NAME Key DATE _____ PERIOD ____

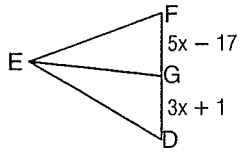
5-1 Skills Practice

MUST Show Work on Separate Paper!!

Bisectors, Medians, and Altitudes

ALGEBRA For Exercises 1–4, use the given information to find each value.

1. Find x if $\overline{EG}$ is a median of $\triangle DEF$.



FG = GD def of median

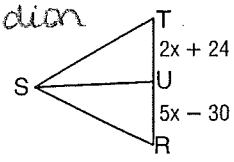
$5x - 17 = 3x + 1$

$2x = 18$

$x = 9$

$x = 9$

2. Find x and RT if $\overline{SU}$ is a median of $\triangle RST$.



TU = UR def of median

$2x + 24 = 5x - 30$

$54 = 3x$

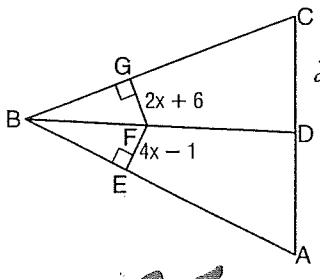
$18 = x$

$RT = 5(18) - 30 + 2(18) + 24$

$RT = 120$

$x = 18$, $RT = 120$

3. Find x and EF if $\overline{BD}$ is an angle bisector.



GF = FE incenter is equidistant to sides

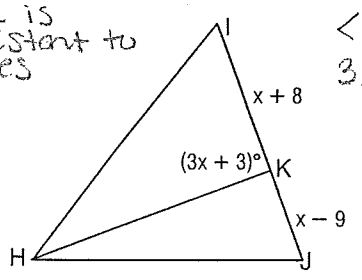
$2x + 6 = 4x - 1$

$7 = 2x$

$\boxed{\frac{7}{2} = x}$

$x = 3.5$ $EF = 13$

4. Find x and IJ if $\overline{HK}$ is an altitude of $\triangle HIJ$.



$\angle HKI = 90$ def of altitude

$3x + 3 = 90$

$3x = 87$

$x = 29$

$IJ = KJ + KI$

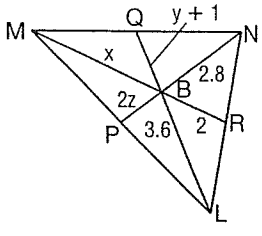
$IJ = 29 - 9 + 29 + 8$

$IJ = 57$

$x = 29$ $IJ = 57$

ALGEBRA For Exercises 5–7, use the following information.

In $\triangle LMN$, P, Q, and R are the midpoints of $\overline{LM}$, $\overline{MN}$, and $\overline{LN}$, respectively.



5. Find x. 4

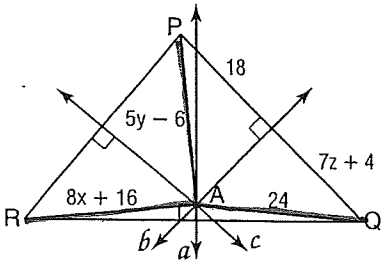
6. Find y. 0.8

7. Find z. 0.7

on paper

B is centroid

ALGEBRA Lines a, b, and c are perpendicular bisectors of $\triangle PQR$ and meet at A.



8. Find x. 1

9. Find y. 6

10. Find z. 2

COORDINATE GEOMETRY The vertices of $\triangle HIJ$ are $J(1, 0)$, $H(6, 0)$, and $I(3, 6)$. Find the coordinates of the points of concurrency of $\triangle HIJ$.

11. orthocenter

(3, 1)

12. centroid

$\left(\frac{10}{3}, 2\right)$

13. circumcenter

$\left(\frac{7}{2}, \frac{5}{2}\right)$

5.) $MB = \frac{2}{3} MR$ centroid theorem

$3 \cdot x = \frac{2}{3}(x+2) \cdot 3$

$3x = 2x + 4$

$\boxed{x = 4}$

6.) $BL = \frac{2}{3} LQ$ Centroid theorem

$3 \cdot 3.6 = \frac{2}{3}(y + 1 + 3.6) \cdot 3$

$10.8 = 2y + 9.2$

$1.6 = 2y$

$\boxed{0.8 = y}$

7.) $NB = \frac{2}{3} NP$ Centroid theorem

$3 \cdot 2.8 = \frac{2}{3}(2z + 2.8) \cdot 3$

$8.4 = 4z + 5.6$

$2.8 = 4z$

$\boxed{0.7 = z}$

8.) $AP = AQ$ circumcenter is equidistant to the vertices

$8x + 16 = 24$

$8x = 8$

$\boxed{x = 1}$

9.) $PA = AQ$

$5y - 6 = 24$

$5y = 30$

$\boxed{y = 6}$

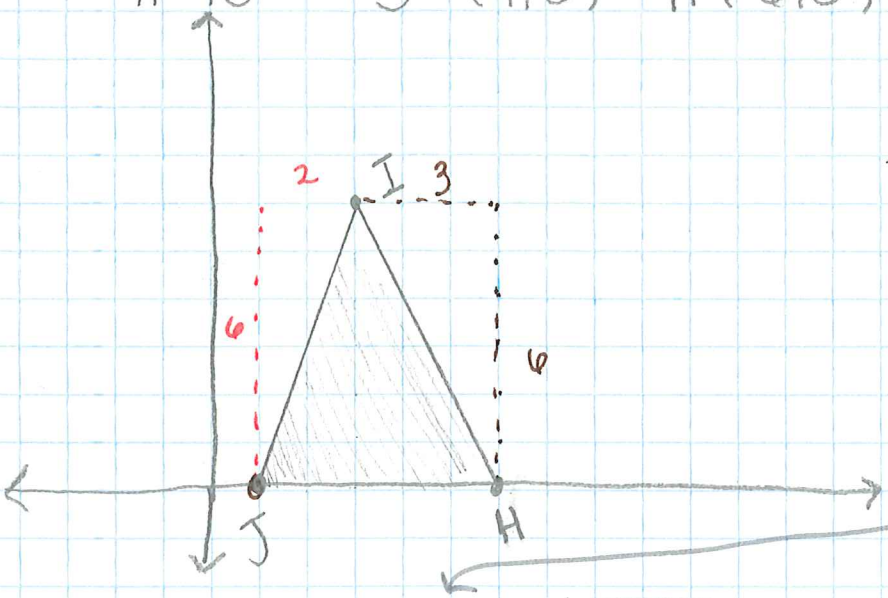
10.) $7z + 4 = 18$ def of bisector b bisects PQ

$7z = 14$

$\boxed{z = 2}$

From Pg 8

11-13 J (1,0) H (6,0) I (3,6)



Orthocenter

Points where alt. meets

Find eq lines for alt.

J (1,0) slope IH $= -\frac{6}{3} = -2$

$\perp = \frac{1}{2}$

$y = \frac{1}{2}(x-1)$

H (6,0) slope JI $= \frac{6}{2} = 3$

$\perp = -\frac{1}{3}$

$y = -\frac{1}{3}(x-6)$

$\frac{3}{2} \cdot \frac{1}{2}(x-1) = -\frac{1}{3}(x-6) \cdot \frac{3}{2}$

$2 \cdot \frac{3}{2}(x-1) = -(x-6) \cdot 2$

$3(x-1) = -2(x-6)$

$3x-3 = -2x+12$

$5x-3 = 12$

$+3 \quad +3$

$5x = 15$

$\boxed{x=3}$

$y = -\frac{1}{3}(3-6)$

$y = -\frac{1}{3}(-3)$

$\boxed{y=1}$

$\boxed{(3,1)}$

$\boxed{\text{orthocenter}}$

Centroid - points where medians meet

mid pt of JH:

$\left(\frac{1+6}{2}, \frac{0+0}{2}\right)$

$\left(\frac{7}{2}, 0\right) = (3.5, 0)$

+ pt I (3,6)

EQ of median

slope: $\frac{6-0}{3-3.5} = 12 = m$

$y-6 = 12(x-3)$

$y-6 = \boxed{12x+30}$

$y = -12x+42$

Midpt IH:

$\left(\frac{6+3}{2}, \frac{6}{2}\right)$

$\left(\frac{9}{2}, 3\right)$ + pt J (1,0)

(4.5, 3)

EQ of median

Slope: $\frac{0-3}{1-4.5} = \frac{-3}{-3.5} = \frac{6}{7}$

$y = \boxed{\frac{6}{7}(x-1)}$

$-12x+42 = \frac{6}{7}x - \frac{6}{7}$

$-\frac{7}{90} \cdot -\frac{90}{7}x = \frac{-300}{7} \cdot -\frac{7}{90}$

$x = 10/3$

13. Circumcenter = where ⊥ bisectors meet

∴ JH midpt (3.5, 0)
JH slope = 0
⊥ = undefined
$x = 3.5$ ← equation of ⊥ bisector

↓

$y = \frac{1}{2}(3.5) + .75$
$y = 1.75 + .75$
$y = 2.5$

∴ (4.5, 3) midpt of IH
IH slope = −2
⊥ = $\frac{1}{2}$
$y - 3 = \frac{1}{2}(x - 4.5)$
$y - 3 = \frac{1}{2}x - 2.25$
$y = \frac{1}{2}x + .75$

↑ EQ of ⊥ bisector

Point of concurrency of ⊥ bisectors is

$\boxed{(3.5, 2.5)}$

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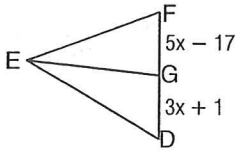

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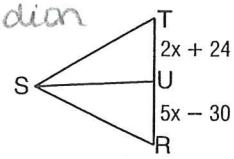

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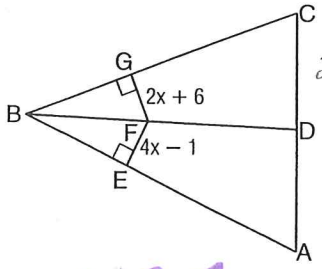

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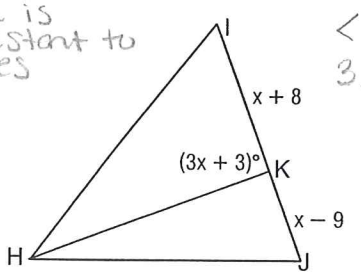

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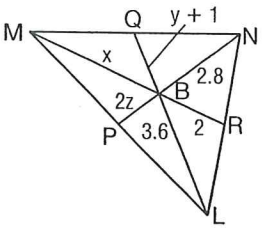

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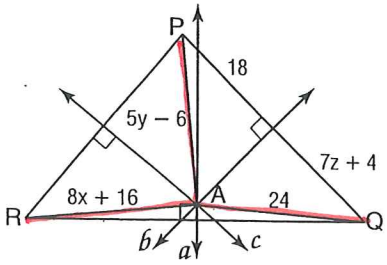

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1.) $CP = \frac{2}{3}CD$ centroid Thm

$30 = \frac{2}{3}(4x - 3 + 30)$

$30 = \frac{2}{3}(4x + 27)$

$90 = 8x + 54$

$36 = 8x$

$\boxed{4.5 = x}$

2.) $AP = \frac{2}{3}AE$ Centroid thm

$y = \frac{2}{3}(y + 18)$

$3y = 2y + 36$

$\boxed{y = 36}$

3.) $BP = \frac{2}{3}BF$ Centroid thm

$42 = \frac{2}{3}(5z + 10 + 42)$

$42 = \frac{2}{3}(5z + 52)$

$126 = 10z + 104$

$22 = 10z$

$\boxed{2.2 = z}$

5.) $RX \cong SX$ def of median

$x + 7 = 3x - 11$

$18 = 2x$

$\boxed{9 = x}$

Find: $RS = RX + XS$

$RS = 9 + 7 + 3(9) - 11$

$\boxed{RS = 32}$

6.) $\angle PTR = 90$ def of altitude

$8x - 6 = 90$

$8x = 96$

$x = 12$

$RT = 12 - 6$

$\boxed{RT = 6}$

7.) $EH \cong FH$ circumcenter is equidistant to the vertices

$16 = 6x - 5$

$19 = 6x$

$\boxed{3.5 = x}$

8. $EG \cong GF$ def of $\perp$ BISECTOR

$3.2y - 1 = 2y + 5$

$1.2y = 6$

$\boxed{y = 5}$

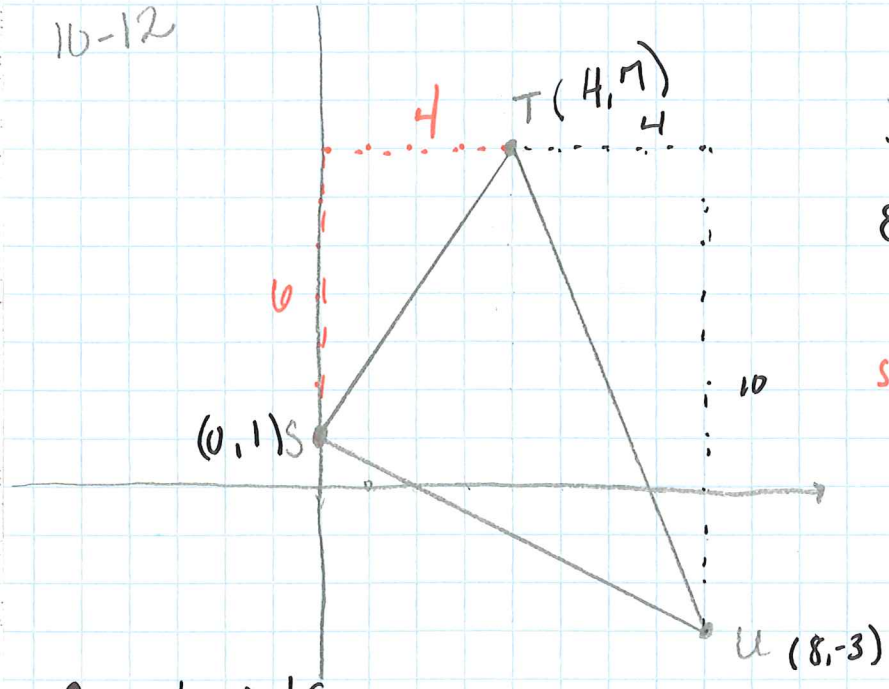
9.) $\angle EGH = 90^\circ$ def of $\perp$

$12z = 90$

$\boxed{z = 7.5}$

From page 9

10-12



orthocenter

slope TU $= -\frac{10}{4} = -\frac{5}{2}$ $(0,1)$

EQ: $y - 1 = \frac{2}{5}(x)$

$\boxed{y = \frac{2}{5}x + 1}$

slope ST $= \frac{6}{4} = 3/2$ $(8,-3)$

$\perp = -2/3$

$y + 3 = -\frac{2}{3}(x - 8)$

$y + 3 = -\frac{2}{3}x + \frac{16}{3}$

-3 $\quad 3$

$\boxed{y = -\frac{2}{3}x + \frac{7}{3}}$

$\frac{2}{5}x + 1 = -\frac{2}{3}x + \frac{7}{3}$ $\quad \frac{2}{5} = 6/15$

$+\frac{10}{15}$ $\quad +\frac{10}{15}$

$\frac{15}{16} \cdot \frac{16}{15}x = \frac{4}{3} \cdot \frac{15}{16}$

$\boxed{x = \frac{5}{4}}$

$y = \frac{2}{5}\left(\frac{5}{4}\right) + 1$

$y = .5 + 1$

$\boxed{y = 1.5}$

$\boxed{\text{ORTHOCENTER } \left(\frac{5}{4}, \frac{3}{4}\right)}$

Centroids

Find midpts $\left(\frac{4+8}{2}, \frac{7-3}{2}\right)$

midpt TU $= (6,2)$

midpt ST $\left(\frac{4}{2}, \frac{7+1}{2}\right) = (2,4)$

EQ Thru $(6,2)$ and $S(0,1)$

slope: $\frac{1-2}{0-6} = \frac{-1}{-6} = \frac{1}{6}$

$y - 1 = \frac{1}{6}(x - 0)$

$y - 1 = \frac{1}{6}x$

$\boxed{y = \frac{1}{6}x + 1}$

EQ thru $(2,4)$ and $U(8,-3)$

slope: $\frac{-3-4}{8-2} = \frac{-7}{6}$

$y + 3 = \frac{-7}{6}(x - 8)$

$\boxed{y = -\frac{7}{6}x + \frac{38}{6}}$

$6\left(\frac{1}{6}x + 1\right) = \left(-\frac{7}{6}x + \frac{38}{6}\right)6$

$x + 6 = -7x + 38$

$8x = 32$

$\boxed{x = 4}$

$y = \frac{1}{6}(4) + 1$

$y = \frac{4}{6} + \frac{6}{6}$

$y = \frac{10}{6} = \frac{5}{3}$ $\quad \boxed{y = 5/3}$

$\boxed{\text{Centroid } (4, 5/3)}$

Circumcenter

$\perp$ bisector of TU

$\perp$ slope $= \frac{2}{5}$

midpt $(6,2)$

$y - 2 = \frac{2}{5}(x - 6)$

$y - 2 = \frac{2}{5}x - \frac{12}{5}$

$\boxed{y = \frac{2}{5}x - 2/5}$

$\perp$ bisector ST

$\perp$ slope: $-2/3$

midpt $(2,4)$

$y - 4 = -\frac{2}{3}(x - 2)$

$y - 4 = -\frac{2}{3}x + \frac{4}{3}$

$\boxed{y = -\frac{2}{3}x + \frac{10}{3}}$

$3\left(\frac{2}{5}x - \frac{2}{5}\right) = \left(-\frac{2}{3}x + \frac{16}{3}\right)3$

$5\left(\frac{6}{5}x - \frac{6}{5}\right) = (-2x + 16)5$

$6x - 6 = -10x + 80$

$16x = 86$

$x = 5.375 = \boxed{\frac{43}{8}}$

$y = -\frac{2}{3}\left(\frac{43}{8}\right) + \frac{16}{3}$

$y = 1.75 \boxed{= 7/4}$

$\boxed{\text{Circumcenter } \left(\frac{43}{8}, \frac{7}{4}\right)}$