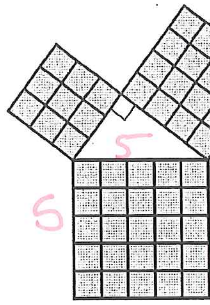
Acc Geometry

8.2 Pythagorean Theorem Notes

Pythagorean Theorem: If a triangle is a right triangle, then the sum of the squares of the legs is equal to the square of the hypotenuse.

Proofs of the Pythagorean Theorem:
Euclidian's Visual Proof

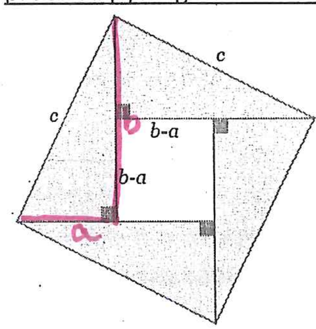
$\boxed{s^2 = 25}$



$3^2 + 4 \cdot 4$
$9 + 16$
$\boxed{= 25}$

$\therefore a^2 + b^2 = c^2$

2. The Chinese or Indian Proof: Bhaskara's proof is also a dissection proof. It is similar to the visual proof provided by Euclid/ Bhaskara was born in India. He was one of the most important Hindu mathematicians of the second century AD. It is commonly disputed whether he or the Chinese Zhou Bi Suan Jing developed this proof. It is known that the Chinese did not complete the proof the way we can algebraically prove it today. Both visuals are very similar. Bhaskara used the following diagrams in proving the Pythagorean Theorem. Using the diagram, write three expressions that represent the area of the large square, the area of the four triangles, and the area of the small square. Set the areas of the large square equal to the sum of the four triangles and the small square. Algebraically manipulate the equation to show the Pythagorean Theorem holds.
http://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-geometry/cc-8th-pythagorean-proofs/v/bhaskara-s-proof-of-pythagorean-theorem-avi



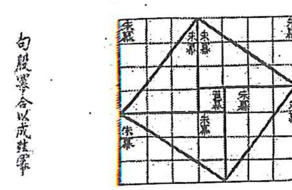
Area of Big Square = Area of 4 △s + Small Square

$C \cdot C = 4\left(\frac{1}{2} a \cdot b\right) + (b-a)(b-a)$

$c^2 = \underline{2 \cdot ab} + b^2 - \underline{2ab} + a^2$

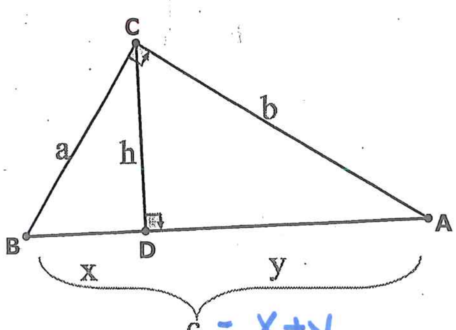
CLT

$c^2 = a^2 + b^2$ ✓

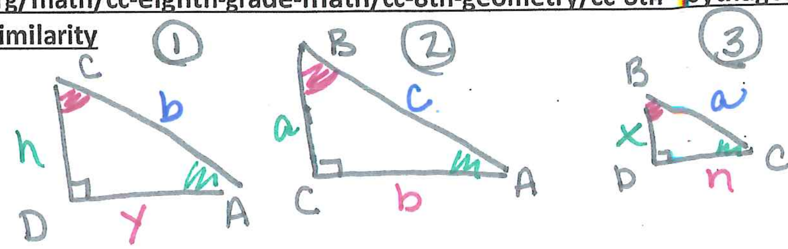


Bhaskara's Second Proof of the Pythagorean Theorem
Watch it online! http://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-geometry/cc-8th-pythagorean-proofs/v/pythagorean-theorem-proof-using-similarity

3. Similarity Proof:
Given △ABC with right angle C
Prove: $a^2 + b^2 = c^2$



$c = x + y$



$\triangle ABC \sim \triangle CBD \sim \triangle ACD$

$\frac{\triangle 2}{\triangle 3}$ $\quad \frac{c}{a} = \frac{a}{x}$ $\quad \frac{\triangle 1}{\triangle 2}$ $\quad \frac{b}{c} = \frac{y}{b}$

$cx = a^2$ $\quad cy = b^2$

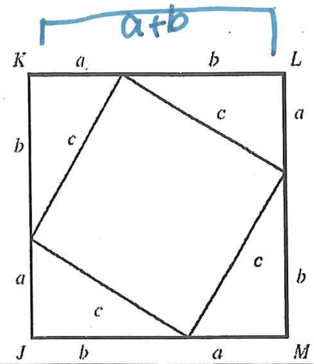
$c = x + y$

$a^2 + b^2 = cx + cy$
$a^2 + b^2 = c(x+y)$

$a^2 + b^2 = c \cdot c$

$\boxed{\therefore a^2 + b^2 = c^2}$

4. Euclid's Pythagorean Theorem Algebraic Proof. Using the diagram, write two different expressions that represent the area of the large square and then set them equal to each other. Algebraically manipulate the equation to show the Pythagorean Theorem.



little □ + 4 Δs = Big □

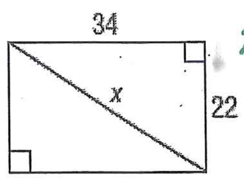
$c \cdot c + 4 \frac{1}{2} a \cdot b = (a+b)(a+b)$

$c^2 + 2ab = a^2 + 2ab + b^2$

$c^2 = a^2 + b^2$ ✓

Practice using the Pythagorean Theorem: Then find area of figure

1.



$22^2 + 34^2 = x^2$

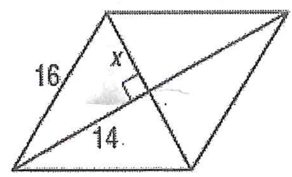
$1640 = x^2$

$\boxed{2\sqrt{410} = x}$

$A = 22 \cdot 34$

$\boxed{A = 748 \text{ units}^2}$

2.



Find x

$x^2 + 14^2 = 16^2$

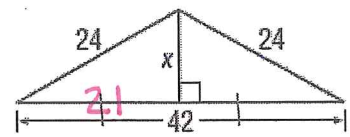
$x^2 = 60$

$\boxed{x = 2\sqrt{15}}$

Area = 4 Δs

$= 4 \frac{1}{2} 14 \cdot 2\sqrt{15}$ $\boxed{A = 2016\sqrt{15} u^2}$

3.



Find x

$x^2 + 21^2 = 24^2$

$x^2 = 135$

$\boxed{x = 3\sqrt{15}}$

$A = \frac{1}{2} 42 \cdot 3\sqrt{15}$

$\boxed{A = 63\sqrt{15} u^2}$

Converse of the Pythagorean Theorem: If the sum of the squares of the two smaller sides is equal to the square of the longest side, then the triangle is a right triangle. Pythagorean Triples are whole numbers which follows the converse of the Pythagorean Theorem. a^2+b^2

If sum > 3rd side ⇒ acute

If sum = 3rd side ⇒ Right

If sum < 3rd side ⇒ obtuse

Given the following lengths, do they form right triangles? Are they Pythagorean Triples?

Ex.1) 9,40,41 $9^2 + 40^2 = 1681$ $41^2 = 1681$

$a^2 + b^2 = c^2$ ∴ Right Δ

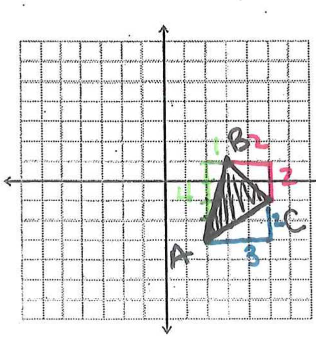
Yes 9,40,41 is a pyth. triple!

Ex2.) 6,7,8 $6^2 + 7^2 = 85$ $8^2 = 64$

$85 > 64$ ∴ Acute

Ex3.) Determine if ΔABC is a right triangle when A(2,-3), B(3,1), and C(5,-1). Explain.

acute or obtuse?



$2^2 + 2^2 = BC^2$

$\boxed{2\sqrt{2} = BC}$

$2^2 + 3^2 = AC^2$

$\boxed{\sqrt{13} = AC}$

$4^2 + 1^2 = AB^2$

$\boxed{\sqrt{17} = AB}$

Check:

$AC^2 + BC^2 =$

$= (\sqrt{13})^2 + (2\sqrt{2})^2$

$= 21$

AB^2

$= \sqrt{17}^2$

$= 17$

$\boxed{21 > 17 \therefore \text{acute}}$