Name ______________________________ Hour __________

ACC: 8.1 GEOMETRIC MEAN NOTES

Geometric Mean When the means of a proportion are the same number, that number is called the geometric mean of the extremes. The **geometric mean** between two numbers is the positive square root of their product.

$$\text{extreme} \rightarrow \frac{a}{x} = \frac{x}{b} \leftarrow \text{mean}$$
$$\text{mean} \rightarrow \qquad \leftarrow \text{extreme}$$

Key Concept **Geometric Mean** For Your FOLDABLE

Words The geometric mean of two positive numbers a and b is the number x such that $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

Example The geometric mean of $a = 9$ and $b = 4$ is 6, because $6 = \sqrt{9 \cdot 4}$.

$\frac{9}{x} = \frac{x}{4}$ $\sqrt{x^2} = \sqrt{36}$ $x = 6$

Example #1: Find the geometric mean between 2 and 8.

$\frac{x}{2} = \frac{8}{x}$ $x^2 = 16$ $\boxed{x = 4}$

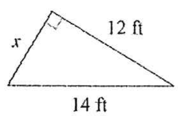
Example #2: Find the geometric mean between $\sqrt{15}$ and $\sqrt{101}$.

$\frac{x}{\sqrt{15}} = \frac{\sqrt{101}}{x}$ $\sqrt{x^{\cancel{2}}} = \sqrt{\sqrt{1515}}$

$x = 1515^{\frac{1}{4}}$

Ex. 3Pythagorean Theorem: Review

Find x.



$a^2 + b^2 = c^2$

$x^2 + 12^2 = 14^2$

$x = \sqrt{52}$

$\boxed{x = 2\sqrt{13}}$

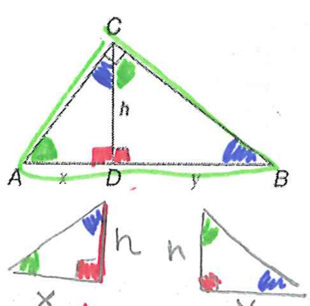
GEOMETRIC MEAN IN RIGHT TRIANGLES

Geometric Means in Right Triangles In a right triangle, an altitude drawn from the vertex of the right angle to the hypotenuse forms two additional right triangles. These three right triangles share a special relationship.

Theorems For Your FOLDABLE

Right Triangle Geometric Mean Theorems

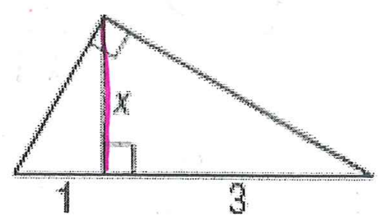
8.2 Geometric Mean (Altitude) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.



Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{x}{h} = \frac{h}{y}$ or $h = \sqrt{xy}$.

$\frac{h}{y} = \frac{x}{h}$

Example 4.

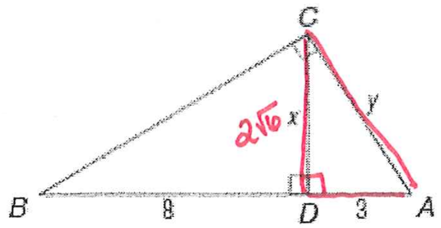


$\frac{x}{1} = \frac{3}{x}$

$x^2 = 3$

$\boxed{x = \sqrt{3}}$

Example #5:



$$\frac{x}{3} = \frac{8}{x}$$

$$x^2 = 24$$

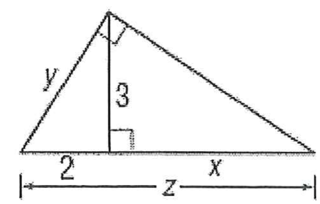
$$\boxed{x = 2\sqrt{6}}$$

$$(2\sqrt{6})^2 + 3^2 = y^2$$

$$2^2 \cdot 6 + 3^2 = y^2$$

$$\boxed{\sqrt{33} = y}$$

Example #6



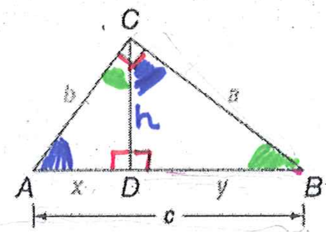
$$\frac{3}{2} = \frac{x}{3} \quad \boxed{x = 4.5}$$

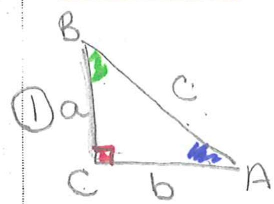
$$\boxed{z = 6.5} \quad z = 2 + 4.5$$

$$y^2 = 3^2 + 2^2$$

$$\boxed{y = \sqrt{13}}$$

8.3 Geometric Mean (Leg) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

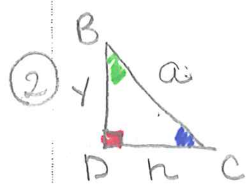




Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{c}{b} = \frac{b}{x}$ or $b = \sqrt{xc}$ and $\frac{c}{a} = \frac{a}{y}$ or $a = \sqrt{yc}$.

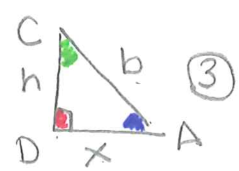
1 + 3

$$\frac{b}{x} = \frac{c}{b}$$

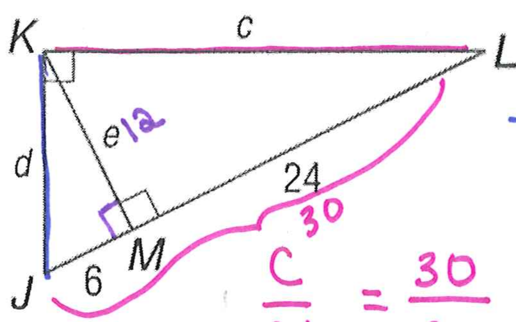


1 + 2

$$\frac{a}{y} = \frac{c}{a}$$



Example 7.



$$\frac{d}{6} = \frac{30}{d}$$

$$d^2 = 180$$

$$\boxed{d = 6\sqrt{5}}$$

$$\frac{c}{24} = \frac{30}{c}$$

$$c^2 = 720$$

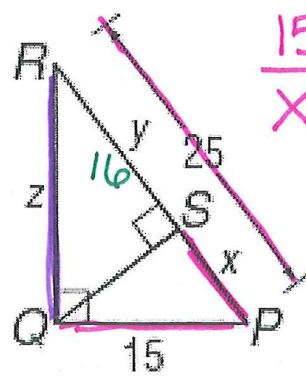
$$\boxed{c = 12\sqrt{5}}$$

$$\frac{e}{6} = \frac{24}{e}$$

$$e^2 = 144$$

$$\boxed{e = 12}$$

Example 8.



$$\frac{15}{x} = \frac{25}{15}$$

$$\boxed{x = 9}$$

$$\boxed{y = 16}$$

$$\frac{z}{16} = \frac{25}{z}$$

$$z^2 = 400$$

$$\boxed{z = 20}$$