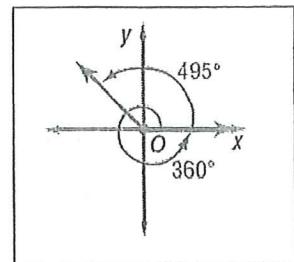
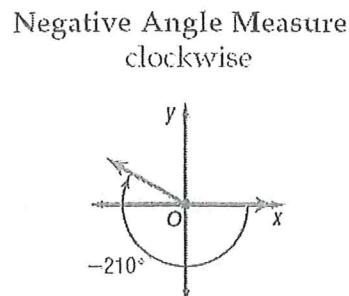
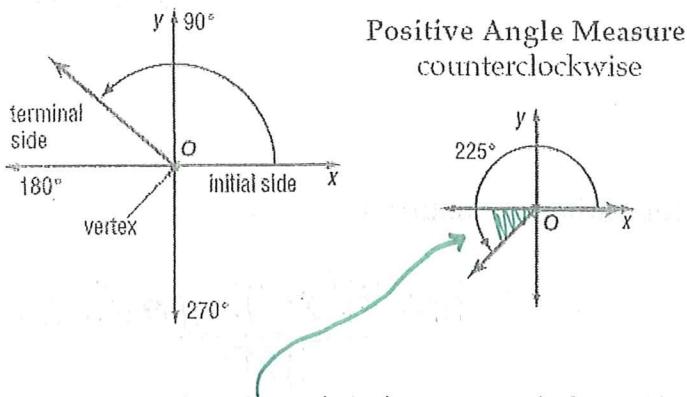


# 13-2/13-3 Angles, Reference Angles & Radians

## Notes

Remember: When sketching an angle, always start at the positive x-axis. Type equation here.  
The positive x-axis represents 0° or 360°.

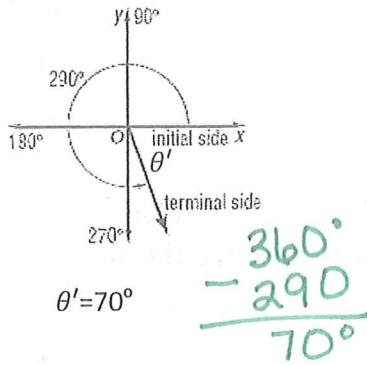
Angle of Rotation  
In trigonometry, an angle is sometimes referred to as an *angle of rotation*.



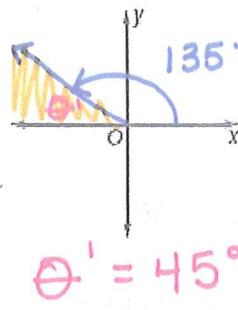
A reference angle is the acute angle formed by the terminal side and the x-axis. (denoted by  $\theta'$ )

Examples: Sketch each angle. Then find its reference angle.

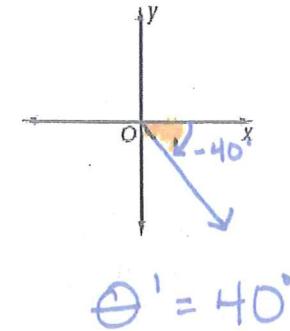
1.  $290^\circ$



2.  $135^\circ$



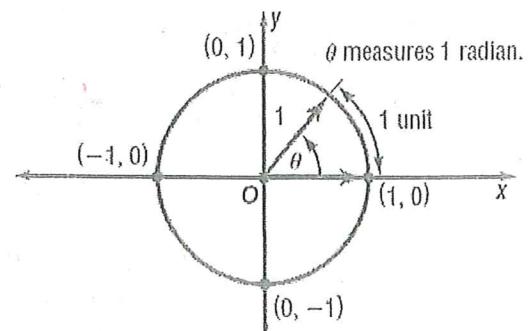
3.  $-40^\circ$



### Radians

The definition of a radian is based on the unit circle, a circle of radius 1 which centers at the origin.

One radian is the measure of angle  $\theta$  in standard position whose rays intercept an arc of length 1 unit.

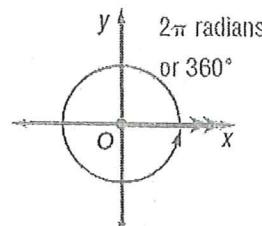


$r=1$  for the unit circle

Circle Review: The circumference of any circle is  $2\pi r$ , where  $r$  is the radius.

The circumference of the unit circle would

be  $2\pi(1) = 2\pi$



O If I walk around the unit circle, I will have walked  $2\pi$  units.

$C =$  be  $2\pi(1)$ , so  $360^\circ = 2\pi$  Radians  $180^\circ = \pi$  Radians.

### Conversions

Converting degrees to radians:

$$R = D \left( \frac{\pi}{180^\circ} \right)$$

Converting radians to degrees:

$$D = R \left( \frac{180}{\pi} \right)$$

Radian Measure The word *radian* is usually omitted when angles are expressed in radian measure. Thus, when no units are given for an angle measure, radian measure is implied.

Examples:

Rewrite the degree measures in radians and the radian measure in degrees.

1.  $60^\circ$

$$R = 60^\circ \cdot \frac{\pi}{180^\circ} = \frac{60\pi}{180} = \frac{\pi}{3}$$

2.  $45^\circ$

$$R = 45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4}$$

$$3. -\frac{7\pi}{4}$$

$$D = -\frac{7\pi}{4} \cdot \frac{180}{\pi} = -\frac{1260}{4} = -315^\circ$$

$$4. \frac{5\pi}{3}$$

$$D = \frac{5\pi}{3} \cdot \frac{180}{\pi} = \frac{900}{3} = 300^\circ$$

Coterminal Angles: The graph shows a  $405^\circ$  angle and a  $45^\circ$  angle. They both share the same terminal side. When two angles in standard position have the same terminal sides, they are called coterminal angles.

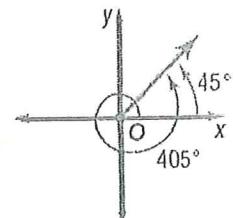
$$405^\circ - 360^\circ = 45^\circ$$

$$\text{Positive} = 45^\circ$$

$$\text{Negative} = 405^\circ - 720^\circ = -315^\circ$$

In degrees, you add/subtract 360

In radians, you would add/subtract  $2\pi$



$$\frac{9\pi}{4} - 4\pi = \frac{9\pi}{4} - \frac{16\pi}{4}$$

$$\text{Neg} = -\frac{7\pi}{4}$$

Examples: Find one angle with positive measure and one angle with negative measure coterminal with each angle.

$$1. 240^\circ$$

$$240 + 360 = -360$$

$$\text{Pos: } 600^\circ$$

$$\text{Neg: } -120^\circ$$

$$3. 15^\circ$$

$$15 - 360$$

$$\text{Neg: } -345^\circ$$

$$\text{Pos: } 15 + 360$$

$$\text{Pos: } 375^\circ$$

$$\frac{9\pi}{4} - 2\pi$$

$$\text{Positive: } \frac{\pi}{4}$$

$$2. \frac{9\pi}{4} - 2\pi$$

$$\frac{9\pi}{4} - \frac{8\pi}{4} = \frac{\pi}{4}$$

$$4. -\frac{\pi}{4} + \frac{8\pi}{4} = \frac{7\pi}{4} = \text{Positive}$$

$$-\frac{\pi}{4} - 2\pi = -\frac{\pi}{4} - \frac{8\pi}{4} = -\frac{9\pi}{4} = \text{Negative}$$