12-2 Notes (Day 1), Permutations

Recall: The Fundamental Counting Principle

# of choices of Event A • # of choices of Event B = # of possible outcomes

Factorial: ! a number multiplied by each previous # to 1.

Example:

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$\frac{6!}{4!} = \frac{6 \cdot 5 \cdot 4!}{4!} = 30$

Permutations: # of ways to arrange n items into r spots - ORDER MATTERS*

nPr P(n,r)

Example 1: ${}_6P_4$ or $P(6,4)$ "6 items arranged in 4 spots"

n items → r spots

$6 \cdot 5 \cdot 4 \cdot 3$

360

$\frac{6!}{2!} = \frac{6!}{(6-4)!}$

Permutation Formula:

$$nPr = \frac{n!}{(n-r)!}$$

* order matters *

Example 2: Permutations

A. How many outcomes are there when two die are rolled?

$6 \cdot 6 = 36$ outcomes

B. How many outcomes are there when a coin is flipped four times?

$2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16$ outcomes

C. How many four-letter arrangements can be made from the word "MATH"? (counting principle)

$4 \cdot 3 \cdot 2 \cdot 1 = 24$

(formula)

$${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 24$$

D. How many five-letter arrangements can be made from the word "COUGARS"?

$${}_7P_5 = \frac{7!}{(7-5)!} = \frac{7!}{2!} = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$$

2520 arrangements

Example 3: Permutations with Repetition

A. How many five-letter arrangements can be made from the word "APPLE"?

PAPLE
PAPLE

$$\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{5!}{2!} = 5 \cdot 4 \cdot 3 = 60$$

↖ repetitions

Permutation w/ repitition Formula:

$$\frac{n!}{p!q!}$$

* p & q are repetitions
* n: # of items

B. How many different ways can the letters of the word "MISSISSIPPI" be arranged?

$$\frac{11!}{(4!4!2!)} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$$

I S P repetitions

34650

C. How many different ways can the letters of the word "BANANA" be arranged?

$$\frac{6!}{2!3!} = \frac{6\cdot5\cdot4\cdot\cancel{3\cdot2\cdot1}}{(2\cdot1)\cdot\cancel{3\cdot2\cdot1}} = \frac{120}{2}$$

N N A

D. If 8 people are running in a race, how many ways can first, second, and third place be awarded?

$$_8P_3 = \frac{8!}{5!} = 336 \text{ ways}$$

12-2 Notes (Day 2), Permutations & Combinations

Recall - Permutations: order matters- arrangement

${}_{\text{items}}nP_{r\ \text{spots}}$ or $P(n,r) = \frac{n!}{(n-r)!}$ n items arranged in r spots

Combinations: order does not matter

nCr or $C(n,r)$ n items selected/chosen in r spots

items, select

Formula: $C(n,r) = \frac{nCr}{(n-r)!r!}$

items not used; repeated groups

Example 1: ${}_6C_4$ *or* $C(6,4)$

items, choosing 4

$6C_4 = \frac{6!}{(6-4)!4!} = \frac{6!}{2!4!} = \frac{6\cdot5\cdot4!}{2!4!} = \frac{6\cdot5}{2\cdot1} = 15$

not used; repeated groups

Example 2: Combinations

A. A group of seven students working on a project needs to choose two students to present the group's report. How many ways can they choose the two students?

order does not matter

$7C_2 = \frac{7!}{(7-2)!2!} = \frac{7!}{5!2!} = \frac{7\cdot6\cdot5!}{5!2!} = \frac{42}{2} = 21$

repeated

B. A family with septuplets assigns different chores to the children each week. How many ways can three children be chosen to help with the laundry?

$7C_3 = \frac{7!}{4!3!} = \frac{7\cdot6\cdot5\cdot4!}{4!3!} = \frac{210}{6} = 35$

not chosen; repetitions; $3\cdot2\cdot1$

Example 3: Permutation or Combination

A. Eight people enter the "Best Pie" contest. How many different ways can blue, red, and yellow ribbons be awarded?

1st 2nd 3rd

Order matters permutation $_8P_3 = 8 \cdot 7 \cdot 6 = 336$ $\frac{8!}{5!}$ not used

B. There are eight people hiking together. They walk single file on a narrow section of the trail. How many ways can they be lined up?

Order matters permutation $_8P_8 = 8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$

C. A group of seven students working on a project needs to choose two students to present the group's report. How many ways can they choose the two students?

Order does not matter combination choosing $_7C_2 = \frac{7!}{5!\,2!} = 21$ (2! repeated)

D. There are six actresses that audition for a part. Only three actresses will move on to the second audition. How many groups of three can be chosen for the second audition?

Order does not matter combination $_6C_3 = \frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$

(3! not chosen; 3! repeated groups)

PERMUTATIONS -vs- COMBINATIONS

$_nP_r$ or $P(n,r)$

(n items, r spots)

n items arranged into r spots

ORDER MATTERS!

$nPr = \frac{n!}{(n-r)!}$ (n−r)! not used

- order (line, row)
- arrangement (letters)
- placement (1st, 2nd, 3rd) - race/competition
- Pres, VP, Secretary

$_nC_r$ or $C(n,r)$

(n items, r choosing)

n items chosen in r spots

ORDER DOES NOT MATTER

$nCr = \frac{n!}{(n-r)!\,r!}$ (n−r)! not used; r! repeated groups

- choosing/chosen
- selection/selecting
- group/grouping

Combinations with Multiple Events

Example 1: In how many ways can you choose 1 vowel and 2 consonants from a set of 26 letter tiles?

Event A · Event B

5 vowels
21 consonants

$_5C_1 \cdot {}_{21}C_2 = \frac{5!}{4!\,1!} \cdot \frac{21!}{19!\,2!}$

(selecting 1 vowel · selecting 2 consonants)

$\frac{5 \cdot 4!}{4!\,1!} \cdot \frac{21 \cdot 20 \cdot 19!}{19!\,2!} = 5 \cdot 210 =$ **1050 ways**

Example 2: From a group of 10 men and 12 women, how many committtees of 5 men and 6 women be formed?

Event A · Event B

(# of ways to choose 5 men) · (# of ways to choose 6 women)

$_{10}C_5 \cdot {}_{12}C_6$

$\frac{10!}{5!\,5!} \cdot \frac{12!}{6!\,6!}$

$252 \cdot 924 =$ **232,848**

$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 5!}$ ($5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$)

$\frac{(12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7)6!}{6!\,6!}$ ($6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$)

Note: A ***standard deck of cards*** means a deck of 52 playing cards. There are 4 suits in a standard deck of cards - clubs (black), spades (black), diamonds (red), hearts (red) - with 13 cards in each suit . The Kings, Queens, and Jacks are considered face cards.

Example 3: Five cards are drawn from a standard deck of cards. How many hands consist of three clubs and two diamonds?

Event A · Event B

$_{13}C_3 \cdot {}_{13}C_2$

$\frac{13!}{10!\,3!} \cdot \frac{13!}{11!\,2!}$

$286 \cdot 78 =$ **22,308**

Warm-up (Permutations)

1. Calculate ${}_7P_3$. Show work!

items ↙ ↘ spots

$7 \cdot 6 \cdot 5 = 210$

$\frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}}$

4! ← not used

2. How many ways can you arrange 6 books on a shelf?

${}_6P_6 = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

6 ← books; 6 ← spots

3. How many three-letter arrangements can you make with the word "HEARTS"?

$6 \cdot 5 \cdot 4 = 120$

4. How many ways can you arrange the letters in the word "ALGEBRA"?

$\frac{7!}{2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2 \cdot 1}}{\cancel{2 \cdot 1}} = 2520$

2! ← repetitions

5. How many ways can you arrange the letters in the word "ARKANSAS"

$\frac{8!}{3!2!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2 \cdot 1}\,2!} = 3360$

3! ← As; 2! ← Ss; 3·2·1