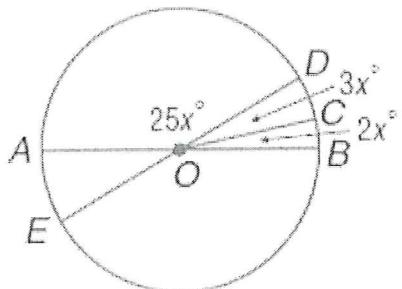


# 10.2 and 10.4 Angles and Arcs

Review Example 1: Find the  $m\angle AOD$ .



$$\angle AOD + \angle DOC + \angle COB = 180$$

$$25x + 3x + 2x = 180$$

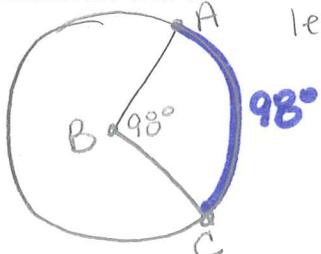
$$30x = 180$$

$$x = 6$$

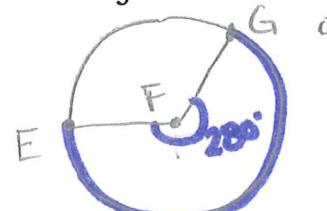
$$\begin{aligned} \angle AOD &= 25x \\ &= 25(6) \\ &= 150^\circ \end{aligned}$$

## Types of Arcs

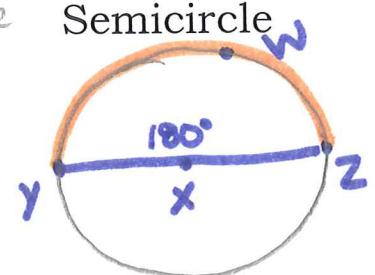
Minor Arc - arc w/ measure less than  $180^\circ$



Major Arc - arc w/ measure of more than  $180^\circ$



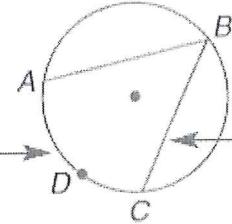
Semicircle



## Inscribed Angles

An inscribed angle is an angle that **has the vertex on the circle and its sides are chords**

$\widehat{ADC}$  is the arc intercepted by  $\angle ABC$ .



Example 1: In Circle O,  $m\widehat{AB} = 140$ ,  $m\widehat{BC} = 100$ ,  $m\widehat{AD} = m\widehat{DC}$ . Find the measures of  $\angle 1$ ,  $\angle 2$ ,  $\angle 3$ ,  $\angle 4$ , and  $\angle 5$ .

$$m\widehat{AB} + m\widehat{BC} + m\widehat{AD} + m\widehat{DC} = 360^\circ$$

$$140 + 100 + x + x = 360$$

$$240 + 2x = 360$$

$$\begin{aligned} 2x &= 120 \\ x &= 60 \end{aligned}$$

$$\angle 1 = \frac{1}{2} m \widehat{AD}$$

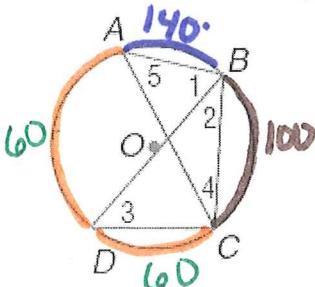
$$\angle 1 = \frac{1}{2} (60)$$

$$\boxed{\angle 1 = 30^\circ}$$

$$\angle 2 = \frac{1}{2} m \widehat{DC}$$

$$\angle 2 = \frac{1}{2} (60)$$

$$\boxed{\angle 2 = 30^\circ}$$



$$\angle 5 = \frac{1}{2} m \widehat{BC}$$

$$\angle 5 = \frac{1}{2} (100)$$

$$\boxed{\angle 5 = 50^\circ}$$

$$\angle 4 = \frac{1}{2} m \widehat{AB}$$

$$\angle 4 = \frac{1}{2} (140)$$

$$\boxed{\angle 4 = 70^\circ}$$

$$\angle 3 = \frac{1}{2} \widehat{BC}$$

$$\angle 3 = \frac{1}{2} (100)$$

$$\boxed{\angle 3 = 50^\circ}$$

What did you notice about  $\angle 3$  and  $\angle 5$ ? Intercept the same arc • are  $\cong$

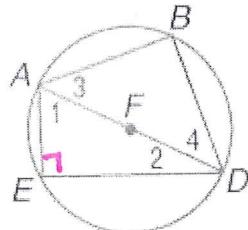
Why? This is because if two inscribed angles of a circle intercept

congruent arcs or the same arc, then the angles are  $\cong$ .

### Angles of Inscribed Polygons

If the inscribed angle of a **triangle** intercepts a **semicircle**, the angle is a **right** angle.

Example 2: Triangles ABD and ADE are inscribed in Circle F with  $\widehat{AB} \cong \widehat{BD}$ . Find the measures of  $\angle 1$  and  $\angle 2$  if  $m\angle 1 = 12x - 8$  and  $m\angle 2 = 3x + 8$ .

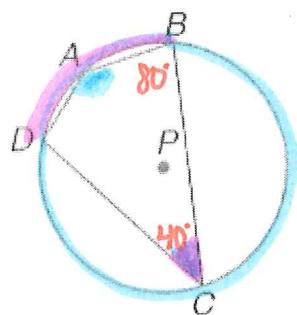


$$\begin{aligned} m\angle 1 + m\angle 2 + 90^\circ &= 180^\circ \\ 12x - 8 + 3x + 8 + 90 &= 180 \\ 15x + 90 &= 180 \\ 15x &= 90 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} m\angle 1 &= 12(6) - 8 \\ m\angle 1 &= 64^\circ \end{aligned}$$

$$\begin{aligned} m\angle 2 &= 3(6) + 8 \\ m\angle 2 &= 26^\circ \end{aligned}$$

Example 3: Quadrilateral ABCD is inscribed in Circle P. If  $m\angle B = 80$  and  $m\angle C = 40$ , find  $m\angle A$  and  $m\angle D$ .



$$\begin{aligned} m\widehat{DAB} &= 2(m\angle C) \\ m\widehat{DAB} &= 80^\circ \\ m\widehat{BCD} + m\widehat{DAB} &= 360^\circ \\ x + 80 &= 360 \\ x &= 280^\circ \\ m\widehat{BCD} &= 2(m\angle A) \\ 280 &= 2(m\angle A) \\ 140^\circ &= m\angle A \end{aligned}$$

$$\begin{aligned} m\widehat{ADC} &= 2(m\angle B) \\ m\widehat{ADC} &= 2(80) = 160^\circ \end{aligned}$$

$$\begin{aligned} m\widehat{ABC} + m\widehat{ADC} &= 360^\circ \\ x + 160 &= 360 \\ x &= 200 \end{aligned}$$

$$\begin{aligned} m\widehat{ABC} &= 2(m\angle D) \\ 200 &= 2(m\angle D) \\ m\angle D &= 100^\circ \end{aligned}$$

If a quadrilateral is inscribed in a circle, then its opposite angles

are supplementary