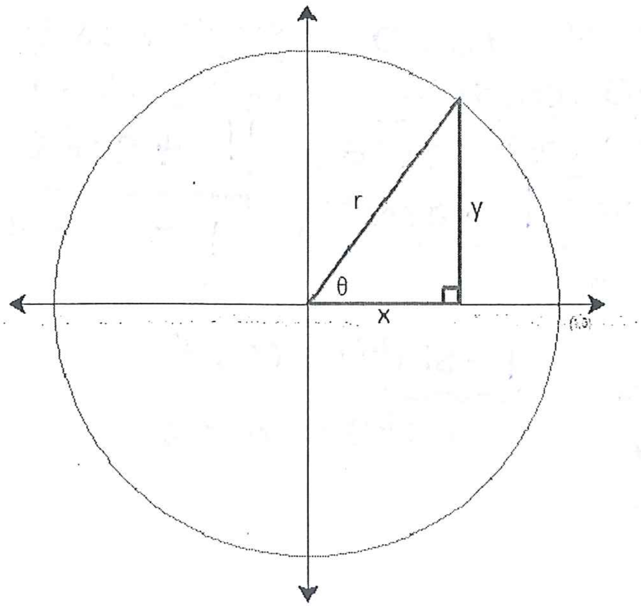
14.3 Day ONE: Trigonometric Identities Notes

Review:



With radius= 1, find:

$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$

$\sin\theta = \frac{y}{1} = y$

$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$

Given: The right triangle above is in the unit circle

Prove: $\sin^2\theta + \cos^2\theta = 1$

1.	1. given
2. $y^2 + x^2 = 1^2$	2. pyth. thm
3. $y = \sin\theta$, $x = \cos\theta$	3. trig Identity
4. $\sin^2\theta + \cos^2\theta = 1$	4. Substitution

__14-3 Notes (Day 2), Simplifying with Trigonometric Identities__

A __trigonometric Identity__ is an equation involving trigonometric functions that is true for all values for which every expression in the equation is defined.

__Quotient Identities__

$\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

__Reciprocal Identities__

Simplifying Expressions Using Trigonometric Identities

We will use the trigonometric identities to simplify expressions. We may have to rewrite the identities so we can use them.

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $\tan^2\theta + 1 = \sec^2\theta$ $\qquad$ $\cot^2 + 1 = \csc^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$1 = \sec^2\theta - \tan^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\boxed{\tan^2\theta + 1 = \sec^2\theta}$

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\boxed{1 + \cot^2\theta = \csc^2\theta}$

$\boxed{1 = \csc^2\theta - \cot^2\theta}$

Example 1: Simplify the expression

$\sin\theta(\csc\theta - \sin\theta) = \cos^2\theta$

$\sin\theta\csc\theta - \sin^2\theta = \cos^2\theta$

$\frac{\sin\theta}{1} \cdot \frac{1}{\sin\theta} - \sin^2\theta = \cos^2\theta$

$\frac{\sin\theta}{\sin\theta} - \sin^2\theta = \cos^2\theta$

$\underbrace{1 - \sin^2\theta}_{} = \cos^2\theta$

$\cos^2\theta = \cos^2\theta$ ✓

Example 2: Simplify the expression

$\frac{\csc^2\theta - \cot^2\theta}{\cos\theta} = \sec\theta$

$\frac{1}{\cos\theta} = \sec\theta$

$\sec\theta = \sec\theta$ ✓

Example 3: Simplify each expression

$\frac{\sec\theta}{\sin\theta}(1 - \cos^2\theta) = \tan\theta$

$\frac{\sec\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{1} = \tan\theta$

$\frac{\sec\theta\,\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}} = \tan\theta$

$\sec\theta\,\sin\theta = \tan\theta$

$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} = \tan\theta$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

$\tan\theta = \tan\theta$ ✓

$\frac{x^4}{x} = \frac{x \cdot \cancel{x} \cdot x \cdot x}{\cancel{x}}$ $\qquad$ $\frac{x^2}{x} = x$

Example 4: Simplify each expression

$\frac{\tan^2\theta\csc^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

$\frac{\left(\frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{1}{\sin^2\theta}\right) - 1}{\sec^2\theta} = \frac{\frac{\cancel{\sin^2\theta}}{\cos^2\theta \cdot \cancel{\sin^2\theta}} - 1}{\sec^2\theta} = \frac{\frac{1}{\cos^2\theta} - 1}{\sec^2\theta} = \frac{\sec^2\theta - 1}{\sec^2\theta}$

$\frac{\sec^2\theta}{\sec^2\theta} - \frac{1}{\sec^2\theta} = 1 - \cos^2\theta = \sin^2\theta$ ✓