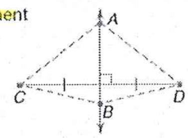
Key

5.1 ACC Notes: Points of Concurrency
Perpendicular Bisectors and Circumcenters

THEOREMS — *Points on Perpendicular Bisectors*

5.1 Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

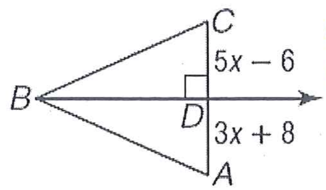
Example: If $\overline{AB} \perp \overline{CD}$ and $\overline{AB}$ bisects $\overline{CD}$, then $AC = AD$ and $BC = BD$.



5.2 Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.

Example: If $AC = AD$, then A lies on the perpendicular bisector of $\overline{CD}$.
If $BC = BD$, then B lies on the perpendicular bisector of $\overline{CD}$.

Example 1 $\overrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$. Find x.



$CD \cong AD$ def of $\perp$ bisector ← this part

$5x-6=3x+8$

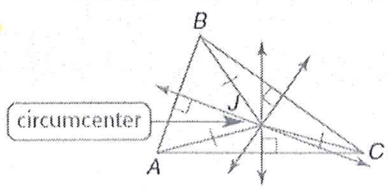
$\boxed{x=7}$

The point of concurrency of all 3 perpendicular bisectors is the circumcenter.

THEOREM 5.3 — *Circumcenter Theorem*

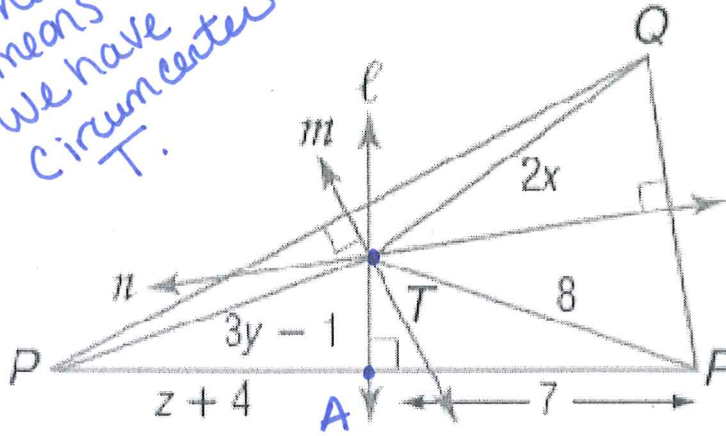
The circumcenter of a triangle is equidistant from the vertices of the triangle.

Example: If J is the circumcenter of $\triangle ABC$, then $AJ = BJ = CJ$.



Example: **ALGEBRA** Lines ℓ, m, and n are perpendicular bisectors of $\triangle PQR$ and meet at T. If $TQ = 2x$, $PT = 3y - 1$, and $TR = 8$, find x, y, and z.

This means we have circumcenter T.



$QT \cong TR$ ← Circumcenter is equidistant to the vertices.

$2x = 8$

$\boxed{x=4}$

$PT \cong TR$ ← same

$3y-1=8$

$\boxed{y=3}$

$PA \cong RA$ def of $\perp$ bisector

$z+4=7$

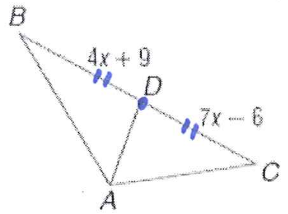
$\boxed{z=3}$

Medians and Centroids

Median: a segment from a vertex to the opposite side's midpoint.

Practice example:

ALGEBRA Find x if $\overline{AD}$ is a median of $\triangle ABC$.



$BD \cong CD$ def of median

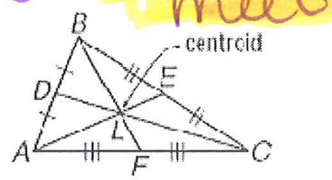
$4x + 9 = 7x - 6$

$\boxed{5 = x}$

Centroid

The point where the medians meet.

Centroid Theorem	The centroid of a triangle is located two thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median.



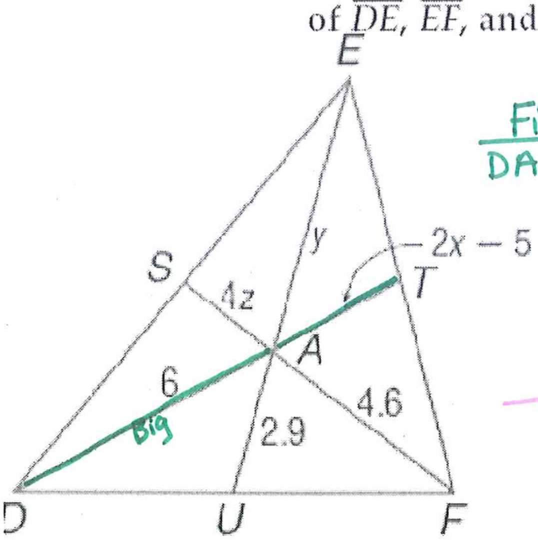
$AL = \frac{2}{3}AE,\ BL = \frac{2}{3}BF,\ CL = \frac{2}{3}CD$

$CL = 2DL$ $\quad DL = \frac{1}{2}CL$

$DL = \frac{1}{3}CD$

Practice example:

ALGEBRA Points S, T, and U are the midpoints of $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$, respectively. Find x, y, and z.



Find x

$DA = 2AT$

$6 = 2(2x - 5)$

$\boxed{4 = x}$

Find y

$EA = 2(AU)$

$y = 2(2.9)$

$\boxed{y = 5.8}$

Find z

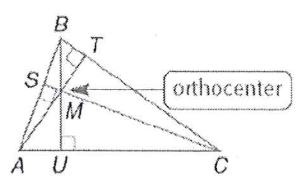
$AF = 2AS$

$4.6 = 2(4z)$

$\boxed{0.575 = z}$

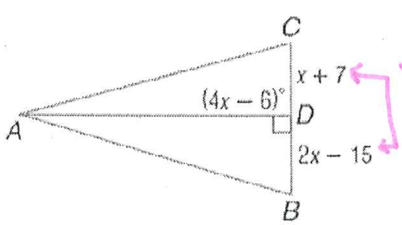
Altitudes and Orthocenters

An **altitude** of a triangle is a segment from a vertex to the line containing the opposite side and perpendicular to the line containing that side. Every triangle has three altitudes. The intersection point of the altitudes of a triangle is called the **orthocenter**.



Practice Example:

ALGEBRA Find x if $\overline{AD}$ is an altitude of $\triangle ABC$.



we don't know these are $\cong$ by altitudes, we do know $4x - 6 = 90°$

$\angle ADC = 90°$ def of altitude

$4x - 6 = 90°$

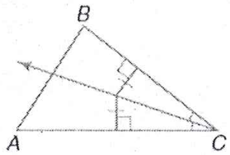
$\boxed{x = 24}$

Angle Bisectors and Incenters

THEOREMS — *Points on Angle Bisectors*

5.4 Any point on the angle bisector is equidistant from the sides of the angle.

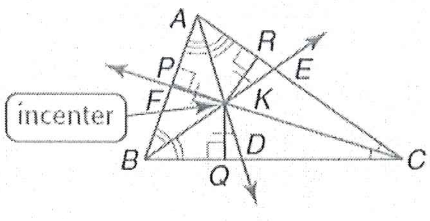
5.5 Any point equidistant from the sides of an angle lies on the angle bisector.



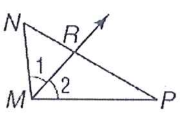
THEOREM 5.6 — *Incenter Theorem*

The incenter of a triangle is equidistant from each side of the triangle.

Example: If K is the incenter of $\triangle ABC$, then $KP = KQ = KR$.



Example 2 $\overrightarrow{MR}$ is the angle bisector of $\angle NMP$. Find x if $m\angle 1 = 5x + 8$ and $m\angle 2 = 8x - 16$.



$\angle 1 \cong \angle 2$ def of $\angle$ bisector

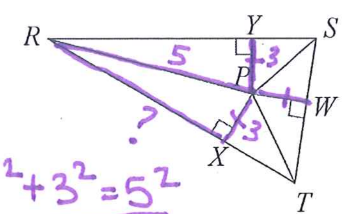
$5x + 8 = 8x - 16$

$\boxed{8 = x}$

If P is the incenter

Example 3.) $PY = 3$ and $RP = 5$. Find RX.

is equidistant to the sides.

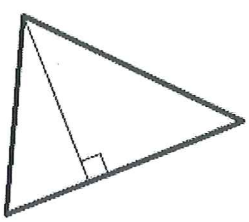


$RX^2 + 3^2 = 5^2$

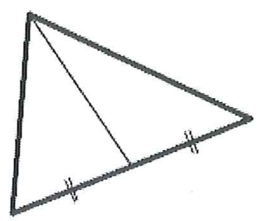
$\boxed{RX = 4}$

Vocab Practice:

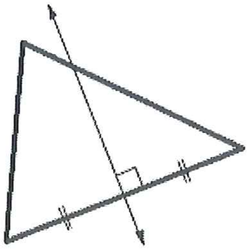
Circle the letter with the name of the segment/line/ray shown.

1. 

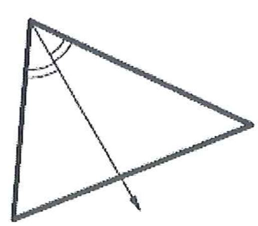
(a) perpendicular bisector
(b) angle bisector
(c) median
(d) altitude

2. 

(a) perpendicular bisector
(b) angle bisector
(c) median
(d) altitude

3. 

(a) perpendicular bisector
(b) angle bisector
(c) median
(d) altitude

4. 

(a) perpendicular bisector
(b) angle bisector
(c) median
(d) altitude

Name of the correct point of concurrency for each.

circumcenter **incenter** **centroid** **orthocenter**

5. The three altitudes of a triangle intersect at the orthocenter.

6. The three medians of a triangle intersect at the centroid.

7. The three perpendicular bisectors of a triangle intersect at the circumcenter.

8. The three angle bisectors of a triangle intersect at the incenter.

9. It is equidistant from the three vertices of the triangle. Circumcenter.

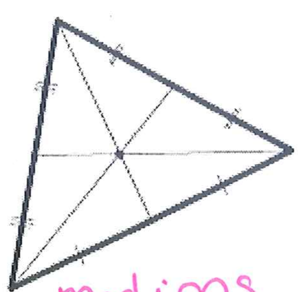
10. It is equidistant from the three sides of the triangle. incenter.

11. It divides each median into two sections at a 2:1 ratio. Centroid.

Name the special segments and the points of concurrency.

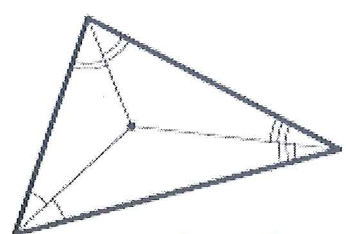
Special lines/segments: Perpendicular bisectors, angle bisectors, medians, altitudes
Points of concurrency: circumcenter, incenter, centroid, orthocenter

12.



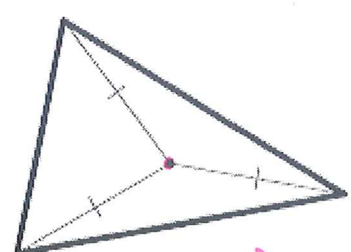
medians
centroid

13.



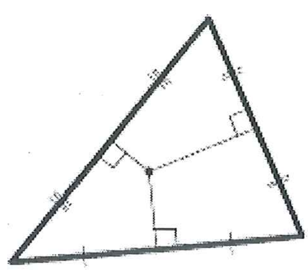
angle bisectors
incenter

14.



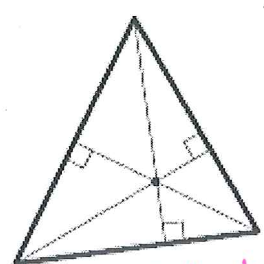
(None)
equidistant to vertices = circumcenter

15.



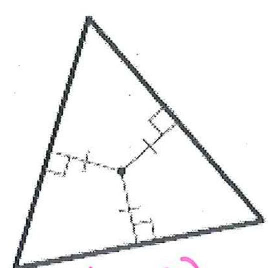
⊥ bisectors
circumcenter

16.



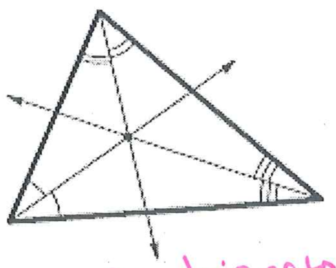
altitudes
orthocenter

17.



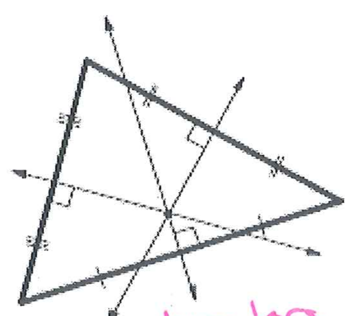
(none)
equidistant to sides = incenter

18.



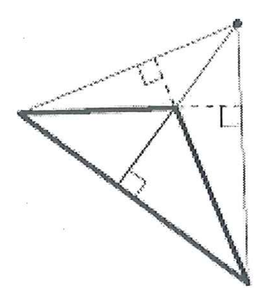
angle bisectors
incenter

19.



⊥ bisetors
circumcenter

20.



altitudes
orthocenter