

ARCS AND CHORDS NOTES (10.3)

THEOREM 10.2

In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Abbreviations:

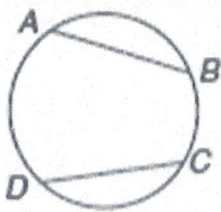
In \odot , 2 minor arcs are \cong , corr. chords are \cong .

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Examples:

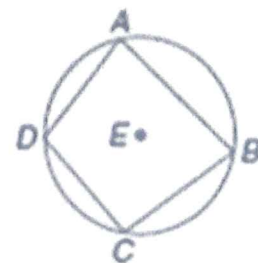
If $\overline{AB} \cong \overline{CD}$,
 $\widehat{AB} \cong \widehat{CD}$.

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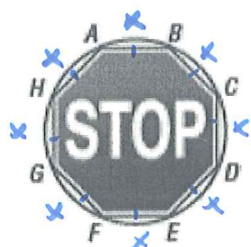
The chords of adjacent arcs can form a poly gon.

Quadrilateral ABCD is an inscribed polygon because all of its vertices lie on the circle.



Circle E is circumscribed about the polygon because it contains all the vertices of the polygon.

Let's see some examples:

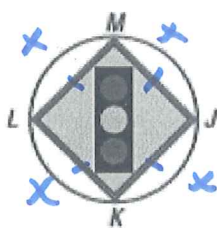


$$8x = 360$$

$$x = \frac{360}{8}$$

$$x = 45^\circ$$

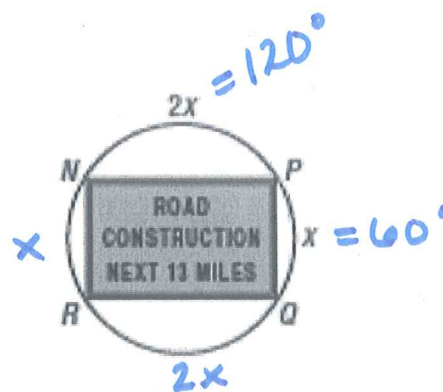
square



$$4x = 360$$

$$x = \frac{360}{4}$$

$$x = 90^\circ$$



$$2x + x + 2x + x = 360$$

$$6x = 360$$

$$x = 60^\circ$$

Key Concept!

Theorem:

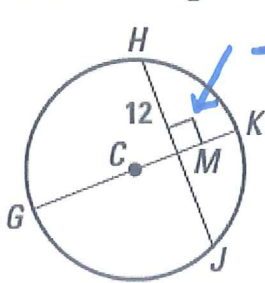
In a circle, if a diameter (or radius) is perpendicular to a chord, then it BISECTS the chord and its arc.

With your shoulder partner, define the word bisect and write it down: to cut

a segment into 2 \cong parts.

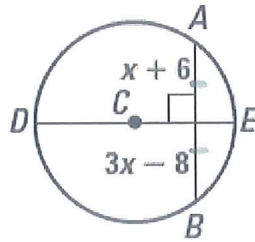
1. Find the length of \overline{JM} .

2. Find x .



\perp to radius or diameter!
So Chord HJ is bisected.

$$\overline{JM} = 12$$

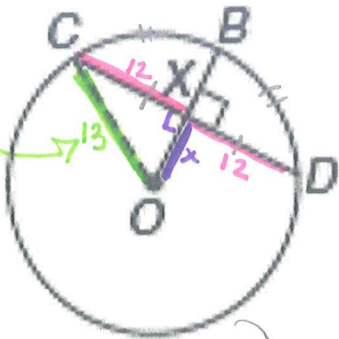


$$\begin{aligned} x+6 &= 3x-8 \\ -x & \quad -x \\ 6 &= 2x-8 \\ +8 & \quad +8 \end{aligned}$$

$$\begin{aligned} 14 &= 2x \\ \boxed{7} &= x \end{aligned}$$

Example 3: Given the information below, find CX, OX, XB, and the $m\widehat{CD}$.

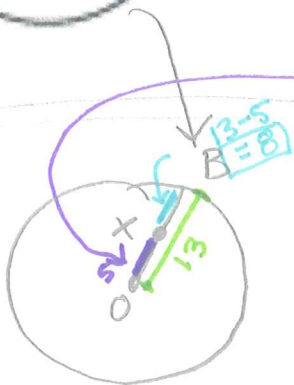
Circle O has a radius of 13 inches. Radius \overline{OB} is perpendicular to chord \overline{CD} , which is 24 inches long.



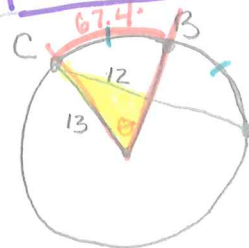
$$CX = \frac{1}{2} 24 = \boxed{12}$$

$$\begin{aligned} OX &= \begin{array}{c} 12 \\ \triangle \\ 13 \end{array} \quad \begin{aligned} x^2 + 12^2 &= 13^2 \\ x^2 + 144 &= 169 \\ x^2 &= 25 \end{aligned} \end{aligned}$$

$$\boxed{OX = 5}$$



$$\boxed{XB = 8}$$



The measure of the arc is the same as the central \angle .

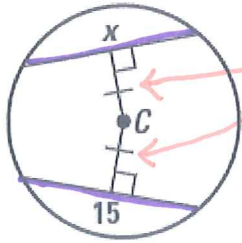
$$\begin{aligned} m\widehat{CD} &= \\ 67.4 + 67.4 & \\ \boxed{134.8^\circ} & \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{12}{13} \\ \theta &= \sin^{-1}\left(\frac{12}{13}\right) \theta = 67.4^\circ \end{aligned}$$

In a circle or in congruent circles, two chords are congruent if and only if they are

equidistant from the center.

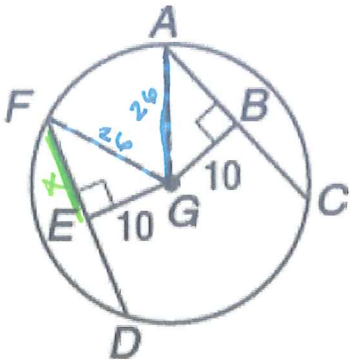
Example 4: Find x .



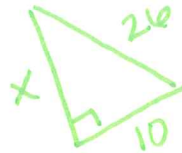
equidistant from the center so the chords are \cong

$$x = 15$$

Example 5: Chords \overline{AC} and \overline{DF} are equidistant from the center. If the radius of Circle G is 26m, find FE, DE, AB and AC.



FE =



$$x^2 + 10^2 = 26^2$$

$$x^2 + 100 = 676$$

$$x^2 = 576$$

$$x = 24$$

$$FE = 24$$

$$DE = 24$$

$$AB = 24$$

$$AC = 2 \cdot 24$$

$$AC = 48$$