

ACC Geometry Booklet for Notes

Introduction to Trig Graphs (Chapter 14)

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Objective: To construct the parent graphs of trigonometric functions for sine, cosine, and tangent using exact values.

Recall: A function whose graph repeats a basic pattern is said to be *periodic*.

Complete the following table for $\sin \theta$.

| θ (degree) x | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° |
|-------------------------------------|-----------|-----------------|----------------------|----------------------|-----------------|----------------------|----------------------|------------------|-------------|
| θ (radian) x | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| $\sin \theta$ (exact) y | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| $\sin \theta$ (nearest tenth), y | 0 | .5 | .7 | .9 | 1 | .9 | .7 | .5 | 0 |

| θ (degree) x | 210° | 225° | 240° | 270° | 300° | 315° | 330° | 360° | 390° |
|-------------------------------------|------------------|-----------------------|-----------------------|------------------|-----------------------|-----------------------|-------------------|-------------|-------------------|
| θ (radian) x | $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{4}$ | $\frac{11\pi}{6}$ | 2π | $\frac{13\pi}{6}$ |
| $\sin \theta$ (exact) y | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | -1 | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ |
| $\sin \theta$ (nearest tenth), y | -.5 | -.7 | -.9 | -1 | -.9 | -.7 | -.5 | 0 | .5 |

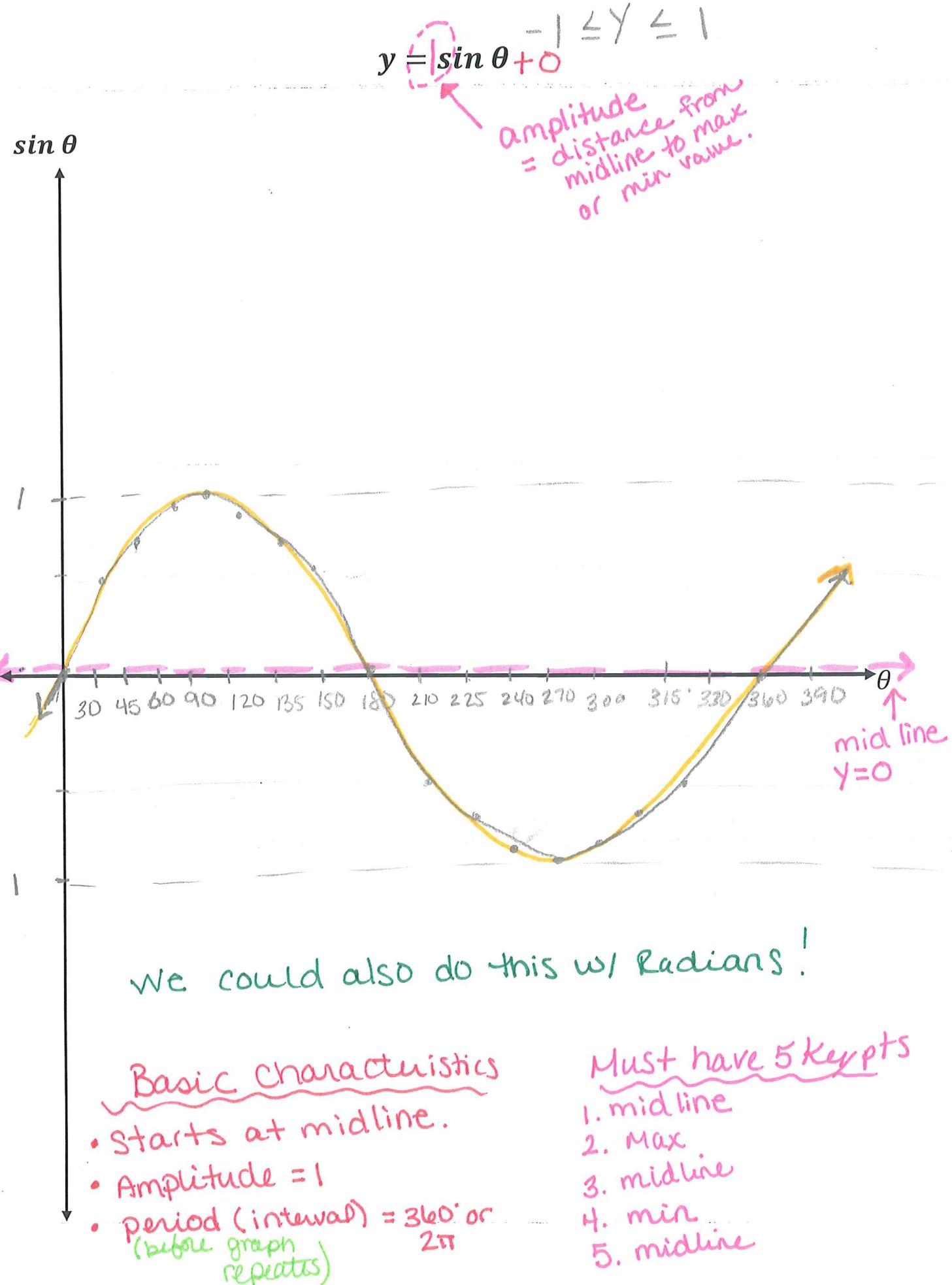
(13.6) Period- Length that is takes before the graph repeats.

To graph the function $y = \sin \theta$, use values of θ expressed in either degrees or radians.

These values represent the x values on a graph. Use the values of $\sin \theta$ expressed as a value rounded to the nearest tenth to represent the y values on the graph.

Ordered pairs for points on these graphs are of the form $(\theta, \sin \theta)$.

On the back plot the points, $(\theta, \sin \theta)$. Connect the points with a smooth curve. This graph represents the graph of the sine function.



Complete the following table for $\cos \theta$.

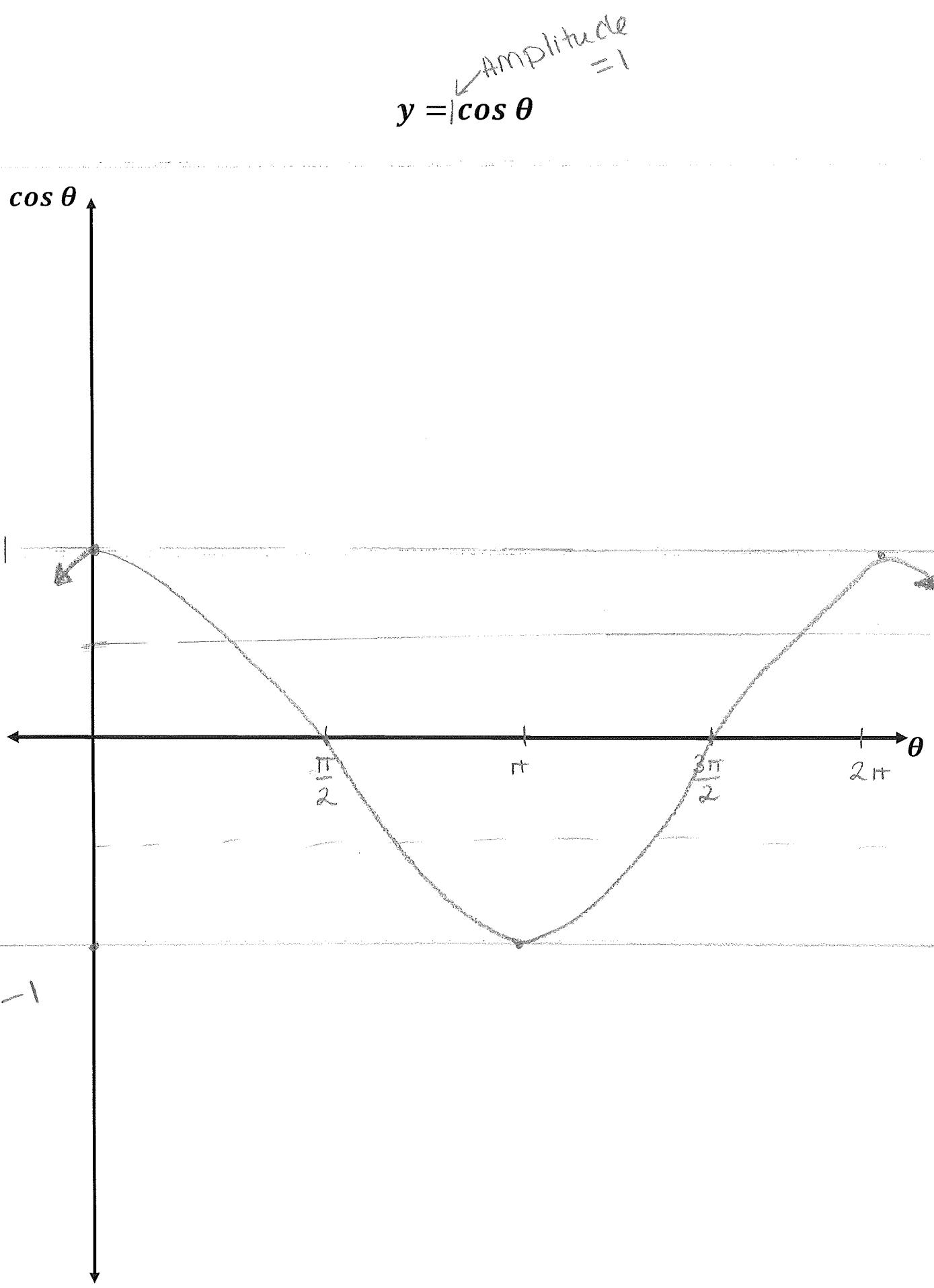
| θ (degree) x | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° |
|-------------------------------------|-----------|----------------------|----------------------|-----------------|-----------------|------------------|-----------------------|-----------------------|-------------|
| θ (radian) x | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| $\cos \theta$ (exact) y | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | -1 |
| $\cos \theta$ (nearest tenth), y | 1 | .9 | .7 | .5 | 0 | -.5 | -.7 | -.9 | -1 |

| θ (degree) x | 210° | 225° | 240° | 270° | 300° | 315° | 330° | 360° | 390° |
|-------------------------------------|-----------------------|-----------------------|------------------|------------------|------------------|----------------------|----------------------|-------------|----------------------|
| θ (radian) x | $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{4}$ | $\frac{11\pi}{6}$ | 2π | $\frac{13\pi}{6}$ |
| $\cos \theta$ (exact) y | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ |
| $\cos \theta$ (nearest tenth), y | -.9 | -.7 | -.5 | 0 | .5 | .7 | .9 | 1 | .9 |

To graph the function $y = \cos \theta$, use values of θ expressed in either degrees or radians. These values represent the x values on a graph. Use the values of $\cos \theta$ expressed as a value rounded to the nearest tenth to represent the y values on the graph.

Ordered pairs for points on these graphs are of the form $(\theta, \cos \theta)$.

On the back plot the points, $(\theta, \cos \theta)$. Connect the points with a smooth curve. This graph represents the graph of the cosine function.



Complete the following table for $\tan \theta$. Remember: $\tan \theta = \frac{\sin \theta}{\cos \theta}$

| θ (degree) x | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° |
|--|----|-----------------|-----------------|-----------------|-----------------|------------------|------------------|------------------|-------|
| θ (radian) x | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| $\tan \theta$ (exact) y | | | | | | | | | |
| $\tan \theta$ (nearest tenth), y | 0 | .6 | 1 | 1.7 | und. | -1.7 | -1 | -0.6 | 0 |

| θ (degree) x | 210° | 225° | 240° | 270° | 300° | 315° | 330° | 360° | 390° |
|--|------------------|------------------|------------------|------------------|------------------|------------------|-------------------|--------|-------------------|
| θ (radian) x | $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{4}$ | $\frac{11\pi}{6}$ | 2π | $\frac{13\pi}{6}$ |
| $\tan \theta$ (exact) y | | | | | | | | | |
| $\tan \theta$ (nearest tenth), y | 0 | 1 | 1.7 | und. | -1.7 | -1 | -0.6 | 0 | -0.6 |

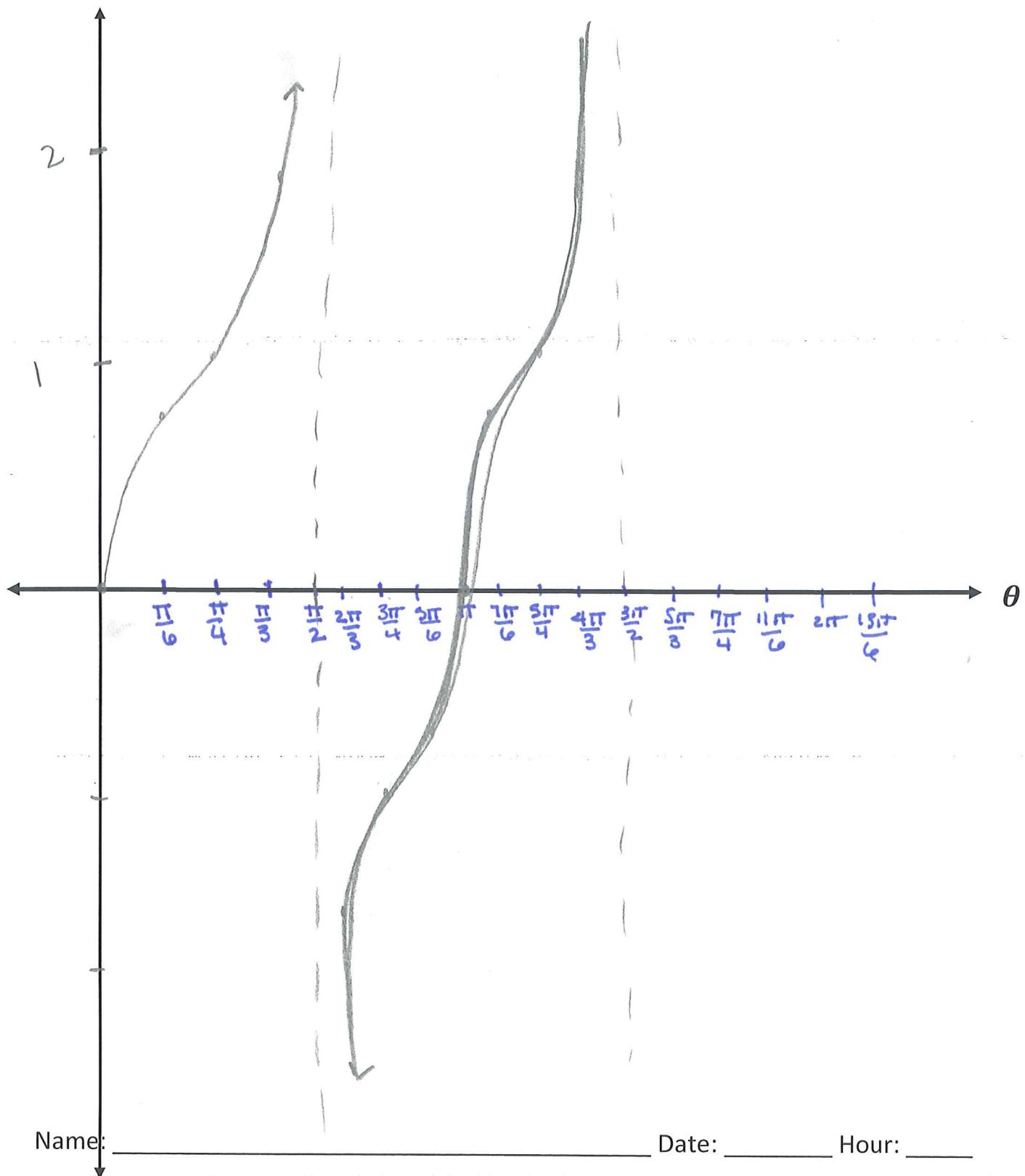
To graph the function $y = \tan \theta$, use values of θ expressed in either degrees or radians. These values represent the x values on a graph. Use the values of $\tan \theta$ expressed as a value rounded to the nearest tenth to represent the y values on the graph.

Ordered pairs for points on these graphs are of the form $(\theta, \tan \theta)$.

On the back plot the points, $(\theta, \tan \theta)$. Connect the points with a smooth curve. This graph represents the graph of the tangent function.

$$y = \tan \theta$$

$\tan \theta$

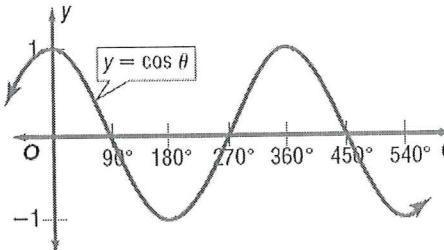
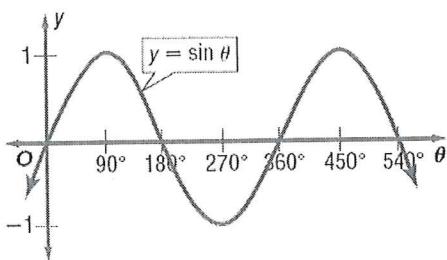


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Changing Amplitude & Period

Sine & Cosine Functions

For every 360° degrees or 2π radians, the sine and cosine functions repeat their values. We say the sine and cosine functions are periodic, each having a period of 360° degrees or 2π radians.



To change the period of a sine and cosine function a value must be placed before theta, b.

For example, $y = \sin 2\theta$. This function would have a period of $\frac{360}{b}$ or $\frac{2\pi}{b}$. $\frac{360}{b} = \frac{360}{2\pi} = \frac{360}{2} = 180^\circ$ or π

The amplitude is the midpoint from the highest point to the lowest point of the function. The graphs we constructed had an amplitude of 1.

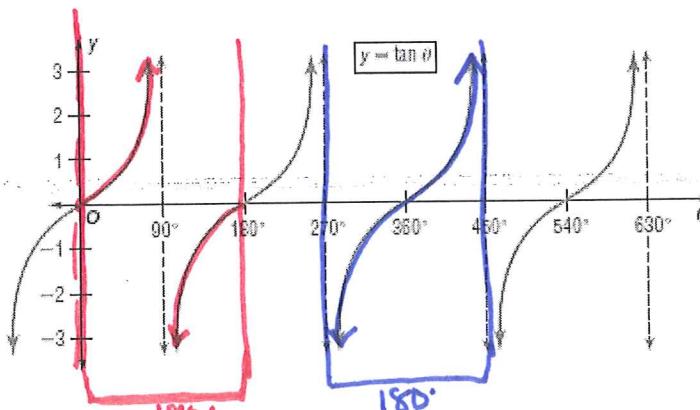
To change the amplitude of a sine or cosine function the coefficient, a, must be changed.

$y = a \sin \theta$ "a" changes Amplitude.

For example, $y = 2\sin \theta$. This function would have an amplitude of 2. In other words, the maximum it would reach is 2 and the minimum it would reach is -2.

Tangent Functions

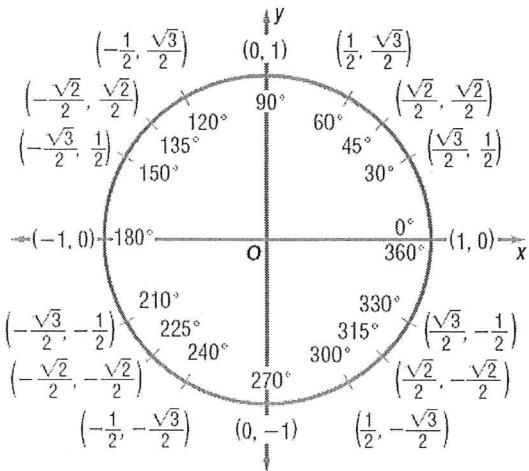
For every 180° degrees or π radians, the tangent functions repeat their values. We say the tangent functions are periodic, each having a period of 180° degrees or π radians.



To change the period of tangent function a value must be placed before theta, b.

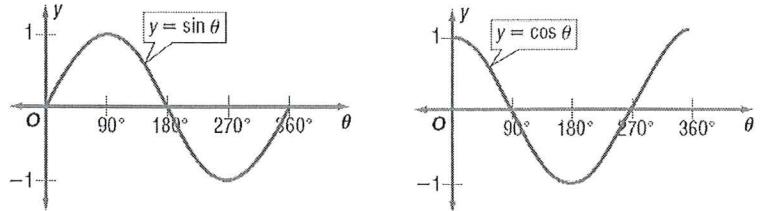
For example, $y = \tan 2\theta$. This function would have a period of $\frac{180}{b}$ or $\frac{180}{2\pi} = \frac{180}{2} = 90^\circ$ or $\frac{\pi}{2}$

Because the tangent is infinite in both directions, the tangent function has no amplitude.



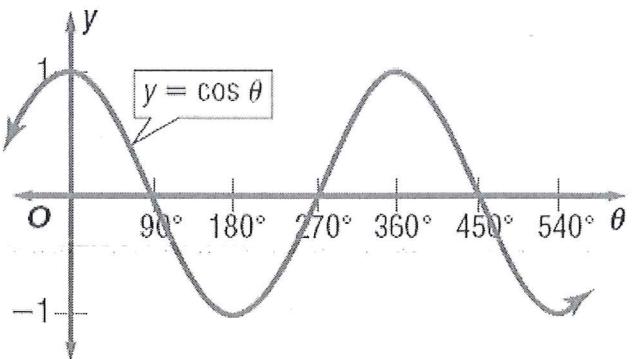
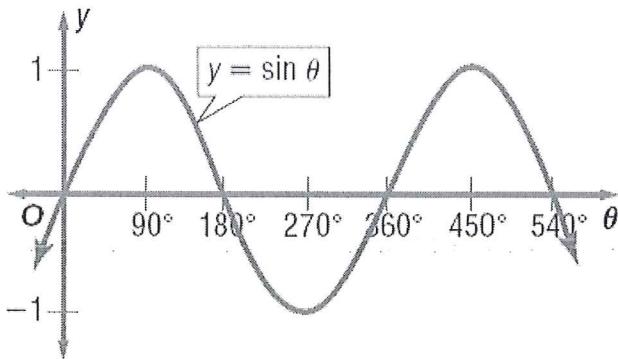
Look at Nspire demonstration of this!

This same information is presented on the graphs of the sine and cosine functions below, where the horizontal axis shows the values of θ and the vertical axis shows the values of $\sin \theta$ or $\cos \theta$.



Key Concept : Periodic Functions

For every 360 degrees or 2π radians, the sine and cosine functions repeat their values. We say the sine and cosine functions are periodic, each having a period of 360 degrees or 2π radians.



Remember:

Both sine and cosine have a maximum value of 1 and a minimum value of -1

Key Concept: Amplitude

The amplitude of the graph of a periodic function is the absolute value of half the difference between its maximum value and its minimum value.

You try: Using the above information and your definition of amplitude, set up and expression as to how to time the amplitude of the graphs of the sine and cosine functions.

13.6/14.1 Homework

Key
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Part 1 What We Discovered

Formal Key Concepts: Amplitude and Period

Words For functions of the form $y = a \sin b\theta$ and $y = a \cos b\theta$, the amplitude is $|a|$, and the period is $\frac{360^\circ}{|b|}$ or $\frac{2\pi}{|b|}$.

For functions of the form $y = a \tan b\theta$, the amplitude is not defined, and the period is $\frac{180^\circ}{|b|}$ or $\frac{\pi}{|b|}$.

Examples $y = 3 \sin 4\theta$ amplitude 3 and period $\frac{360^\circ}{4}$ or 90°
 $y = -6 \cos 5\theta$ amplitude $|-6|$ or 6 and period $\frac{2\pi}{5}$
 $y = 2 \tan \frac{1}{3}\theta$ no amplitude and period 3π

1. What is the unit circle? How is a unit circle related to the graphed sine and cosine functions? **Unit circle where $r=1$**

X coordinate is $\cos\theta$
Y coordinate is $\sin\theta$



#2-9 Tell whether each statement describes a characteristic of the sine function, cosine function, both functions or neither functions.

2. The function has a period of 360°

Sine & Cosine

3. The function has an amplitude of 2.

none

4. The y-intercept is 1.

Cosine

5. The y-intercept is 0.

Sine (and tangent)

6. The range of the function is

$$-1 \leq y \leq 1.$$

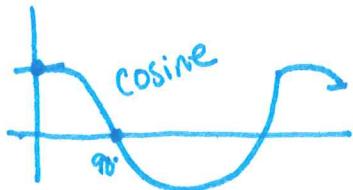
Sine + Cosine

7. The horizontal intercepts occur only at multiples of 90°

Sine + cosine

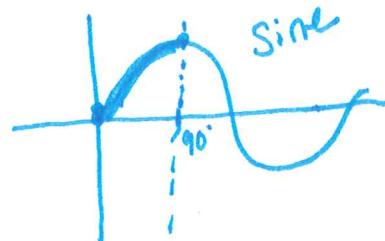
8. The function decreases in the interval

$$0^\circ \leq \theta \leq 90^\circ$$

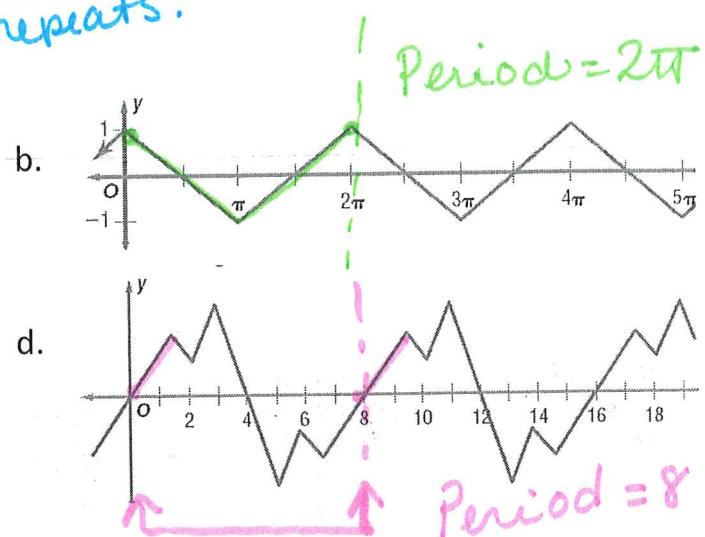
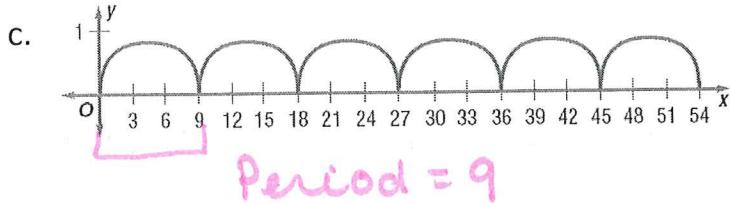
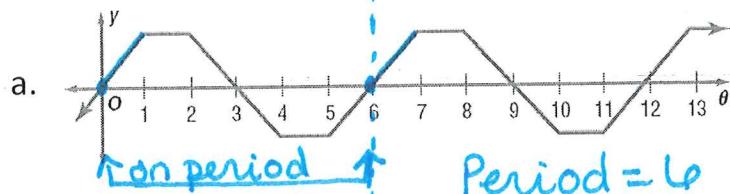


9. The function increases in the interval

$$0^\circ \leq \theta \leq 90^\circ$$



10. Determine the period of each function.



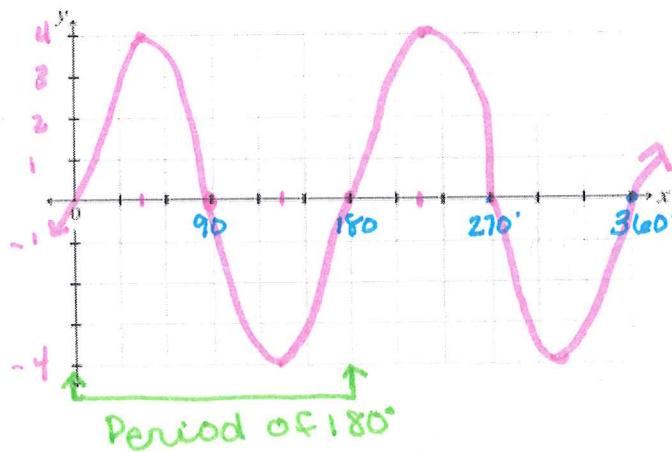
Find the amplitude, if it exists, and the period for each function. Then graph each function.

$$\text{Amp: } 4$$

$$\text{Period: } 360^\circ \div 2 = 180^\circ \text{ or } \pi$$

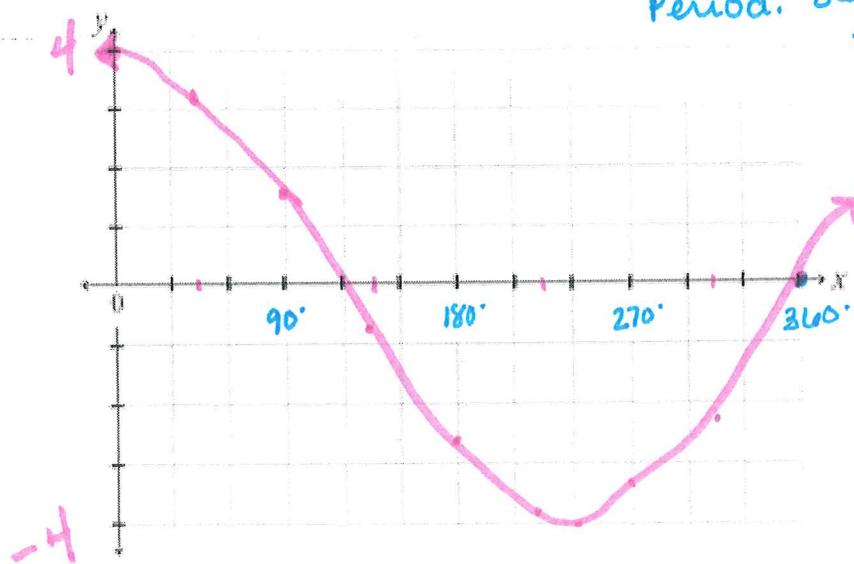
$$11. \ y = 4\sin(2\theta)$$

| θ | y |
|-------------|-----|
| 0° | 0 |
| 45° | 4 |
| 90° | 0 |
| 135° | -4 |
| 180° | 0 |
| 225° | 4 |
| 270° | 0 |
| 315° | -4 |
| 360° | 0 |



$$12. \ y = 4\cos\left(\frac{3}{4}\theta\right)$$

| θ | y |
|-------------|-------|
| 0° | 4 |
| 45° | 3.3 |
| 90° | 1.5 |
| 135° | -0.78 |
| 180° | -2.8 |
| 225° | -3.9 |
| 270° | -4.7 |
| 315° | -2.2 |
| 360° | 0 |



$$\text{Amp} = 4$$

$$\text{Period: } 360^\circ \div \left(\frac{3}{4}\right) = 480^\circ$$