

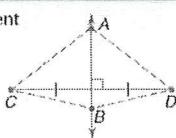
5.1 ACC Notes

THEOREMS

Points on Perpendicular Bisectors

5.1 Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Example: If $\overline{AB} \perp \overline{CD}$ and \overline{AB} bisects \overline{CD} , then $AC = AD$ and $BC = BD$.

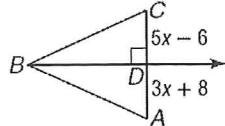


5.2 Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.

Example: If $AC = AD$, then A lies on the perpendicular bisector of \overline{CD} . If $BC = BD$, then B lies on the perpendicular bisector of \overline{CD} .

Key

Example 1 \overrightarrow{BD} is the perpendicular bisector of \overline{AC} . Find x .



$$CD = AD \text{ def of } \perp \text{ bisector}$$

$$5x - 6 = 3x + 8$$

$$2x = 14$$

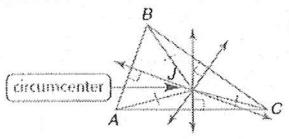
$$\boxed{x = 7}$$

THEOREM 5.3

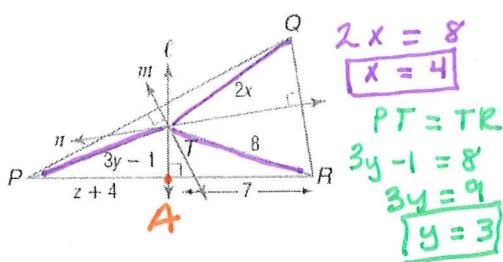
Circumcenter Theorem

The circumcenter of a triangle is equidistant from the vertices of the triangle.

Example: If J is the circumcenter of $\triangle ABC$, then $AJ = BJ = CJ$.



ALGEBRA Lines ℓ , m , and n are perpendicular bisectors of $\triangle PQR$ and meet at T. If $TQ = 2x$, $PT = 3y - 1$, and $TR = 8$, find x , y , and z .



$QT \cong TR$ circumcenter is equidistant from vertices

circumcenter is equidistant from vertices

$$PA = RA \text{ def of } \perp \text{ bisector}$$

$$z+4=7$$

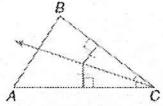
$$\boxed{z=3}$$

THEOREMS

Points on Angle Bisectors

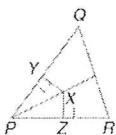
5.4 Any point on the angle bisector is equidistant from the sides of the angle.

5.5 Any point equidistant from the sides of an angle lies on the angle bisector.



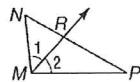
Given: \overline{PX} bisects $\angle QPR$, $\overline{XY} \perp \overline{PQ}$, and $\overline{XZ} \perp \overline{PR}$.

Prove: $\overline{XY} \cong \overline{XZ}$



See attached paper

Example 2 \overrightarrow{MR} is the angle bisector of $\angle NMP$. Find x if $m\angle 1 = 5x + 8$ and $m\angle 2 = 8x - 16$.



$$\angle 1 \cong \angle 2 \text{ def of } \angle \text{ bisector}$$

$$5x + 8 = 8x - 16$$

$$24 = 3x$$

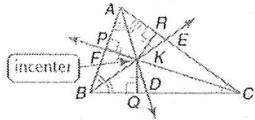
$$\boxed{x = 8}$$

THEOREM 5.6

Incenter Theorem

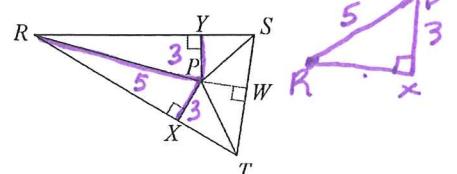
The incenter of a triangle is equidistant from each side of the triangle.

Example: If K is the incenter of $\triangle ABC$, then $KP = KQ = KR$.



$PY = 3$ and $RP = 5$. Find RX .

so now:



$$YP = PR$$

$$3 = PR$$

incenter is equidistant to the sides

$$PR^2 + 3^2 = 5^2$$

$$RX^2 + 9 = 25$$

$$RX^2 = 16$$

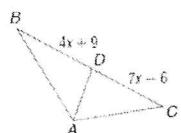
$$\boxed{RX = 4}$$

Median: a segment from a vertex to the opposite side's midpoint.

Here: If YM is a median then M is midpt of XZ

Practice example:

ALGEBRA Find x if \overline{AD} is a median of $\triangle ABC$.



$$BD = DC \text{ def of median}$$

$$4x+9 = 7x-6$$

$$15 = 3x$$

$$5 = x$$

Centroid Theorem	The centroid of a triangle is located two thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median.
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Practice example:

ALGEBRA Points S, T, and U are the midpoints of \overline{DE} , \overline{EF} , and \overline{DF} , respectively. Find x, y, and z.

$$AD = \frac{2}{3} DT \quad \text{Centroid Thm}$$

$$6 = \frac{2}{3}(2x - 5 + 6)$$

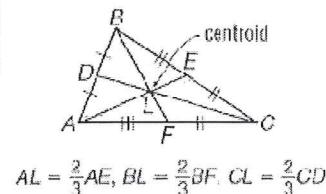
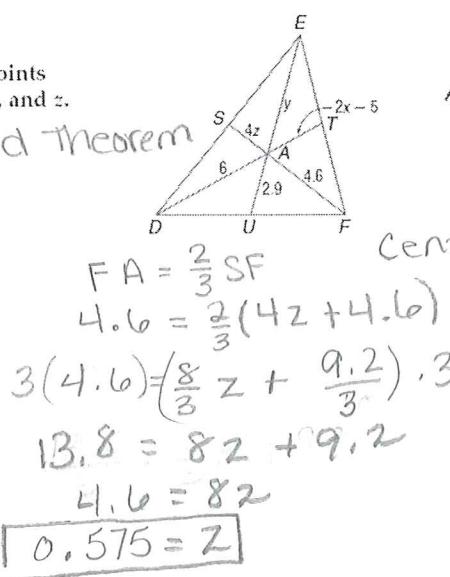
$$6 = \frac{2}{3}(2x + 1)$$

$$6 = \frac{4}{3}x + \frac{2}{3}$$

$$\frac{18}{3} = \frac{4}{3}x + \frac{2}{3}$$

$$- \frac{2}{3} = - \frac{2}{3}$$

$$3 \cdot \frac{16}{3} = \frac{4}{3}x + \frac{3}{4} \quad \boxed{X=4}$$

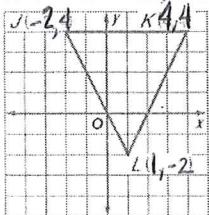


$$AL = \frac{2}{3}AE, BL = \frac{2}{3}BF, CL = \frac{2}{3}CD$$

An altitude of a triangle is a segment from a vertex to the line containing the opposite side and perpendicular to the line containing that side. Every triangle has three altitudes. The intersection point of the altitudes of a triangle is called the orthocenter.

Practice Example:

COORDINATE GEOMETRY The vertices of $\triangle JKL$ are $J(-2, 4)$, $K(4, 4)$, and $L(1, -2)$. Find the coordinates of the orthocenter of $\triangle JKL$.



$$KL \text{ slope} = 2 \text{ use pt } J(-2, 4)$$

$$\text{Slope of altitude is } -\frac{1}{2}$$

$$Y - 4 = -\frac{1}{2}(X + 2)$$

$$Y = -\frac{1}{2}X + 3$$

Find an equation of the altitude from K to \overline{JL} . The slope of \overline{JL} is -2 , so the slope of the altitude is $\frac{1}{2}$.

$$Y - 4 = \frac{1}{2}(X - 4)$$

$$Y - 4 = \frac{1}{2}X - 2$$

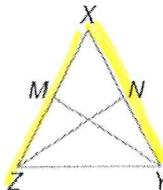
$$Y = \frac{1}{2}X + 2$$

PROOF Write a two-column proof.

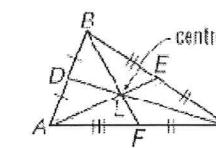
Given: $\overline{XY} \cong \overline{XZ}$

\overline{YM} and \overline{ZN} are medians.

Prove: $\overline{YM} \cong \overline{ZN}$



See attached
paper



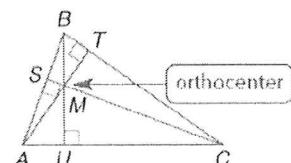
$$AL = \frac{2}{3}AE, BL = \frac{2}{3}BF, CL = \frac{2}{3}CD$$

$$EA = \frac{2}{3}EU \text{ Centroid thm}$$

$$Y = \frac{2}{3}(4+2.9) \text{ thm}$$

$$3y = 2y + 5.8$$

$$Y = 5.8$$



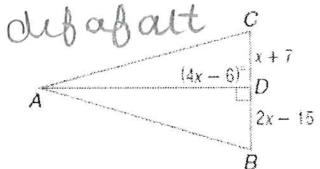
Practice Example: ALGEBRA Find x if \overline{AD} is an altitude of $\triangle ABC$.

$$\angle ADC = 90^\circ \text{ def of alt}$$

$$4x - 6 = 90$$

$$4x = 96$$

$$\boxed{X=24}$$



Plug in for x

$$CD = 24 + 7$$

$$\boxed{CD = 31}$$

$$DB = 2(24) - 15$$

$$\boxed{DB = 33}$$

use a system

$$-\frac{1}{2}x + 3 = \frac{1}{2}x + 2$$

$$3 = x + 2$$

$$\boxed{1 = x}$$

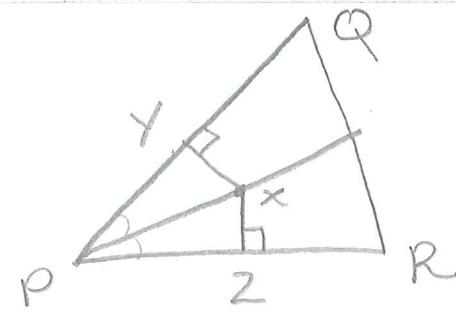
$$Y = -\frac{1}{2}(1) + 3$$

$$Y = -\frac{1}{2} + 3$$

$$Y = \frac{5}{2} = 2\frac{1}{2}$$

$$(1, 2\frac{1}{2})$$

Given: PX bisects $\angle QPR$
 $XY \perp PQ$ and $XZ \perp PR$



Prove: $XY \cong XZ$

1. Given

2. $\angle QPX \cong \angle RPX$

3. $\angle XZP = 90^\circ$
 $\angle XYP = 90^\circ$

4. $\angle XZP \cong \angle XYP$

5. $PX \cong PX$

6. $\triangle PYX \cong \triangle PZX$

7. $XY \cong XZ$

1.

2. $BZ \cong ZC$

$AX \cong XB$

$AY \cong YC$

3. $\angle YJ \cong \angle YJ$, $\angle JZ \cong \angle JZ$, $\angle JX \cong \angle JX$

4. $\angle BXJ = 90^\circ$, $\angle AXJ = 90^\circ$

$\angle JYC = 90^\circ$, $\angle JYA = 90^\circ$

$\angle JZB = 90^\circ$, $\angle JZC = 90^\circ$

1.

2. def of \perp
 bisector

3. Reflexive

4. def of \perp

5. substitution

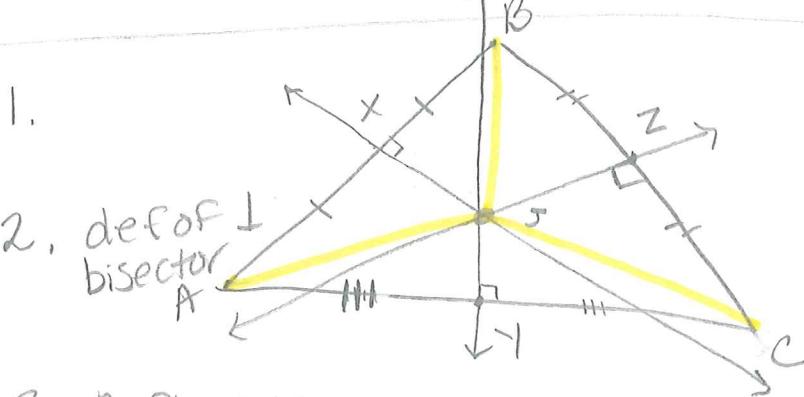
5. $\angle BXJ \cong \angle AXJ$, $\angle JYC \cong \angle JYA$

$\angle JZB \cong \angle JZC$

6. $\triangle JYA \cong \triangle JYC$, $\triangle JCZ \cong \triangle JBZ$
 $\triangle JA X \cong \triangle JBX$

7. $JB \cong JC$, $JC \cong JA$, $JA \cong JB$

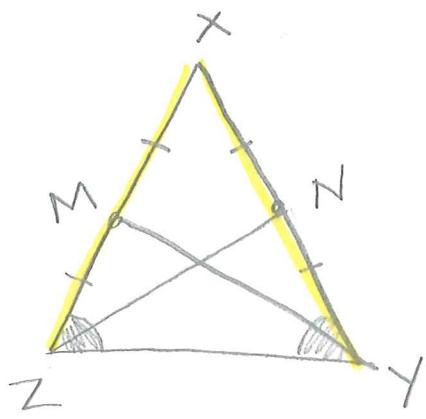
8. $AJ \cong BJ \cong CJ$



6. SAS

7. CPCTC

8. Substitution



Given: $XY \cong XZ$
 YM and ZN are medians
 Prove: $YM \cong ZN$

1. $XY \cong XZ$

YM and ZN are
medians

1.5 M is midpt of XZ and
N is midpt of XY

2. $XM \cong MZ$

$XN \cong NY$

3. $XY = XN + NY$

$XZ = XM + MZ$

4. $XN + NY = XM + MZ$

5. $XN + NY = XM + XM$

6. $2XN = 2XM$

7. $XN = XM$

8. $MZ \cong NY$

9. $\angle XYZ \cong \angle XZY$

10. $ZY \cong ZY$

11. $\triangle YNZ \cong \triangle ZMY$

12. $YM \cong ZN$

1. given

1.5 def of median

2. def of median
midpt

3. segment addition

4. substitution

5. substitution

6. CLT

7. division

8. substitution

9. base ls of isosceles Δs
are \cong

10 Reflexive

11. SAS

12. CPCTC