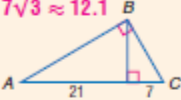
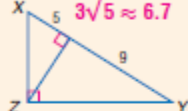
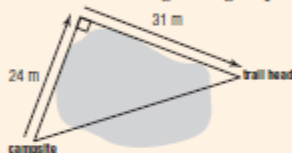


Find the measure of the altitude drawn to the hypotenuse. (Lesson 8-1)

1. $7\sqrt{3} \approx 12.1$  2. $3\sqrt{5} \approx 6.7$ 

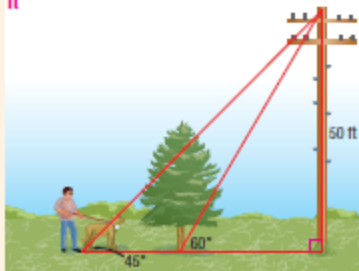
3. Determine whether $\triangle ABC$ with vertices $A(2, 1)$, $B(4, 0)$, and $C(5, 7)$ is a right triangle. Explain. (Lesson 8-2) **yes**; $AB = \sqrt{5}$, $BC = \sqrt{50}$, $AC = \sqrt{45}$; $(\sqrt{5})^2 + (\sqrt{45})^2 = (\sqrt{50})^2$
4. **MULTIPLE CHOICE** To get from your campsite to a trail head, you must take the path shown below to avoid walking through a pond.



About how many meters would be saved if it were possible to walk through the pond? (Lesson 8-2) **D**


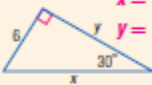
- A 55.0 C 24.7
B 39.2 D 15.8

5. **DOG WALKING** A man is walking his dog on level ground in a straight line with the dog's favorite tree. The angle from the man's present position to the top of a nearby telephone pole is 45° . The angle from the tree to the top of the telephone pole is 60° . If the telephone pole is 50 feet tall, about how far is the man with the dog from the tree? (Lesson 8-3) **21 ft**

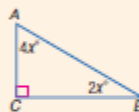


6. **WOODWORKING** Ginger made a small square table for her workshop with a diagonal that measures 55 inches. What are the measures of the sides? Recall that a square has right angles at the corners and congruent sides. (Lesson 8-3) ≈ 38.9 in. on each side

Find x and y . (Lesson 8-3)

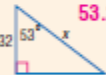


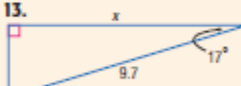
7.  $x = 3$; $y = 3\sqrt{2}$ 8.  $x = 12$; $y = 6\sqrt{3}$

9. **MULTIPLE CHOICE** In the right triangle, what is AB if $BC = 6$? (Lesson 8-3) **H**



- F 12 units H $4\sqrt{3}$ units
G $6\sqrt{2}$ units J $2\sqrt{3}$ units

Find x to the nearest tenth. (Lesson 8-4)

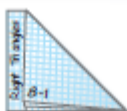
10.  **53.2** 11.  **58.0**
12.  **21.8°** 13.  **9.3**

14. **GARDENING** The lengths of the sides of a triangular garden are 32 feet, 24 feet, and 40 feet. What are the measures of the angles formed on each side of the garden? (Lesson 8-4) **36.9° , 53.1° , 90°**

Find the measure of each angle to the nearest tenth of a degree. (Lesson 8-4)

15. $\sin T = 0.5299$ **32.0**
16. $\cos W = 0.0175$ **89.0**

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Geometric Mean (Lesson 8-1)

- For two positive numbers a and b , the geometric mean is the positive number x where the proportion $a : x = x : b$ is true. This proportion can be written using fractions as $\frac{a}{x} = \frac{x}{b}$ or with cross products as $x^2 = ab$ or $x = \sqrt{ab}$.

Pythagorean Theorem (Lesson 8-2)

- In a right triangle, the sum of the squares of the measures of the legs equals the square of the hypotenuse.

Special Right Triangles (Lesson 8-3)

- The measures of the sides of a 45° - 45° - 90° triangle are x , x , and $x\sqrt{2}$.
- The measures of the sides of a 30° - 60° - 90° triangle are x , $x\sqrt{3}$, and $2x$.

Trigonometry (Lesson 8-4)

- Trigonometric Ratios:

$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

Laws of Sines and Cosines

(Lessons 8-6 and 8-7)

Let $\triangle ABC$ be any triangle with a , b , and c representing the measures of the sides opposite the angles with measures A , B , and C , respectively.

- Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

- Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cos A$
 $b^2 = a^2 + c^2 - 2ac \cos B$
 $c^2 = a^2 + b^2 - 2ab \cos C$

Key Vocabulary

angle of depression (p. 465)	sine (p. 456)
angle of elevation (p. 464)	solving a triangle (p. 472)
cosine (p. 456)	tangent (p. 456)
geometric mean (p. 432)	trigonometric ratio (p. 456)
Pythagorean triple (p. 443)	trigonometry (p. 456)

3. false, a right triangle 6. false, 45° - 45° - 90°

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a *true* sentence.

- To solve a triangle means to find the measures of all its sides and angles. **true**
- The Law of Sines can be applied if you know the measures of two sides and an angle opposite one of these sides of the triangle. **true**
- In any triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.
- An angle of depression is the angle between the line of sight and the horizontal when an observer looks upward. **false, elevation**
- The geometric mean between two numbers is the positive square root of their product. **true**
- A 30° - 60° - 90° triangle is isosceles.
- Looking at a city while flying in a plane is an example that uses an angle of elevation. **false, depression**
- The numbers 3, 4, and 5 form a Pythagorean identity. **false, triple**



Lesson-by-Lesson Review

8-1 Geometric Mean (pp. 432–438)

Find the geometric mean between each pair of numbers.

9. 4 and 16 **8** 10. 4 and 81 **18**

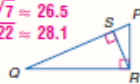
11. 20 and 35 12. 18 and 44

13. In $\triangle PQR$, $PS = 8$, and $QS = 14$.

Find RS . $4\sqrt{7} \approx 10.6$

11. $10\sqrt{7} \approx 26.5$

12. $6\sqrt{22} \approx 28.1$



14. **INDIRECT MEASUREMENT** To estimate the height of the Space Needle in Seattle, Washington, James held a book up to his eyes so that the top and bottom of the building were in line with the bottom edge and binding of the cover. If James' eye level is 6 feet from the ground and he is standing 60 feet from the tower, how tall is the tower? ≈ 600 ft

Example 1 Find the geometric mean between 10 and 30.

$$\frac{10}{x} = \frac{x}{30}$$

Definition of geometric mean

$$x^2 = 300$$

Cross products

$$x = \sqrt{300} \text{ or } 10\sqrt{3}$$

Simplify.

Example 2 Find NG in $\triangle TGR$.

The measure of the altitude is the geometric mean between the measures of the two hypotenuse segments.



$$\frac{TN}{GN} = \frac{GN}{RN}$$

Definition of geometric mean

$$\frac{2}{GN} = \frac{GN}{4}$$

$$TN = 2, RN = 4$$

$$8 = (GN)^2$$

Cross products

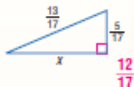
$$\sqrt{8} \text{ or } 2\sqrt{2} = GN$$

Take the square root of each side.

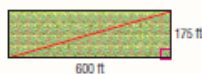
8-2 The Pythagorean Theorem and Its Converse (pp. 440–446)

Find x .

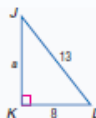
15. **25** 16. **12**



17. **FARMING** A farmer wishes to create a maze in his corn field. He cuts a path 625 feet across the diagonal of the rectangular field. Did the farmer create two right triangles? Explain. **See margin.**



Example 3 Use $\triangle JKL$ to find a .



$$a^2 + (LK)^2 = (JL)^2$$

Pythagorean Theorem

$$a^2 + 8^2 = 13^2$$

$$LK = 8 \text{ and } JL = 13$$

$$a^2 + 64 = 169$$

Simplify.

$$a^2 = 105$$

Subtract 64 from each side.

$$a = \sqrt{105}$$

Take the square root of each side.

$$a \approx 10.2$$

Use a calculator.

Lesson-by-Lesson Review

Intervention If the given examples are not sufficient to review the topics covered by the questions, remind students that the page references tell them where to review that topic in their textbooks.

Two-Day Option Have students complete the Lesson-by-Lesson Review on pages 487–490. Then you can use ExamView® Assessment Suite to customize another review worksheet that practices all the objectives of this chapter or only the objectives on which your students need more help.

ExamView Assessment Suite For more information on ExamView® Assessment Suite, see page 430C.

Differentiated Instruction

Super DVD: MindJogger Plus

Use this DVD as an alternative format of review for the test. For more information on this game show format, see page 430D.

Additional Answer

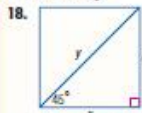
17. Yes; the farmer created two right triangles each with a hypotenuse of 625 feet. $625^2 = 175^2 + 600^2$.

Additional Answers

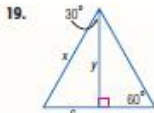
23. $\sin F = \frac{3}{5} = 0.60$, $\cos F = \frac{4}{5} = 0.80$,
 $\tan F = \frac{3}{4} = 0.75$, $\sin G = \frac{4}{5} = 0.80$,
 $\cos G = \frac{3}{5} = 0.60$, $\tan G = \frac{4}{3} \approx 1.33$
24. $\sin F = \frac{7}{25} = 0.28$, $\cos F = \frac{24}{25} = 0.96$,
 $\tan F = \frac{7}{24} \approx 0.29$, $\sin G = \frac{24}{25} = 0.96$,
 $\cos G = \frac{7}{25} = 0.28$, $\tan G = \frac{24}{7} \approx 3.43$
25. $\sin F = \frac{9}{41} \approx 0.22$, $\cos F = \frac{40}{41} \approx 0.98$,
 $\tan F = \frac{9}{40} \approx 0.23$, $\sin G = \frac{40}{41} \approx 0.98$,
 $\cos G = \frac{9}{41} \approx 0.22$, $\tan G = \frac{40}{9} \approx 4.44$

8-3 Special Right Triangles (pp. 448–454)

Find x and y .



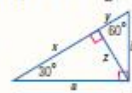
$$x = 9, y = 9\sqrt{2}$$



$$x = 12, y = 6\sqrt{3}$$

For Exercises 20 and 21, use the figure.

20. If $y = 18$, find z and a .



21. If $x = 14$, find a , z , b , and y .

22. **ORIGAMI** To create a bird, Michelle first folded a square piece of origami paper along one of the diagonals. If the diagonal measured 8 centimeters, find the length of one side of the square.

$$20. z = 18\sqrt{3}, a = 36\sqrt{3}$$

$$21. a = \frac{28\sqrt{3}}{3}, z = \frac{14\sqrt{3}}{3}, b = \frac{28}{3}, y = \frac{14}{3}$$

$$22. 4\sqrt{2} \text{ cm}$$

Example 4 Find x .

The shorter leg, \overline{XZ} , of $\triangle XYZ$ is half the measure of the hypotenuse \overline{XY} .



Therefore, $XZ = \frac{1}{2}(26)$ or 13. The longer leg is $\sqrt{3}$ times the measure of the shorter leg. So, $x = 13\sqrt{3}$.

Example 5 Find x .

The hypotenuse of a 45° - 45° - 90° triangle is $\sqrt{2}$ times the length of a leg.



$$x\sqrt{2} = 4$$

$$x = \frac{4}{\sqrt{2}}$$

$$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \text{ or } 2\sqrt{2}$$

8-4 Trigonometry (pp. 456–462)

Use $\triangle FGH$ to find $\sin F$, $\cos F$, $\tan F$, $\sin G$, $\cos G$, and $\tan G$. Express each ratio as a fraction and as a decimal to the nearest hundredth. 23–25. See margin.

23. $f = 9, g = 12, h = 15$

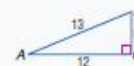
24. $f = 7, g = 24, h = 25$

25. $f = 9, g = 40, h = 41$



26. **SPACE FLIGHT** A space shuttle is directed towards the Moon but drifts 0.8° from its calculated path. If the distance from Earth to the Moon is 240,000 miles, how far has the space shuttle drifted from its path when it reaches the Moon? **3351.25 mi**

Example 6 Find $\sin A$, $\cos A$, and $\tan A$. Express as a fraction and as a decimal.



$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{5}{13} \text{ or about } 0.38$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13} \text{ or about } 0.92$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12} \text{ or about } 0.42$$

8-5 Angles of Elevation and Depression (pp. 464–470)

Determine the angles of elevation or depression in each situation.

27. Upon takeoff, an airplane must clear a 60-foot pole at the end of a runway 500 yards long. **2.3°**
28. An escalator descends 100 feet for each horizontal distance of 240 feet. **22.6°**
29. A hot-air balloon ascends 50 feet for every 1000 feet traveled horizontally. **2.9°**
30. **EAGLES** An eagle, 1350 feet in the air, notices a rabbit on the ground. If the horizontal distance between the eagle and the rabbit is 700 feet, at what angle of depression must the eagle swoop down to catch the rabbit and fly in a straight path? **62.6°**

Example 7 The ramp of a loading dock measures 12 feet and has a height of 3 feet. What is the angle of elevation?

Make a drawing.



Let x represent $m\angle BAC$.

$$\sin x^\circ = \frac{BC}{AB} \quad \sin x = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\sin x^\circ = \frac{3}{12} \quad BC = 3 \text{ and } AB = 12$$

$$x = \sin^{-1}\left(\frac{3}{12}\right) \quad \text{Find the inverse.}$$

$$x \approx 14.5 \quad \text{Use a calculator.}$$

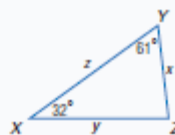
The angle of elevation for the ramp is about 14.5° .

8-6 The Law of Sines (pp. 471–477)

Find each measure using the given measures of $\triangle FGH$. Round angle measures to the nearest degree and side measures to the nearest tenth.

31. Find f if $g = 16$, $m\angle G = 48$, and $m\angle F = 82$. **21.3**
32. Find $m\angle H$ if $h = 10.5$, $g = 13$, and $m\angle G = 65$. **47**
33. **GARDENING** Elena is planning a triangular garden. She wants to build a fence around the garden to keep out the deer. The length of one side of the garden is 26 feet. If the angles at the end of this side are 78° and 44° , find the length of fence needed to enclose the garden. **≈ 77.3 ft**

Example 8 Find x if $y = 15$. Round to the nearest tenth.



To find x and z , use proportions involving $\sin Y$ and y .

$$\frac{\sin Y}{y} = \frac{\sin X}{x} \quad \text{Law of Sines}$$

$$\frac{\sin 61^\circ}{15} = \frac{\sin 32^\circ}{x} \quad \text{Substitute.}$$

$$x \sin 61^\circ = 15 \sin 32^\circ \quad \text{Cross Products}$$

$$x = \frac{15 \sin 32^\circ}{\sin 61^\circ} \quad \text{Divide.}$$

$$x \approx 9.1 \quad \text{Use a calculator.}$$

The Law of Cosines (pp. 479–485)

In $\triangle XYZ$, given the following measures, find the measures of the missing side.

34. $x = 7.6$, $y = 5.4$, $m\angle Z = 51$ **5.9**

35. $x = 21$, $m\angle Y = 73$, $z = 16$ **22.4**

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

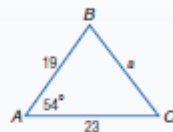
$a \approx 17.0$, $m\angle B \approx 44$, $m\angle C \approx 72$

36. $c = 18$, $b = 13$, $m\angle A = 64$

37. $b = 5.2$, $m\angle C = 53$, $c = 6.7$
 $m\angle B \approx 38$, $m\angle A \approx 89$, $a \approx 8.4$

38. **ART** Adelina is creating a piece of art that is in the shape of a parallelogram. Its dimensions are 35 inches by 28 inches and one angle is 80° . Find the lengths of both diagonals.
 ≈ 40.85 in.; ≈ 48.47 in.

Example 9 Find a .



$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{Law of Cosines}$$

$$a^2 = 23^2 + 19^2 - 2(23)(19) \cos 54^\circ \quad b = 23, \\ c = 19, \text{ and } m\angle A = 54$$

$$a^2 = 890 - 874 \cos 54^\circ \quad \text{Simplify.}$$

$$a = \sqrt{890 - 874 \cos 54^\circ} \quad \text{Take the square root of each side.}$$

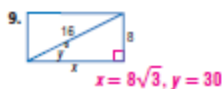
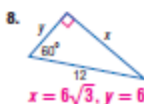
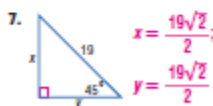
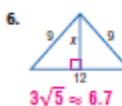
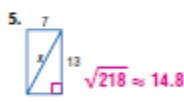
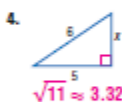
$$a \approx 19.4 \quad \text{Use a calculator.}$$

8 Practice Test

Find the geometric mean between each pair of numbers.

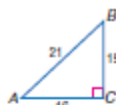
1. 7 and 63 2. 6 and 24 3. 10 and 50
 21 12 $10\sqrt{5}$

Find the missing measures.



Use the figure to find each trigonometric ratio. Express answers as a fraction.

10. $\cos B$ 10. $\frac{5}{7}$
 11. $\tan A$ 11. $\frac{15}{16}$
 12. $\sin A$ 12. $\frac{5}{7}$



Find each measure using the given measures from $\triangle FGH$. Round to the nearest tenth.

13. Find g if $m\angle F = 59$, $f = 13$, and $m\angle G = 71$. **14.3**
 14. Find $m\angle H$ if $m\angle F = 52$, $f = 10$, and $h = 12.5$. **80.1**
 15. Find f if $g = 15$, $h = 13$, and $m\angle F = 48$. **11.5**
 16. Find h if $f = 13.7$, $g = 16.8$, and $m\angle H = 71$. **17.9**

Solve each triangle. Round each angle measure to the nearest degree and each side measure to the nearest tenth.

17. $a = 15$, $b = 17$, $m\angle C = 45$
 18. $a = 12.2$, $b = 10.9$, $m\angle B = 48$
 19. $a = 19$, $b = 23.2$, $c = 21$
 17. $m\angle A \approx 59$, $m\angle B \approx 76$, $c \approx 12.4$
 18. $m\angle A \approx 56$, $m\angle C \approx 76$, $c \approx 14.2$
 19. $m\angle A \approx 51$, $m\angle B \approx 72$, $m\angle C \approx 57$

20. **TRAVEL** From an airplane, Janara looked down to see a city. If she looked down at an angle of 9° and the airplane was half a mile above the ground, what was the horizontal distance to the city? **3.2 mi**

21. **CIVIL ENGINEERING** A section of freeway has a steady incline of 10° . If the horizontal distance from the beginning of the incline to the end is 5 miles, how high does the incline reach? **0.9 mi**

22. **MULTIPLE CHOICE** Find $\tan X$. **D**



- A $\frac{5}{12}$ C $\frac{17}{12}$
 B $\frac{12}{13}$ D $\frac{12}{5}$

23. **COMMUNICATIONS** To secure a 500-foot radio tower against high winds, guy wires are attached to the tower 5 feet from the top. The wires form a 15° angle with the tower. Find the distance from the centerline of the tower to the anchor point of the wires. **132.6 ft**

24. Solve $\triangle DEF$.

$m\angle D \approx 41$,
 $m\angle E \approx 57$,
 $e \approx 10.2$



25. **MULTIPLE CHOICE** The top of the Boone Island Lighthouse in Boone Island, Maine, is 137 feet above sea level. The angle of depression from the light on the top of the tower to a passing ferry is 37° . How many feet from the foot of the lighthouse is the ferry? **F**

- F 181.8 ft H 109.4 ft
 G 171.5 ft J 103.2 ft