2	5	7	3	1	6	4	8
4	6	3	8	5	2	7	9	1
5	9	2	1	7	3	4	8	6
8	3	1	4	6	5	9	2	7
9	7	4	2	8	9	5	1	3
3	4	9	5	1	7	8	6	2
2	8	7	3	4	6	1	5	9
1	5	6	9	2	8	3	7	4

For a game of Sudoku, how many different possibilities are there for the first row of numbers?
362,880

4. **NAMES** Hannah is curious to know how many different 6 letter sequences she can make using each of the letters of her name exactly once. For example, "HANNAH," "AAHHNN," and "NAHNAH" are all possible sequences. How many total sequences are possible?
90

METEORITES For Exercises 5 and 6, use the following information.

Over the course of several years, Kendra managed to collect 7 meteorites. Each one is unique.

5. For a school science fair, Kendra displays her meteorites in a row. How many ways are there to order the meteorites?
5040

6. She decides to trade three of her meteorites for a telescope after the fair. How many ways can she pick out 3 meteorites from her collection?
35

Answers