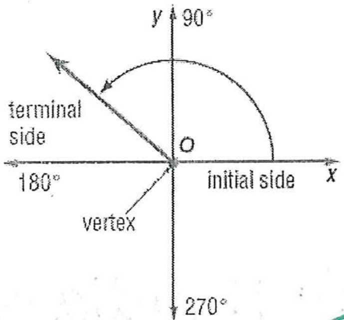
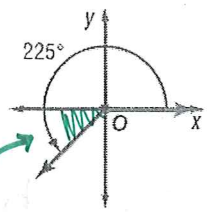
13-2/13-3 Angles, Reference Angles & Radians Notes

Remember: When sketching an angle, always start at the positive x-axis. ~~Type equation here.~~
The positive x-axis represents __0__ ° or __360__ °.

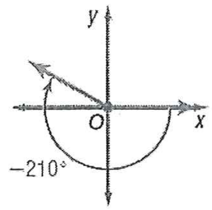
Angle of Rotation In trigonometry, an angle is sometimes referred to as an *angle of rotation.*

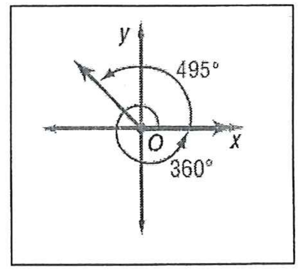


Positive Angle Measure
counterclockwise



Negative Angle Measure
clockwise

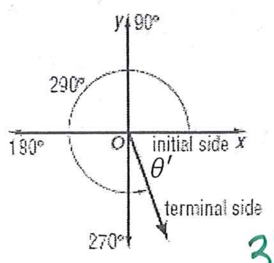




A **reference angle** is the acute angle formed by the terminal side and the x-axis. (denoted by θ')

Examples: Sketch each angle. Then find its reference angle.

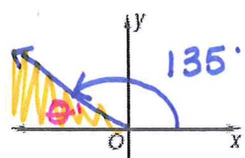
1. 290°



θ'=70°

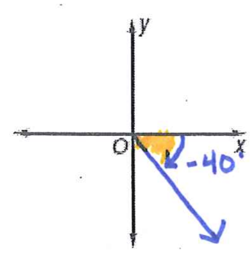
$$\begin{array}{r} 360^\circ \\ -\,290 \\ \hline 70^\circ \end{array}$$

2. 135°



$\theta' = 45^\circ$

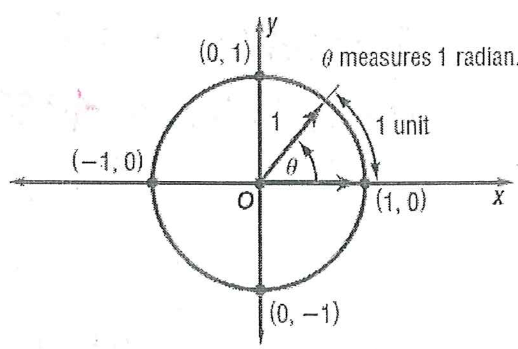
3. -40°



$\theta' = 40^\circ$

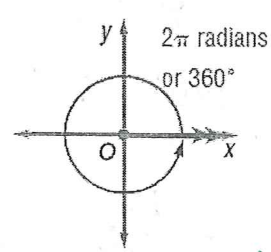
Radians

The definition of a radian is based on the unit circle, a circle of radius 1 which centers at the origin.
One radian is the measure of angle θ in standard position whose rays intercept and arc of length 1 unit.



$r = 1$ for the unit circle

Circle Review: The circumference of any circle is __$2\pi r$__, where r is the radius.



The circumference of the unit circle would be $2\pi(1) = 2\pi$

If I walk around the unit circle, I will have walked 2π units

$C =$ be $2\pi(1)$, so 360° = 2π Radians 180° = π Radians.

Radian Measure The word *radian* is usually omitted when angles are expressed in radian measure. Thus, when no units are given for an angle measure, radian measure is implied.

Conversions

Converting degrees to radians:

$$R=D\left(\frac{\pi}{180°}\right)$$

Converting radians to degrees:

$$D=R\left(\frac{180}{\pi}\right)$$

Examples:

Rewrite the degree measures in radians and the radian measure in degrees.

1. 60°

$$R = 60° \cdot \frac{\pi}{180°} = \frac{60\pi}{180} = \frac{\pi}{3}$$

2. 45°

$R = 45 \cdot \frac{\pi}{180}$ radians

$\boxed{= \pi/4 \text{ Radians}}$

3. $-\frac{7\pi}{4}$

$$D = -\frac{7\pi}{4} \cdot \frac{180}{\pi} = -\frac{1260}{4} = -315°$$

4. $\frac{5\pi}{3}$

$D = \frac{5\pi}{3} \cdot \frac{180}{\pi} = \frac{900\pi}{3\pi}$

$\boxed{D = 300°}$

Coterminal Angles: The graph shows a 405° angle and a 45° angle. They both share the same terminal side. When two angles in standard position have the same terminal sides, they are called co terminal angles.

405°- 360°=45°

Positive $= 45°$

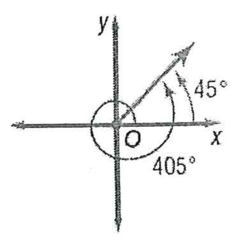
Negative $= 405° - 720° = -315°$

In degrees, you add/subtract 360

In radians, you would add/subtract 2π

$\frac{9\pi}{4} - 4\pi = \frac{9\pi}{4} - \frac{16\pi}{4}$

$\boxed{\text{Neg} = -\frac{7\pi}{4}}$



Examples: *Find one angle with positive measure and one angle with negative measure coterminal with each angle.*

1. 240°

$240 + 360 =$ $\boxed{\text{Pos: } 600°}$

$240 - 360$ $\boxed{\text{Neg: } -120}$

2. $\frac{9\pi}{4} - 2\pi$

$\frac{9\pi}{4} - \frac{8\pi}{4} = \frac{\pi}{4}$

$\boxed{\text{Positive: } \pi/4}$

3. 15°

$15 - 360$ $\boxed{\text{Neg: } -345°}$

Pos: $15 + 360$ $\boxed{\text{Pos} = 375°}$

4. $-\frac{\pi}{4} + \frac{8\pi}{4} = \boxed{\frac{7\pi}{4} = \text{Positive}}$

$-\frac{\pi}{4} - 2\pi = -\frac{\pi}{4} - \frac{8\pi}{4} = \boxed{-\frac{9\pi}{4} : \text{Neg}}$