

5.3 Notes-Indirect Proof

Indirect proof- A way to prove a statement true by assuming its conclusion is false and showing that this assumption leads to a contradiction of the given, definition, theorem or postulate known to be true.

Steps for writing an indirect proof:

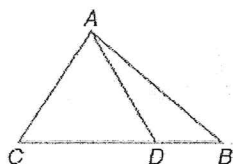
1. Assume the Conjecture is false aka "opposite"
what you want to Prove
2. Show work or write explanations which lead to a contradiction of the given info, theorem, definition, or postulate.
3. State that the assumption must be false and the original conjecture must be true.

Example 1

Write an indirect proof.

Given: $m\angle ADC \neq m\angle ADB$

Prove: \overline{AD} is not an altitude of $\triangle ABC$.



Step 1: Assume \overline{AD} is an altitude

Step 2: $AD \perp CB$ must be the case if AD is an altitude. We then know $\angle ADC = 90$ and $\angle ADB = 90$ by the def of an altitude.
 $\angle ADC \cong \angle ADB$ by substitution, this contradicts the given.

Step 3:

So, the assumption that \overline{AD} is an altitude must be false. \therefore
 \overline{AD} is not an altitude of $\triangle ABC$.

Example 2

Write an indirect proof.

Given: $\triangle ABC$

Prove: A triangle can contain only one obtuse angle.

Step 1: A triangle can have more than one obtuse angle.

Step 2: Let $\angle A > 90^\circ$ and $\angle B > 90^\circ$ by def of obtuse angles. $\angle A + \angle B + \angle C \neq 180$ because $\angle A + \angle B + \angle C > 180$ this contradicts the \triangle sum theorem.

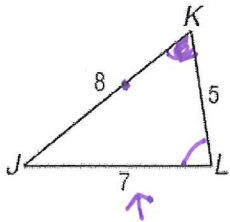
Step 3: Our assumption that a \triangle can have more than one obtuse \angle is false. \therefore a \triangle can contain only ONE obtuse angle.

Example 3

Write an indirect proof.

Given: $\triangle JKL$ with side lengths 5, 7, and 8 as shown

Prove: $m\angle K < m\angle L$



Step 1: $\angle K \geq \angle L$

Step 2: If $\angle K \geq \angle L$ then

$JL \geq KJ$, because op. greatest \angle is the greatest side, this contradicts the given that $JL = 7$ $KJ = 8$.

Step 3: Our assumption that $\angle K \geq \angle L$ is false $\therefore \angle K < \angle L$