10-5 Lesson Notes

1 Focus

Vertical Alignment

Before Lesson 10-5
Make conjectures about circles.

Lesson 10-5
Formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models.

After Lesson 10-5
Find areas of circles.

2 Teach

Scaffolding Questions

Have students observe the picture and read the adjacent caption in *Get Ready for the Lesson.*

Ask:

- What happens to the hammer when the athlete releases it? The hammer flies in a straight line.
- What geometric term represents the location of the release? point of tangency

(continued on the next page)

10-5 Tangents

Main Ideas

- Use properties of tangents.
- Solve problems involving circumscribed polygons.

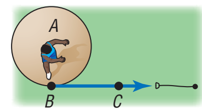
New Vocabulary

tangent
point of tangency

GET READY for the Lesson

In April 2004, Yipsi Moreno of Cuba set the hammer throw record for North America, Central America, and the Carribean with a throw of 75.18 meters in La Habana, Cuba. The hammer is a metal ball, usually weighing 16 pounds, attached to a steel wire at the end of which is a grip. The ball is spun around by the thrower and then released, with the greatest distance thrown winning the event.

Tangents The figure at the right models the hammer throw event. Circle A represents the circular area containing the spinning thrower. Ray BC represents the path the hammer takes when released. $\overrightarrow{BC}$ is **tangent** to $\odot A$, because the line containing $\overrightarrow{BC}$ intersects the circle in exactly one point. This point is called the **point of tangency**.

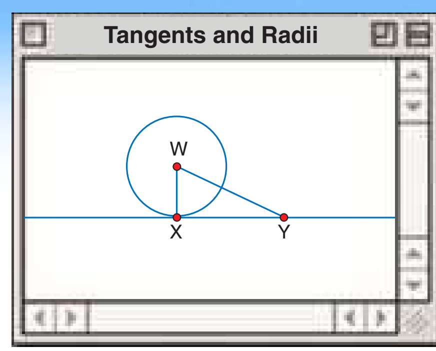


GEOMETRY SOFTWARE LAB

Tangents and Radii

MODEL

- Use The Geometer's Sketchpad to draw a circle with center W. Then draw a segment tangent to $\odot W$. Label the point of tangency as X.
- Choose another point on the tangent and name it Y. Draw $\overline{WY}$.



THINK AND DISCUSS 5. See margin.

1. What is $\overline{WX}$ in relation to the circle? radius
2. Measure $\overline{WY}$ and $\overline{WX}$. Write a statement to relate WX and WY. $WX < WY$
3. Move point Y. How does the location of Y affect the statement you wrote in Exercise 2? 3. It doesn't, unless Y and X coincide.
4. Measure $\angle WXY$. What conclusion can you make? $\overline{WX} \perp \overline{WY}$
5. **Make a conjecture** about the shortest distance from the center of the circle to a tangent of the circle.

Lesson 10-5 Resources

Chapter 10 Resource Masters

Lesson Reading Guide, p. 33 BL OL
Study Guide and Intervention, pp. 34–35 BL OL
Skills Practice, p. 36 BL OL
Practice, p. 37 OL AL
Word Problem Practice, p. 38 BL OL AL
Enrichment, p. 39 OL AL
Cabri Jr, p. 40 OL AL
Geometer's Sketchpad, p. 41 BL OL AL

Transparencies

5-Minute Check Transparency 10-5

Additional Print Resources

Noteables™ Interactive Study Notebook with Foldables™
Teaching Geometry with Manipulatives

Technology

geometryonline.com
Interactive Classroom CD-ROM
AssignmentWorks CD-ROM
Graphing Calculator Easy Files

The lab suggests that the shortest distance from a tangent to the center of a circle is the radius drawn to the point of tangency. Since the shortest distance from a point to a line is a perpendicular, the radius and the tangent must be perpendicular.

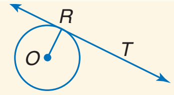
Study Tip

Tangent Lines
All of the theorems applying to tangent lines also apply to parts of the line that are tangent to the circle.

THEOREM 10.9

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

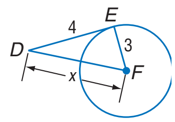
Example: If $\overleftrightarrow{RT}$ is a tangent, $\overline{OR} \perp \overleftrightarrow{RT}$.



You will prove Theorem 10.9 in Exercise 24.

EXAMPLE 1 Find Lengths

ALGEBRA $\overline{ED}$ **is tangent to ⊙*F* at point *E*. Find *x*.**



Because the radius is perpendicular to the tangent at the point of tangency, $\overline{EF} \perp \overline{DE}$. This makes $\angle DEF$ a right angle and $\triangle DEF$ a right triangle. Use the Pythagorean Theorem to find x.

$(EF)^2 + (DE)^2 = (DF)^2$ Pythagorean Theorem

$3^2 + 4^2 = x^2$ $EF = 3$, $DE = 4$, $DF = x$

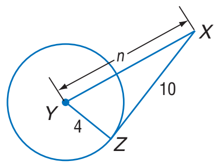
$25 = x^2$ Simplify.

$\pm 5 = x$ Take the square root of each side.

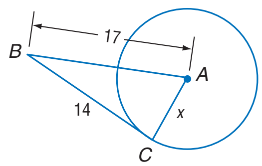
Because x is the length of $\overline{DF}$, ignore the negative result. Thus, $x = 5$.

CHECK Your Progress

1A. $\overline{XZ}$ is tangent to ⊙Y at point Z. Find n. $2\sqrt{29} \approx 10.77$



1B. $\overline{BC}$ is tangent to ⊙A at point C. Find x. $\sqrt{93} \approx 9.64$



The converse of Theorem 10.9 is also true.

- What other situations can be modeled by a circle and a tangent? Sample answers: fishing line unrolling from a spool, the string of a yo-yo, a line of paint being applied to a wall by a roller

Tangents

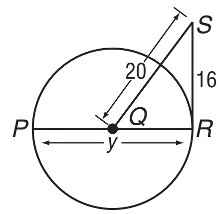
Examples 1–3 show how to use the theorems of tangents to solve problems involving tangents.

Formative Assessment

Use the Check Your Progress exercises after each example to determine students' understanding of concepts.

ADDITIONAL EXAMPLE

1 **ALGEBRA** $\overline{RS}$ is tangent to ⊙Q at point R. Find y. 24



Additional Examples also in:

- Noteables™ Interactive Study Notebook with Foldables™
- Interactive Classroom PowerPoint® Presentations

Geometry Lab

Tell students that since a radius is perpendicular to a tangent at the point of tangency, the diameter containing that radius is also perpendicular to the tangent at the same point. Students can also repeat the activity for other points of tangency. They can start with a new circle, or you can ask students where they could place another tangent on ⊙W that is perpendicular to $\overline{WY}$.

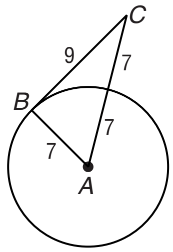
Additional Answer (Geometry Lab)

5. Sample answer: The shortest distance from the center of a circle to the tangent is the radius of the circle, which is perpendicular to the tangent.

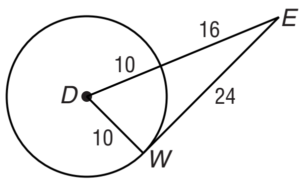
Lesson 10-5 Tangents **589**

ADDITIONAL EXAMPLE

2 **a.** Determine whether $\overline{BC}$ is tangent to $\odot A$. no



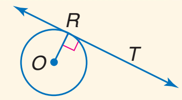
b. Determine whether $\overline{WE}$ is tangent to $\odot D$. yes



THEOREM 10.10

If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

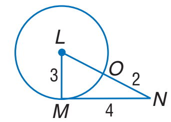
Example: If $\overline{OR} \perp \overleftrightarrow{RT}$, $\overleftrightarrow{RT}$ is a tangent.



You will prove this theorem in Exercise 25.

EXAMPLE Identify Tangents

a. Determine whether $\overline{MN}$ is tangent to $\odot L$. Justify your reasoning.



First determine whether $\triangle LMN$ is a right triangle by using the converse of the Pythagorean Theorem.

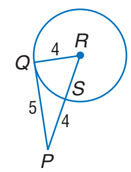
$(LM)^2 + (MN)^2 \stackrel{?}{=} (LN)^2$ Converse of Pythagorean Theorem

$3^2 + 4^2 \stackrel{?}{=} 5^2$ $LM = 3$, $MN = 4$, $LN = 3 + 2$ or 5

$25 = 25$ ✓ Simplify.

Because $3^2 + 4^2 = 5^2$, the converse of the Pythagorean Theorem allows us to conclude that $\triangle LMN$ is a right triangle and $\angle LMN$ is a right angle. Thus, $\overline{LM} \perp \overline{MN}$, making $\overline{MN}$ a tangent to $\odot L$.

b. Determine whether $\overline{PQ}$ is tangent to $\odot R$. Justify your reasoning.



Since $RQ = RS$, $RP = 4 + 4$ or 8 units.

$(RQ)^2 + (PQ)^2 \stackrel{?}{=} (RP)^2$ Converse of Pythagorean Theorem

$4^2 + 5^2 \stackrel{?}{=} 8^2$ $RQ = 4$, $PQ = 5$, $RP = 8$

$41 \neq 64$ Simplify.

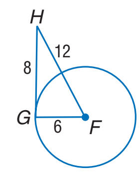
Because $RQ^2 + PQ^2 \neq RP^2$, $\triangle RQP$ is not a right triangle. So, $\overline{PQ}$ is not tangent to $\odot R$.

Study Tip

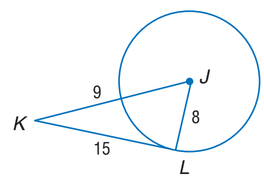
Identifying Tangents

Never assume that a segment is tangent to a circle by appearance unless told otherwise. The figure must either have a right angle symbol or include the measurements that confirm a right angle.

2A. Determine whether $\overline{GH}$ is tangent to $\odot F$. Justify your reasoning. no, $180 \neq 196$



2B. Determine whether $\overline{KL}$ is tangent to $\odot J$. Justify your reasoning. yes, $289 = 289$



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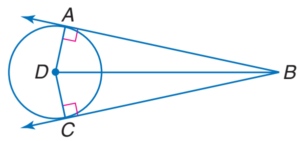
More than one line can be tangent to the same circle. In the figure, $\overline{AB}$ and $\overline{BC}$ are tangent to $\odot D$. So, $(AB)^2 + (AD)^2 = (DB)^2$ *and* $(BC)^2 + (CD)^2 = (DB)^2$.



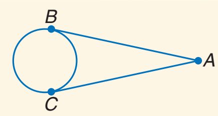
$(AB)^2 + (AD)^2 = (BC)^2 + (CD)^2$	Substitution
$(AB)^2 + (AD)^2 = (BC)^2 + (AD)^2$	$AD = CD$
$(AB)^2 = (BC)^2$	Subtract $(AD)^2$ from each side.
$AB = BC$	Take the square root of each side.

The last statement implies that $\overline{AB} \cong \overline{BC}$. This is a proof of Theorem 10.11.

THEOREM 10.11

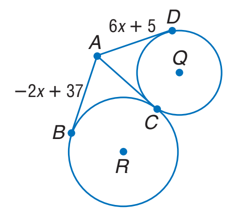
If two segments from the same exterior point are tangent to a circle, then they are congruent.

Example: $\overline{AB} \cong \overline{AC}$



EXAMPLE Congruent Tangents

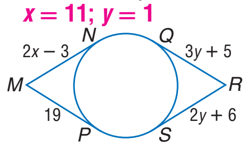
1 **ALGEBRA** **Find x. Assume that segments that appear tangent to circles are tangent.**



$\overline{AD}$ and $\overline{AC}$ are drawn from the same exterior point and are tangent to $\odot Q$, so $\overline{AD} \cong \overline{AC}$. $\overline{AC}$ and $\overline{AB}$ are drawn from the same exterior point and are tangent to $\odot R$, so $\overline{AC} \cong \overline{AB}$. By the Transitive Property, $\overline{AD} \cong \overline{AB}$.

$AD = AB$	Definition of congruent segments
$6x + 5 = -2x + 37$	Substitution
$8x + 5 = 37$	Add $2x$ to each side.
$8x = 32$	Subtract 5 from each side.
$x = 4$	Divide each side by 8.

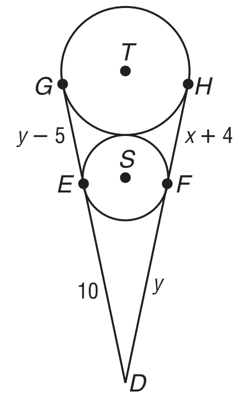
3. Find x and y.
$x = 11$; $y = 1$



In the construction that follows, you will learn how to construct a line tangent to a circle through a point exterior to the circle.

ADDITIONAL EXAMPLE

3 **ALGEBRA** Find x. Assume that segments that appear tangent to circles are tangent. **1**



Focus on Mathematical Content

Explain that even though a tangent intersects a circle, there is never any part of a tangent contained inside a circle. The only point that the tangent and the circle have in common is the point of tangency.

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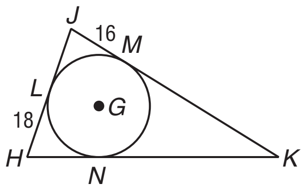
Circumscribed Polygons

Polygons can also be circumscribed about a circle. **Example 4** shows how to find the perimeter of a triangle using theorems learned in this lesson.

ADDITIONAL EXAMPLE

4 Triangle *HJK* is circumscribed about ⊙*G*. Find the perimeter of △*HJK* if $NK = JL + 29$.

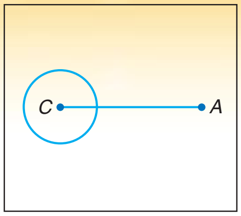


158 units

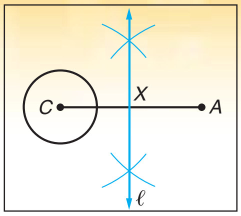
CONSTRUCTION

Line Tangent to a Circle Through a Point Exterior to the Circle

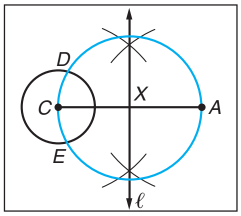
Step 1 Construct a circle. Label the center *C*. Draw a point outside ⊙*C*. Then draw $\overline{CA}$.



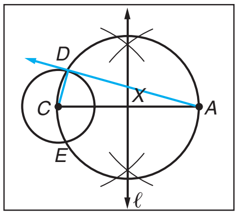
Step 2 Construct the perpendicular bisector of $\overline{CA}$ and label it line ℓ. Label the intersection of ℓ and $\overline{CA}$ as point *X*.



Step 3 Construct circle *X* with radius $\overline{XC}$. Label the points where the circles intersect as *D* and *E*.

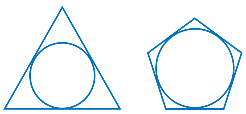


Step 4 Draw $\overleftrightarrow{AD}$. △*ADC* is inscribed in a semicircle. So ∠*ADC* is a right angle, and $\overleftrightarrow{AD}$ is a tangent.

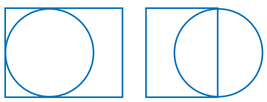


You will construct a line tangent to a circle through a point on the circle in Exercise 23.

Circumscribed Polygons In Lesson 10-3, you learned that circles can be circumscribed about a polygon. Likewise, polygons can be circumscribed about a circle, or the circle is inscribed in the polygon. Notice that the vertices of the polygon *do not* lie on the circle, but every side of the polygon is tangent to the circle.



Polygons are circumscribed.



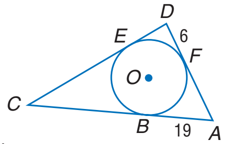
Polygons are *not* circumscribed.

Study Tip

Common Misconceptions
Just because the circle is tangent to one or more of the sides of a polygon does not mean that the polygon is circumscribed about the circle, as shown in the second pair of figures.

EXAMPLE 4 Triangles Circumscribed About a Circle

Triangle *ADC* is circumscribed about ⊙*O*. Find the perimeter of △*ADC* if $EC = DE + AF$.



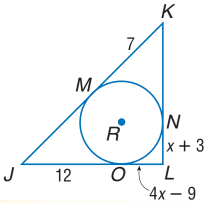
Use Theorem 10.10 to determine the equal measures:
$AB = AF = 19$, $FD = DE = 6$, and $EC = CB$.
We are given that $EC = DE + AF$, so $EC = 6 + 19$ or 25.

$P = AB + BC + EC + DE + FD + AF$ Definition of perimeter

$= 19 + 25 + 25 + 6 + 6 + 19$ or 100 Substitution

The perimeter of △*ADC* is 100 units.

4. Triangle *JKL* is circumscribed about ⊙*R*. Find *x* and the perimeter of △*JKL*. 4; 52 units



Interactive Lab
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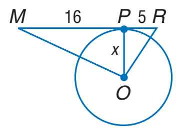
★ indicates multi-step problem

Check Your Understanding

Examples 1 and 2 (pp. 589–590)

For Exercises 1 and 2, use the figure at the right.

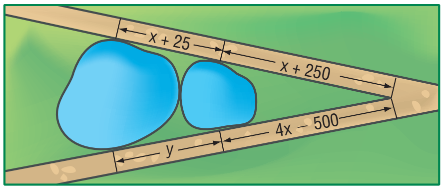


1. Tangent $\overline{MP}$ is drawn to ⊙O. Find x if $MO = 20$. **12**

2. If $RO = 13$, determine whether $\overline{PR}$ is tangent to ⊙O. **Yes; $5^2 + 12^2 = 13^2$.**

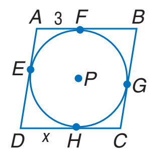
Example 3 (p. 591)

★ **3. LANDSCAPE ARCHITECT** A landscape architect is planning to pave two walking paths beside two ponds, as shown. Find the values of x and y. What is the total length of the walking paths? **$x = 250$; $y = 275$; 1550 units**



Example 4 (p. 592)

4. Rhombus $ABCD$ is circumscribed about ⊙P and has a perimeter of 32 units. Find x. **5**



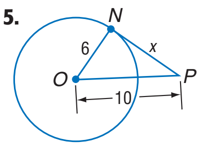
Exercises

HOMEWORK HELP For Exercises	See Examples
5–10	1
11–14	2
15–16	3
17–22	4

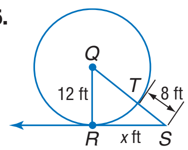
Exercise Levels
A: 5–22
B: 23–31
C: 32–36

Find x. Assume that segments that appear to be tangent are tangent.

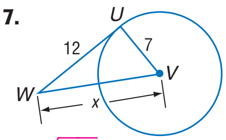
5. **8**



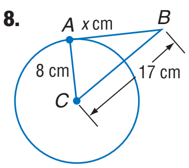
6. **16**



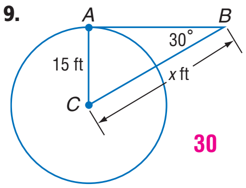
7. **$\sqrt{193}$**



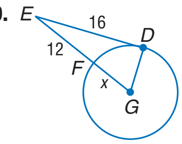
8. **15**



9. **30**

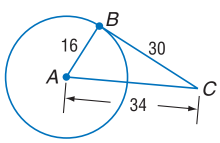


10. **$4\frac{2}{3}$**

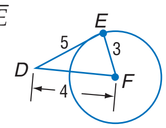


Determine whether each segment is tangent to the given circle.

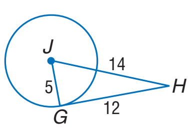
11. $\overline{BC}$ **yes**



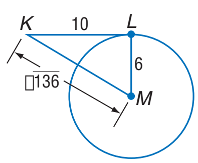
12. $\overline{DE}$ **no**



13. $\overline{GH}$ **no**



14. $\overline{KL}$ **yes**



3 Practice

Formative Assessment

Use Exercises 1–4 to check for understanding.

Then use the chart at the bottom of this page to customize assignments for your students.

Odd/Even Assignments

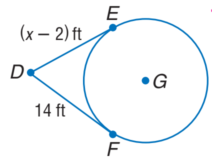
Exercises 5–22 are structured so that students practice the same concepts whether they are assigned odd or even problems.

DIFFERENTIATED HOMEWORK OPTIONS

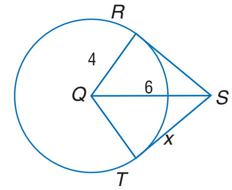
Level	Assignment	Two-Day Option	
BL Basic	5–22, 32–33, 35–46	5–21, 37–38	6–22 even, 32–33, 35–36, 39–46
OL Core	5–21 odd, 23–33, 35–46	5–22, 37–38	23–33, 35–36, 39–46
AL Advanced /Pre-AP	23–42 (optional: 43–46)		

Find x. Assume that segments that appear to be tangent are tangent.

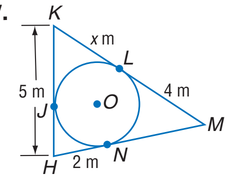
15. 16



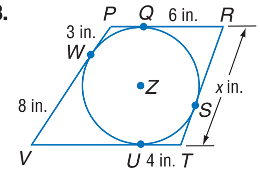
16. $\sqrt{20} \approx 4.47$



17. 3

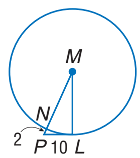


18. 10

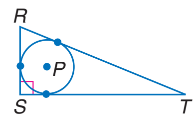


Find the perimeter of each polygon for the given information.

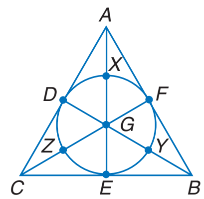
19. 60 units



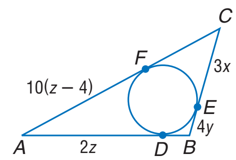
20. $ST = 18$, radius of $\odot P = 5$ 58.5 units



21. $BY = CZ = AX = 2$
radius of $\odot G = 3$ 24 units



22. $CF = 6(3 - x)$, $DB = 12y - 4$ 36 units



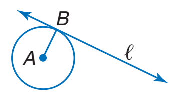
23. CONSTRUCTION Construct a line tangent to a circle through a point on the circle following these steps.

- Construct a circle with center T and locate a point P on $\odot T$ and draw $\overrightarrow{TP}$.
- Construct a perpendicular to $\overleftrightarrow{TP}$ through point T. See students' work.

Study Tip

Look Back
To review **constructing perpendiculars to a line**, see Lesson 3-6.

★ **24. PROOF** Write an indirect proof of Theorem 10.9 by assuming that ℓ is not perpendicular to $\overline{AB}$. See margin.
Given: ℓ is tangent to $\odot A$ at B, $\overline{AB}$ is a radius of $\odot A$.
Prove: Line ℓ is perpendicular to $\overline{AB}$.

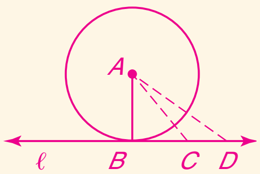


★ **25. PROOF** Write an indirect proof of Theorem 10.10 by assuming that ℓ is not tangent to $\odot A$. See margin.
Given: $\ell \perp \overline{AB}$, $\overline{AB}$ is a radius of $\odot A$.
Prove: Line ℓ is tangent to $\odot A$.

★ **26. PROOF** Write a two-column proof to show that if a quadrilateral is circumscribed about a circle, then the sum of the measures of the two opposite sides is equal to the sum of the measures of the two remaining sides. See margin.

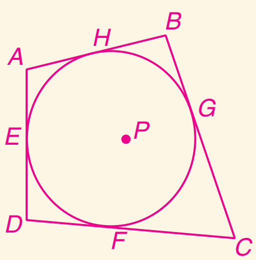
Additional Answers

24. Assume that ℓ is not perpendicular to $\overline{AB}$. If ℓ is not perpendicular to $\overline{AB}$, some other segment $\overline{AC}$ must be perpendicular to ℓ. Also, there is a point D on $\overrightarrow{BD}$ as shown in the diagram such that $\overline{CB} \cong \overline{CD}$. $\angle ACB$ and $\angle ACD$ are right angles by the definition of perpendicular. $\angle ACB \cong \angle ACD$ and $\overline{AC} \cong \overline{AC}$. $\triangle ACB \cong \triangle ACD$ by SAS, so $\overline{AB} \cong \overline{AD}$ by CPCTC. Thus, both B and D are on $\odot A$. For two points of ℓ to also be on $\odot A$ contradicts the given fact that ℓ is tangent to $\odot A$ at B. Therefore, $\ell \perp \overline{AB}$ must be true.



25. Proof: Assume ℓ is not tangent to circle A. Since ℓ intersects circle A at B, it must intersect the circle in another place. Call this point C. Then $AB = AC$. But if $\overline{AB} \perp \ell$, then $\overline{AB}$ must be the shortest segment from A to ℓ. If $AB = AC$, then $\overline{AC}$ is the shortest segment from A to ℓ. Since B and C are two different points on ℓ, this is a contradiction. Therefore, ℓ is tangent to circle A.

26. Given: Quadrilateral $ABCD$ is circumscribed about $\odot P$.
Prove: $AB + CD = AD + BC$



Proof:

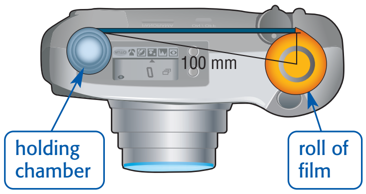
Statements (Reasons)

1. Quadrilateral $ABCD$ is circumscribed about circle P. (Given)
2. Sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are tangent to $\odot P$ at points H, G, F, and E, respectively. (Def. of circumscribed)
3. $\overline{EA} \cong \overline{AH}$; $\overline{HB} \cong \overline{BG}$, $\overline{GC} \cong \overline{CF}$, $\overline{FD} \cong \overline{DE}$. (Two segments tangent to a circle from the same exterior point are congruent.)
4. $AB = AH + HB$, $BC = BG + GC$, $CD = CF + FD$, $DA = DE + EA$ (Segment Addition)
5. $AB + CD = AH + HB + CF + FD$; $DA + BC = DE + EA + BG + GC$ (Substitution)
6. $AB + CD = AH + BG + GC + FD$; $DA + BC = FD + AH + BG + GC$ (Substitution)
7. $AB + CD = FD + AH + BG + GC$ (Commutative Prop of Add.)
8. $AB + CD = DA + BC$ (Substitution)

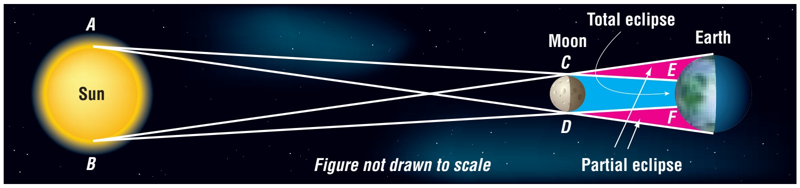
32a. Two; from any point outside the circle, you can draw only two tangents.

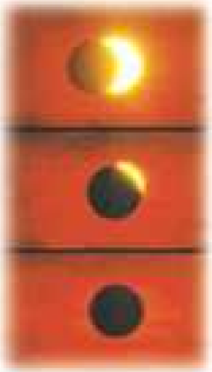
32b. None; a line containing a point inside the circle would intersect the circle in two points. A tangent can only intersect a circle in one point.

★ **27. PHOTOGRAPHY** The film in a 35-mm camera unrolls from a cylinder, travels across an opening for exposure, and then goes into another circular chamber as each photograph is taken. The roll of film has a diameter of 25 millimeters, and the distance from the center of the roll to the intake of the chamber is 100 millimeters. To the nearest millimeter, how much of the film would be exposed if the camera were opened before the roll had been used up? **99 mm**



ASTRONOMY For Exercises 28 and 29, use the following information.
A solar eclipse occurs when the Moon blocks the Sun's rays from hitting Earth. Some areas of the world will experience a total eclipse, others a partial eclipse, and some no eclipse at all, as shown in the diagram below.





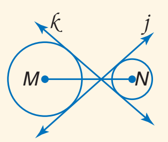
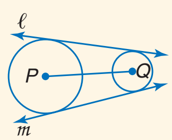
Real-World Link
During the 20th century, there were 78 total solar eclipses, but only 15 of these affected parts of the United States. The next total solar eclipse visible in the U.S. will be in 2017.

Source: *World Almanac*

28. The blue section denotes a total eclipse on that part of Earth. Which tangents define the blue area? **$\overline{AE}$ and $\overline{BF}$**

29. The pink section denotes the area that will have a partial eclipse. Which tangents define the northern and southern boundaries of the partial eclipse? **$\overline{AD}$ and $\overline{BC}$**

COMMON TANGENTS A line that is tangent to two circles in the same plane is called a *common tangent.*

Common internal tangents **intersect the segment connecting the centers.**	***Common external tangents*** **do not intersect the segment connecting the centers.**
 Lines k and j are common internal tangents.	 Lines ℓ and m are common external tangents.

EXTRA PRACTICE
See pages 820, 837.
Self-Check Quiz at geometryonline.com

Refer to the diagram of the eclipse above.

30. Name two common internal tangents. **$\overline{AD}$ and $\overline{BC}$**

31. Name two common external tangents. **$\overline{AE}$ and $\overline{BF}$ or $\overline{AC}$ and $\overline{BD}$**

H.O.T. Problems

32. REASONING Determine the number of tangents that can be drawn to a circle for each point. Explain your reasoning. **See margin.**

a. containing a point outside the circle

b. containing a point inside the circle

c. containing a point on the circle

33. OPEN ENDED Draw an example of a circumscribed polygon and an example of an inscribed polygon, and give real-life examples of each. **See Ch. 10 Answer Appendix.**

Study Guide and Intervention
CRM pp. 34–35 BL OL ELL

10-5 Study Guide and Intervention *(continued)*

Tangents

Tangents A tangent to a circle intersects the circle in exactly one point, called the **point of tangency**. There are three important relationships involving tangents.

- If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.
- If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent to the circle.
- If two segments from the same exterior point are tangent to a circle, then they are congruent.

$\overline{RP} \perp \overline{SR}$ if and only if $\overline{SR}$ is tangent to $\odot P$.

If $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.

Example **$\overline{AB}$ is tangent to $\odot C$. Find x.**

$\overline{AB}$ is tangent to $\odot C$, so $\overline{AB}$ is perpendicular to radius $\overline{BC}$. $\overline{CD}$ is a radius, so $CD = 8$ and $AC = 9 + 8$ or 17. Use the Pythagorean Theorem with right $\triangle ABC$.

$(AB)^2 + (BC)^2 = (AC)^2$ Pythagorean Theorem
$x^2 + 8^2 = 17^2$ Substitution
$x^2 + 64 = 289$ Multiply.
$x^2 = 225$ Subtract 64 from each side.
$x = 15$ Take the positive square root of each side.

Exercises

Find x. Assume that segments that appear to be tangent are tangent.

1. **19**
2. **25**
3. **12**
4. **20**
5. **20**
6. **12**

Chapter 10 34 *Glencoe Geometry*

Practice
CRM p. 37 OL AL

10-5 Practice

Tangents

Determine whether each segment is tangent to the given circle.

1. $\overline{MP}$ **no**
2. $\overline{QR}$ **yes**

Find x. Assume that segments that appear to be tangent are tangent.

3. **2**
4. **$5\sqrt{13}$**

Find the perimeter of each polygon for the given information. Assume that segments that appear to be tangent are tangent.

5. $CD = 52$, $CU = 18$, $TB = 12$ **128 units**
6. $KG = 32$, $HG = 56$ **154 units**

CLOCKS For Exercises 7 and 8, use the following information.
The design shown in the figure is that of a circular clock face inscribed in a triangular base. AF and FC are equal.

7. Find AB. **9.5 in.**
8. Find the perimeter of the clock. **34 in.**

Chapter 10 37 *Glencoe Geometry*

Lesson 10-5

Word Problem Practice
CRM p. 38 OL AL

10-5 Word Problem Practice

Tangents

1. **CANALS** The concrete canal in Landtown is shaped like a "V" at the bottom. One day, Maureen accidentally dropped a cylindrical tube as she was walking and it rolled to the bottom of the dried out concrete canal. The figure shows a cross section of the tube at the bottom of the canal.
Compare the lengths AV and BV.
They are equal.

2. **PACKAGING** Taylor packed a sphere inside a cubic box. He had painted the sides of the box black before putting the sphere inside. When the sphere was later removed, he discovered that the black paint had not completely dried and there were black marks on the sides of the sphere at the points of tangency with the sides of the box. If the black marks are used as the vertices of a polygon, what kind of polygon results?
a square

3. **TRIANGLES** A circle is inscribed in a 40°-60°-80° triangle. The points of tangency form the vertices of a triangle inscribed in the circle. What are the angles of the inscribed triangle?
50°, 60°, 70°

4. **ROLLING** A wheel is rolling down an incline. Twelve evenly spaced diameters form spokes of the wheel.
When spoke 2 is vertical, which spoke will be perpendicular to the incline?
spoke 10

DESIGN For Exercises 5 and 6, use the following information.
Amanda wants to make this design of circles inside an equilateral triangle.

5. What is the radius of the large circle to the nearest hundredth of an inch? **2.89 in.**
6. What are the radii of the smaller circles to the nearest hundredth of an inch? **0.96 in.**

Chapter 10 38 *Glencoe Geometry*

BL = *Below Grade Level*
OL = *On Grade Level*
AL = *Above Grade Level*
ELL = *English Language Learner*

Additional pages not shown:

CRM ***Lesson Reading Guide***, p. 33 BL OL ELL
CRM ***Skills Practice***, p. 36 BL OL

Enrichment
CRM p. 39 OL AL

10-5 Enrichment

Tangent Circles

Two circles in the same plane are **tangent circles** if they have exactly one point in common. Tangent circles with no common interior points are **externally tangent**. If tangent circles have common interior points, then they are **internally tangent**. Three or more circles are **mutually tangent** if each pair of them is tangent.

Externally Tangent Circles
Internally Tangent Circles

4 Assess

Ticket Out the Door Provide an example on the board with a triangle formed by a tangent, a radius, and the line from the center of the circle to a point on the tangent. Assign lengths to the figure and ask students to write the equation necessary to solve the problem. Have them state the answer as they leave the classroom.

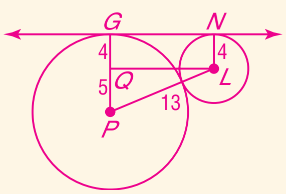
Foldables™ Follow-Up

Remind students to use the fifth flap in their Foldables to record notes on what they have learned about tangents.

Additional Answers

32c. Since a tangent intersects a circle in exactly one point, there is one tangent containing a point on the circle.

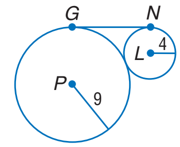
34. 12: Draw $\overline{PG}$, $\overline{NL}$, and $\overline{PL}$. Construct $\overline{LQ} \perp \overline{GP}$, thus $LQGN$ is a rectangle. $GQ = NL = 4$, so $QP = 5$. Using the Pythagorean theorem, $(QP)^2 + (QL)^2 = (PL)^2$. So, $QL = 12$. Since $GN = QL$, $GN = 12$.



35. If the lines are tangent at the endpoints of a diameter, they are parallel and thus, not intersecting.

36. Sample answer: Many of the field events have the athlete moving in a circular motion and releasing an object (discus, hammer, and shot). The movement of the athlete models a circle and the path of the released object models a tangent. In the hammer throw, the arm of the thrower, the handle, the wire, and hammer form the radius defining the circle when the hammer is spun around. The tangent is the path of the hammer when it is released. In this case, the distance the hammer was from Moreno was about 75.15 meters.

34. CHALLENGE Find the measure of tangent $\overline{GN}$. Explain. **See margin.**

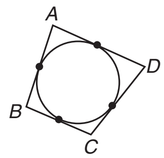


35. REASONING Write an argument to support this statement or provide a counterexample. *If two lines are tangent to the same circle, the lines intersect.* **See margin.**

36. *Writing in Math* Using the information about tangents and track and field on page 588, explain how the hammer throw models a tangent. Determine the distance the hammer landed from Moreno if the wire and handle are 1.2 meters long and her arm is 0.8 meter long. **See margin.**

STANDARDIZED TEST PRACTICE

37. Quadrilateral $ABCD$ is circumscribed about a circle. If $AB = 19$, $BC = 6$, and $CD = 14$, what is the measure of $\overline{AD}$? **D**



A 11 **C** 25

B 20 **D** 27

38. REVIEW A paper company ships reams of paper in a box that weighs 1.3 pounds. Each ream of paper weighs 4.4 pounds, and a box can carry no more than 12 reams of paper. Which inequality best describes the total weight in pounds w to be shipped in terms of the number of reams of paper r in each box? **G**

F $w \geq 1.3 + 4.4r, r \geq 12$

G $w = 1.3 + 4.4r, r \leq 12$

H $w \leq 1.3 + 4.4r, r \leq 12$

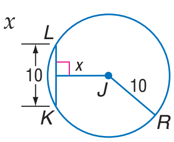
J $w = 1.3 + 4.4r, r \geq 12$

Spiral Review

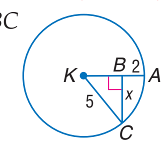
39. ADVERTISING Circles are often used in logos for commercial products. The logo at the right shows two inscribed angles and two central angles. If $\widehat{AC} \cong \widehat{BD}$, $m\widehat{AF} = 90$, $m\widehat{FE} = 45$, and $m\widehat{ED} = 90$, find $m\angle AFC$ and $m\angle BED$. (Lesson 10-4) **45, 45**

Find each measure to the nearest tenth. (Lesson 10-3)

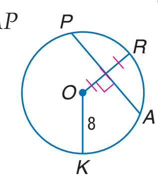
40. x **8.7**



41. BC **4**



42. AP **$8\sqrt{3} \approx 13.9$**



PREREQUISITE SKILL Solve each equation. (pages 781 and 782)

43. $x + 3 = \frac{1}{2}[(4x + 6) - 10]$ **5**

44. $2x - 5 = \frac{1}{2}[(3x + 16) - 20]$ **6**

45. $2x + 4 = \frac{1}{2}[(x + 20) - 10]$ **$\frac{2}{3}$**

46. $x + 3 = \frac{1}{2}[(4x + 10) - 45]$ **20.5**

Pre-AP Activity

Use as an Extension.

A circle is circumscribed about a square. The radius of the circle is r. Have the students write an expression for the perimeter of the square in terms of r.

$4r\sqrt{2} \approx 5.66r$