EXTEND 10-5

Geometry Lab
Inscribed and Circumscribed Triangles

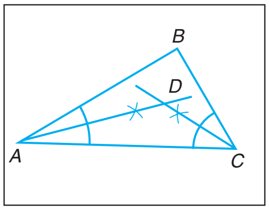
In Lesson 5-1, you learned that there are special points of concurrency in a triangle. Two of these will be used in these activities.

- The *incenter* is the point at which the angle bisectors meet. It is equidistant from the sides of the triangle.
- The *circumcenter* is the point at which the perpendicular bisectors of the sides intersect. It is equidistant from the vertices of the triangle.

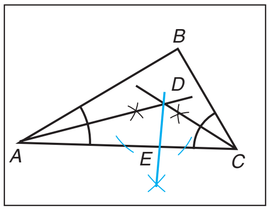
Concepts in Motion
Animation
geometryonline.com

ACTIVITY 1 **Construct a circle inscribed in a triangle.** The triangle is circumscribed about the circle.

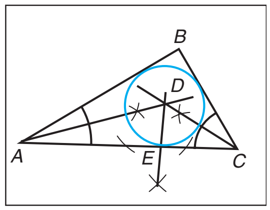
Step 1 Draw a triangle and label its vertices *A*, *B*, and *C*. Construct two angle bisectors of the triangle to locate the incenter. Label it *D*.



Step 2 Construct a segment perpendicular to a side of $\triangle ABC$ through the incenter. Label the intersection *E*.

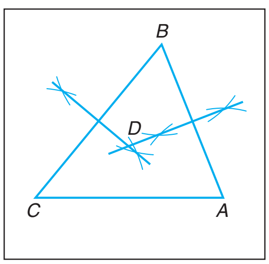


Step 3 Use the compass to measure *DE*. Then put the point of the compass on *D*, and draw a circle with that radius.

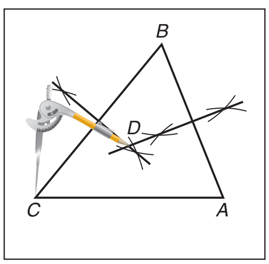


ACTIVITY 2 **Construct a circle through any three noncollinear points.**
This construction may be referred to as circumscribing a circle about a triangle.

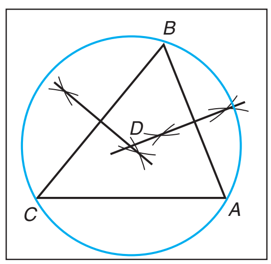
Step 1 Draw a triangle and label its vertices *A*, *B*, and *C*. Construct perpendicular bisectors of two sides of the triangle to locate the circumcenter. Label it *D*.



Step 2 Use the compass to measure the distance from the circumcenter *D* to any of the three vertices.



Step 3 Using that setting, place the compass point at *D*, and draw a circle about the triangle.



EXTEND 10-5
Lesson Notes

1 Focus

Objective Construct inscribed and circumscribed triangles.

Materials for Each Group
- straightedge, compass
- pencil, paper

Explain that students will use the incenter of a triangle to construct a circle so that the triangle is circumscribed about the circle, and they will use the circumcenter of a triangle to construct a circle in which the triangle is inscribed. They will also learn how to construct an equilateral triangle circumscribed about a circle.

2 Teach

Working in Cooperative Groups
Arrange students in groups of 3 or 4, mixing abilities. Then have groups complete **Activities 1–3** and **Exercises 1–3**.

Ask:
- How many angle bisectors need to be made when finding the incenter of a triangle? Only two bisectors are needed because the third angle bisector would pass through the same point.
- What needs to be accurate for these activities to work successfully? the positioning of the incenter and the circumcenter
- In **Activity 3**, how do you know what setting is needed to construct six congruent arcs on the circle? The radius will construct six congruent arcs on the circle.

Practice Have students individually complete Exercises 4–10.

Extend 10-5 Geometry Lab: Inscribed and Circumscribed Triangles **597**

3 Assess

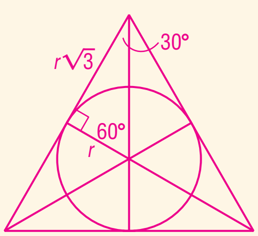
Formative Assessment

Use Exercises 4–10 to analyze their constructions and form a conjecture about the terms *incenter* and *circumcenter*.

Extending the Concept Lead students to repeat the activities for different types of triangles and practice the constructions.

Additional Answers

4. The incenter is equidistant from each side. The perpendicular to one side should be the same length as it is to the other two sides.
5. The incenter is equidistant from all the sides. The radius of the circle is perpendicular to the tangent sides and all radii are congruent, matching the distance from the incenter to the sides.
6. The circumcenter is equidistant from all three vertices, so the distance from the circumcenter to one vertex is the same as the distance to each of the others.
7. The circumcenter is equidistant from the vertices and all of the vertices must lie on the circle. So, this distance is the radius of the circle containing the vertices.
9. Suppose all six radii are drawn. Each central angle measures 60°. Thus, six 30°-60°-90° triangles are formed. Each triangle has a side which is a radius r units long. Using 30°-60°-90° side ratios, the segment tangent to the circle has length $r\sqrt{3}$, making each side of the circumscribed triangle $2r\sqrt{3}$. If all three sides have the same measure, then the triangle is equilateral.

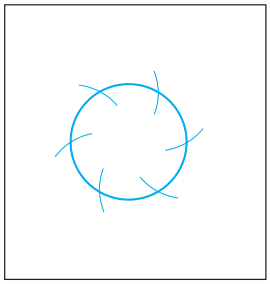


10. The incenter is the point from which you can construct a circle "in" the triangle. Circum means *around.* So the circumcenter is the point from which you can construct a circle "around" the triangle.

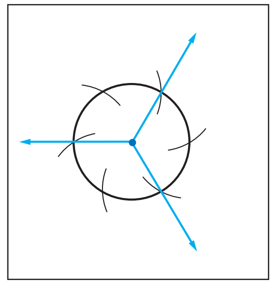
For the next activity, refer to the construction of an inscribed regular hexagon on page 576.

ACTIVITY 3 **Construct an equilateral triangle circumscribed about a circle.**

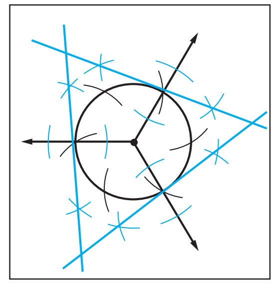
Step 1 Construct a circle and divide it into six congruent arcs.



Step 2 Place a point at every other arc. Draw rays from the center through these points.



Step 3 Construct a line perpendicular to each of the rays through the points.



ANALYZE THE RESULTS **1–3. See students' work.**

1. Draw an obtuse triangle and inscribe a circle in it.
2. Draw a right triangle and circumscribe a circle about it.
3. Draw a circle of any size and circumscribe an equilateral triangle about it.

Refer to Activity 1. **4–5. See margin.**

4. Why do you only have to construct the perpendicular to one side of the triangle?
5. How can you use the Incenter Theorem to explain why this construction is valid?

Refer to Activity 2. **6–7. See margin.**

6. Why do you only have to measure the distance from the circumcenter to any one vertex?
7. How can you use the Circumcenter Theorem to explain why this construction is valid?

Refer to Activity 3.

8. What is the measure of each of the six congruent arcs? **60**
9. Write a convincing argument as to why the lines constructed in Step 3 form an equilateral triangle. **See margin.**
10. Why do you think the terms *incenter* and *circumcenter* are good choices for the points they define? **See margin.**

10-6 Secants, Tangents, and Angle Measures

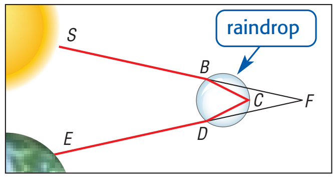
Main Ideas

- Find measures of angles formed by lines intersecting on or inside a circle.
- Find measures of angles formed by lines intersecting outside the circle.

New Vocabulary

secant

GET READY for the Lesson

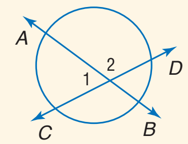


Droplets of water in the air refract or bend sunlight as it passes through them, creating a rainbow. The various angles of refraction result in an arch of colors. In the figure, the sunlight from point S enters the raindrop at B and is bent. The light proceeds to the back of the raindrop, and is reflected at C to leave the raindrop at point D heading to Earth. Angle F represents the measure of how the resulting ray of light deviates from its original path.

Intersections on or Inside a Circle A line that intersects a circle in exactly two points is called a **secant**. In the figure above, $\overline{SF}$ and $\overline{EF}$ are secants of the circle. When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.

10.12

If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.



Examples: $m\angle 1 = \frac{1}{2}(m\widehat{AC} + m\widehat{BD})$

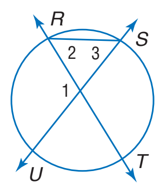
$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

PROOF **Theorem 10.12**

Given: secants $\overleftrightarrow{RT}$ and $\overleftrightarrow{SU}$

Prove: $m\angle 1 = \frac{1}{2}(m\widehat{ST} + m\widehat{RU})$

Proof:



Statements	Reasons
1. $m\angle 1 = m\angle 2 + m\angle 3$	1. Exterior Angle Theorem
2. $m\angle 2 = \frac{1}{2}m\widehat{ST}$, $m\angle 3 = \frac{1}{2}m\widehat{RU}$	2. The measure of the inscribed ∠ = half the measure of the intercepted arc.
3. $m\angle 1 = \frac{1}{2}m\widehat{ST} + \frac{1}{2}m\widehat{RU}$	3. Substitution
4. $m\angle 1 = \frac{1}{2}(m\widehat{ST} + m\widehat{RU})$	4. Distributive Property

10-6 Lesson Notes

1 Focus

Vertical Alignment

Before Lesson 10-6
Make conjectures about circles.

Lesson 10-6
Use numeric patterns to make generalizations of angle relationships in circles.
Formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models.

After Lesson 10-6
Find areas of circles.

2 Teach

Scaffolding Questions

Have students observe the diagram and read the caption in *Get Ready for the Lesson*.

Ask:

- If you were to connect B and D in the figure, what would you have? a triangle inscribed in the circle
- After connecting B and D in the figure, what can you say about $\triangle BCD$ and $\triangle BFD$? They share the base segment $\overline{BD}$.

(continued on the next page)

Lesson 10-6 Resources

Chapter 10 Resource Masters

Lesson Reading Guide, p. 42 BL OL
Study Guide and Intervention, pp. 43–44 BL OL
Skills Practice, p. 45 BL OL
Practice, p. 46 OL AL
Word Problem Practice, p. 47 BL OL AL
Enrichment, p. 48 OL AL
Quiz 3, p. 66

Transparencies

5-Minute Check Transparency 10-6

Additional Print Resources

Noteables™ Interactive Study Notebook with Foldables™

Technology

geometryonline.com
Interactive Classroom CD-ROM
AssignmentWorks CD-ROM
Graphing Calculator Easy Files

- Name some situations that allow you to see a rainbow formed by segments of a circle. Sample answers: a rainy, misty day when the sun is low in the sky; a bright sunny day when you are spraying a mist of water from a hose

Intersections On or Inside a Circle

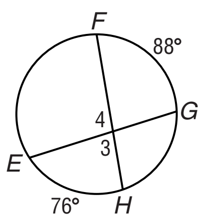
Examples 1 and 2 show how to use the theorems in this lesson to find the measure of angles of secants intersecting on or inside a circle.

Formative Assessment

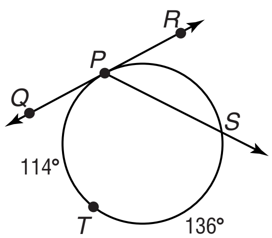
Use the Check Your Progress exercises after each example to determine students' understanding of concepts.

ADDITIONAL EXAMPLES

1 Find $m\angle 4$ if $m\overset{\frown}{FG} = 88$ and $m\overset{\frown}{EH} = 76$. 98



2 Find $m\angle RPS$ if $m\overset{\frown}{PT} = 114$ and $m\overset{\frown}{TS} = 136$. 55

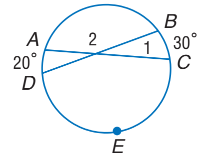


Additional Examples also in:

- Noteables™ Interactive Study Notebook with Foldables™
- Interactive Classroom PowerPoint® Presentations

EXAMPLE 1 Secant-Secant Angle

Find $m\angle 2$ if $m\overset{\frown}{BC} = 30$ and $m\overset{\frown}{AD} = 20$.



Method 1 Find $m\angle 1$.

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AD})$ Theorem 10.12

$= \frac{1}{2}(30 + 20)$ or 25 Substitution

$m\angle 2 = 180 - m\angle 1$

$= 180 - 25$ or 155

Method 2 Find $m\overset{\frown}{AB}$ and $m\overset{\frown}{DEC}$ first.

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{DEC})$ Theorem 10.12

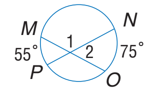
$= \frac{1}{2}[360 - (m\overset{\frown}{BC} + m\overset{\frown}{AD})]$ $m\overset{\frown}{AB} + m\overset{\frown}{DEC} = 360 - (m\overset{\frown}{BC} + m\overset{\frown}{AD})$

$= \frac{1}{2}[360 - (30 + 20)]$ Substitution

$= \frac{1}{2}(310)$ or 155 Simplify.

Check Your Progress

1. Find $m\angle 1$. 115

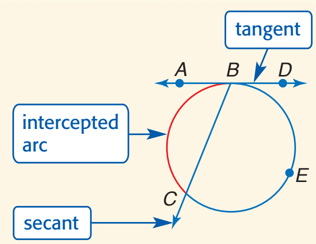


THEOREM 10.13

If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

Examples: $m\angle ABC = \frac{1}{2}m\overset{\frown}{BC}$

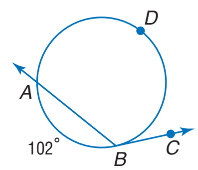
$m\angle DBC = \frac{1}{2}m\overset{\frown}{BEC}$



You will prove Theorem 10.13 in Exercise 41.

EXAMPLE 2 Secant-Tangent Angle

Find $m\angle ABC$ if $m\overset{\frown}{AB} = 102$.



$m\overset{\frown}{ADB} = 360 - m\overset{\frown}{AB}$

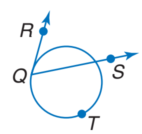
$= 360 - 102$ or 258

$m\angle ABC = \frac{1}{2}m\overset{\frown}{ADC}$

$= \frac{1}{2}(258)$ or 129

Check Your Progress

2. Find $m\angle RQS$ if $m\overset{\frown}{QTS} = 238$. 61



Differentiated Instruction

Naturalist Learners Explain that the relationships presented in this chapter are naturally occurring relationships that have been mathematically defined and explained. Tell students that scientists from all fields can use these relationships to examine everything from raindrops and soap bubbles to cells and microorganisms.

Intersections Outside a Circle Secants and tangents can also meet outside a circle. The measure of the angle formed also involves half of the measures of the arcs they intercept.

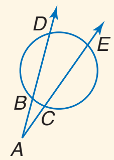
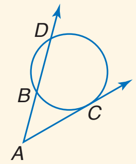
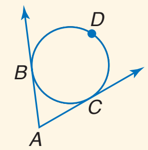
Study Tip

Absolute Value

The measure of each $\angle A$ can also be expressed as one-half the absolute value of the difference of the arc measures. In this way, the order of the arc measures does not affect the outcome of the calculation.

THEOREM 10.14

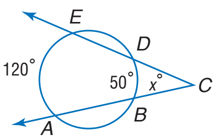
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

Two Secants	Secant-Tangent	Two Tangents
		
$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$	$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$	$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

You will prove Theorem 10.14 in Exercise 40.

EXAMPLE **Secant-Secant Angle**

Find x.

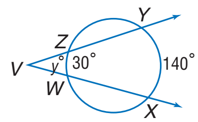


$m\angle C = \frac{1}{2}(m\widehat{EA} - m\widehat{DB})$

$x = \frac{1}{2}(120 - 50)$ Substitution

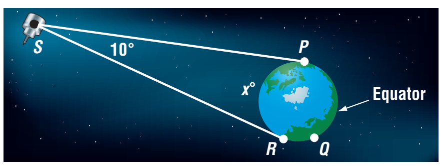
$x = \frac{1}{2}(70)$ or 35 Simplify.

3. Find y. **55**



Real-World EXAMPLE

SATELLITES Suppose a satellite S orbits above Earth rotating so that it appears to hover directly over the equator. Use the figure to determine the arc measure on the equator visible to this satellite.



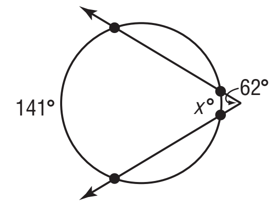
$\widehat{PR}$ represents the arc along the equator visible to the satellite S. If $x = m\widehat{PR}$, then $m\widehat{PQR} = 360 - x$. Use the measure of the given angle to find $m\widehat{PR}$.

Intersections Outside a Circle

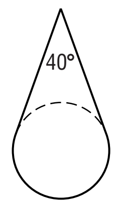
Examples 3–5 show how to use the theorem about secants and tangents to find the measure of an angle that intersects outside a circle.

ADDITIONAL EXAMPLES

3 Find x. **17**



4 **JEWELRY** A jeweler wants to craft a pendant with the shape shown. Use the figure to determine the measure of the arc at the bottom of the pendant. **220**

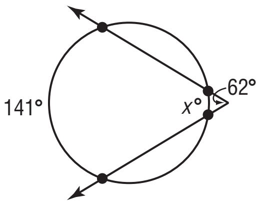


Focus on Mathematical Content

Some students may ask you what the difference is between chords and secants and why there are two names for something that intersects a circle at two points. You may want to review how segments are parts of lines and explain that chords are segments of secants, which are lines that intersect circles. Tell students that every chord lies on a secant and that every secant contains a chord.

ADDITIONAL EXAMPLE

Find x. 17



3 Practice

Formative Assessment

Use Exercises 1–5 to check for understanding.

Then use the chart at the bottom of this page to customize assignments for your students.

Odd/Even Assignments

Exercises 6–24 are structured so that students practice the same concepts whether they are assigned odd or even problems.

$m\angle S = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$

$10 = \frac{1}{2}[(360 - x) - x]$ Substitution

$20 = 360 - 2x$ Multiply each side by 2 and simplify.

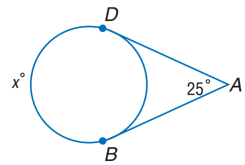
$-340 = -2x$ Subtract 360 from each side.

$170 = x$ Divide each side by -2.

The measure of the arc on Earth visible to the satellite is 170.

CHECK Your Progress

4. Find x. **205**

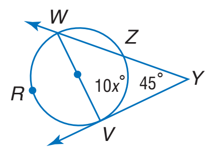


Personal Tutor at geometryonline.com

EXAMPLE Secant-Tangent Angle

Find x.

$\widehat{WRV}$ is a semicircle because $\overline{WV}$ is a diameter.
So, $m\widehat{WRV} = 180$.



$m\angle Y = \frac{1}{2}(m\widehat{WV} - m\widehat{ZV})$

$45 = \frac{1}{2}(180 - 10x)$ Substitution

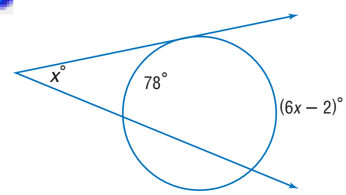
$90 = 180 - 10x$ Multiply each side by 2.

$-90 = -10x$ Subtract 180 from each side.

$9 = x$ Divide each side by -10.

CHECK Your Progress

5. Find x. **20**



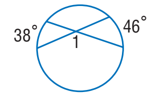
★ indicates multi-step problem

CHECK Your Understanding

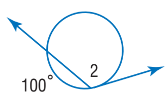
Examples 1, 2 (p. 600)

Find each measure.

1. $m\angle 1$ **138**



2. $m\angle 2$ **130**



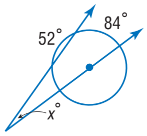
DIFFERENTIATED HOMEWORK OPTIONS

Level	Assignment	Two-Day Option	
BL Basic	6–24, 42–43, 45–57	7–23 odd, 46–47	6–24 even, 42–43, 45, 48–57
OL Core	7–25 odd, 27–43, 45–57	6–24, 46–47	25–43, 45, 48–57
AL Advanced /Pre-AP	25–57		

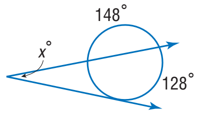
Examples 3 to 5
(pp. 601, 602)

Find x.

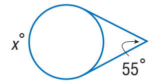
3. **20**



4. **22**



5. **235**



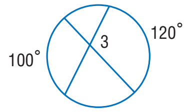
Exercises

HOMEWORK HELP	
For Exercises	See Examples
6–9, 14	1
10–13	2
15–18	3
19–20	4
21–24	5

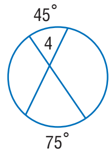
Exercise Levels
A: 6–24
B: 25–41
C: 42–45

Find each measure.

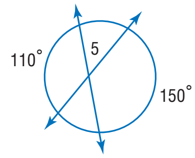
6. $m\angle 3$ **110**



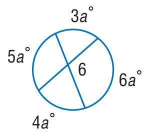
7. $m\angle 4$ **60**



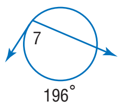
8. $m\angle 5$ **50**



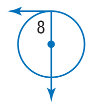
9. $m\angle 6$ **110**



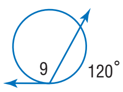
10. $m\angle 7$ **98**



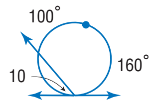
11. $m\angle 8$ **90**



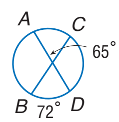
12. $m\angle 9$ **120**



13. $m\angle 10$ **50**

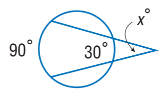


14. $m\widehat{AC}$ **58**

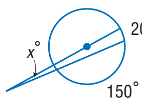


Find x. Assume that any segment that appears to be tangent is tangent.

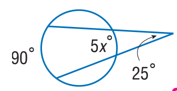
15. **30**



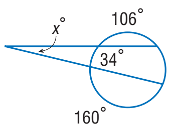
16. **5**



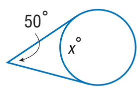
17. **8**



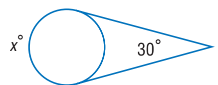
18. **13**



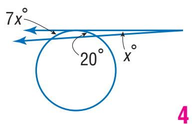
19. **130**



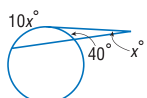
20. **210**



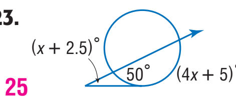
21. **4**



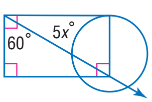
22. **5**



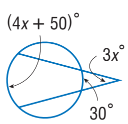
23. **25**



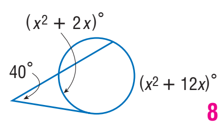
24. **9**



25. **10**



26. **8**



Study Guide and Intervention

CRM pp. 43–44 BL OL ELL

10-6 Study Guide and Intervention
Secants, Tangents, and Angle Measures

Lesson 10-6

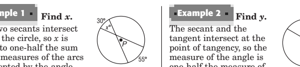
Intersections On or Inside a Circle A line that intersects a circle in exactly two points is called a **secant**. The measures of angles formed by secants and tangents are related to intercepted arcs.

- If two secants intersect in the interior of a circle, then the measure of the angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.
- If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

$m\angle 1 = \frac{1}{2}(m\widehat{PR} + m\widehat{QS})$

$m\angle XTV = \frac{1}{2}m\widehat{TUV}$

$m\angle YTV = \frac{1}{2}m\widehat{TV}$



Example 1 **Find x.**
The two secants intersect inside the circle, so x is equal to one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

$x = \frac{1}{2}(30 + 55)$
$= \frac{1}{2}(85)$
$= 42.5$

Example 2 **Find y.**
The secant and the tangent intersect at the point of tangency, so the measure of the angle is one-half the measure of its intercepted arc.

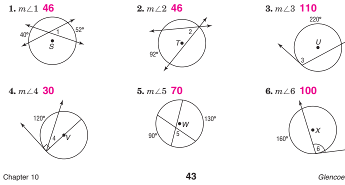
$y = \frac{1}{2}(168)$
$= 84$

Exercises

Find each measure.

1. $m\angle 1$ **46** **2.** $m\angle 2$ **46** **3.** $m\angle 3$ **110**

4. $m\angle 4$ **30** **5.** $m\angle 5$ **70** **6.** $m\angle 6$ **100**



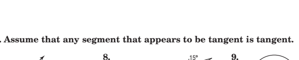
Practice

CRM p. 46 OL AL

10-6 Practice
Secants, Tangents, and Angle Measures

Find each measure.

1. $m\angle 1$ **2.** $m\angle 2$ **3.** $m\angle 3$



79 **113** **72**

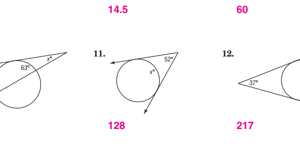
Find x. Assume that any segment that appears to be tangent is tangent.

7. **8.** **9.**

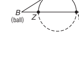
31 **14.5** **60**

10. **11.** **12.**

21 **128** **217**



9. RECREATION In a game of kickball, Rickie has to kick the ball through a semicircular goal to score. If $m\widehat{XZ} = 58$ and the $m\widehat{XY} = 122$, at what angle must Rickie kick the ball to score? Explain.
Rickie must kick the ball at an angle less than 32° since the measure of the angle from the ground that a tangent would make with the goal post is 32°.



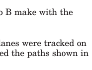
Word Problem Practice

CRM p. 47 OL AL

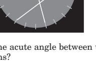
10-6 Word Problem Practice
Secants, Tangents, and Angle Measures

Lesson 10-6

1. TELESCOPES Vanessa looked through her telescope at a mountainous landscape. The figure shows what she saw. Based on the view, approximately what angle does the side of the mountain that runs from A to B make with the horizontal? **60°**



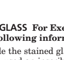
2. RADAR Two airplanes were tracked on radar. They followed the paths shown in the figure.



What is the acute angle between their flight paths?
67.5°

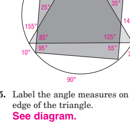
3. EASELS Francisco is a painter. He places a circular canvas on his A-frame easel and carefully centers it. The apex of the easel is 30° and the measure of arc BC is 22°. What is the measure of arc AB?
128°

4. FLYING When flying at an altitude of 5 miles, the lines of sight to the horizon looking north and south make about a 173.7° angle. How much of the longitude line directly under the plane is visible from 5 miles high?



6.3

STAINED GLASS **For Exercises 5 and 6, use the following information.**
Pablo made the stained glass window shown. He used an inscribed square and equilateral triangle for the design.



5. Label the angle measures on the outer edge of the triangle.
See diagram.

6. Label all of the arcs with their degree measure.
See diagram.

BL = Below Grade Level
OL = On Grade Level
AL = Above Grade Level
ELL = English Language Learner

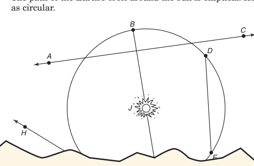
Enrichment

CRM p. 48 OL AL

10-6 Enrichment

Orbiting Bodies

The path of the Earth's orbit around the sun is elliptical. However, it is often viewed as circular.



Additional pages not shown:

CRM *Lesson Reading Guide,* p. 42 BL OL ELL

CRM *Skills Practice,* p. 45 BL OL

Real World Connection

Refer to Exercises 27–30. A circus, which is often inside a large tent, is performed in an oval or circle arena with spectators seated around its circumference. It is believed that the circus originated in Ancient Rome. The modern circus was established in the late 18th century in Britain by Philip Astley.

Additional Answers

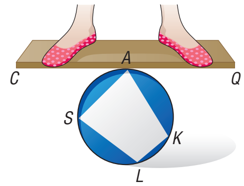
40a. Statements (Reasons)

1. $\overleftrightarrow{AC}$ and $\overleftrightarrow{AT}$ are secants to the circle. (Given)
2. $m\angle CRT = \frac{1}{2}m\widehat{CT}$, $m\angle ACR = \frac{1}{2}m\widehat{BR}$ (The meas. of an inscribed ∠ = $\frac{1}{2}$ the meas. of its intercepted arc.)
3. $m\angle CRT = m\angle ACR + m\angle CAT$ (Exterior △ Theorem)
4. $\frac{1}{2}m\widehat{CT} = \frac{1}{2}m\widehat{BR} + m\angle CAT$ (Substitution)
5. $\frac{1}{2}m\widehat{CT} - \frac{1}{2}m\widehat{BR} = m\angle CAT$ (Subtraction Prop.)
6. $\frac{1}{2}(m\widehat{CT} - m\widehat{BR}) = m\angle CAT$ (Distributive Prop.)

40b. Statements (Reasons)

1. $\overleftrightarrow{DG}$ is a tangent to the circle. $\overleftrightarrow{DF}$ is a secant to the circle. (Given)
2. $m\angle DFG = \frac{1}{2}m\widehat{GE}$, $m\angle FGH = \frac{1}{2}m\widehat{FG}$ (The meas. of an inscribed ∠ = $\frac{1}{2}$ the meas. of its intercepted arc.)
3. $m\angle FGH = m\angle DFG + m\angle FDG$ (Exterior △ Theorem)
4. $\frac{1}{2}m\widehat{FG} = \frac{1}{2}m\widehat{GE} + m\angle FDG$ (Substitution)
5. $\frac{1}{2}m\widehat{FG} - \frac{1}{2}m\widehat{GE} = m\angle FDG$ (Subtraction Prop.)
6. $\frac{1}{2}(m\widehat{FG} - m\widehat{GE}) = m\angle FDG$ (Distributive Prop.)

CIRCUS **For Exercises 27–30, refer to the figure and the information below.**
One of the acrobatic acts in the circus requires the artist to balance on a board that is placed on a round drum, as shown at the right. Find each measure if $\overline{SA} \parallel \overline{LK}$, $m\angle SLK = 78$, and $m\widehat{SA} = 46$.



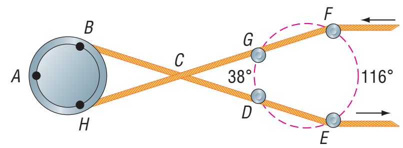
27. $m\angle CAS$ **23**

28. $m\angle QAK$ **55**

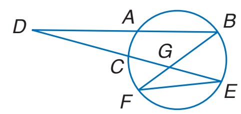
29. $m\widehat{KL}$ **94**

30. $m\widehat{SL}$ **110**

31. **WEAVING** Once yarn is woven from wool fibers, it is often dyed and then threaded along a path of pulleys to dry. One set of pulleys is shown below. Note that the yarn appears to intersect itself at C, but in reality it does not. Use the information from the diagram to find $m\widehat{BH}$. **141**



Find each measure if $m\widehat{FE} = 118$, $m\widehat{AB} = 108$, $m\angle EGB = 52$, and $m\angle EFB = 30$.



32. $m\widehat{AC}$ **30**

33. $m\widehat{CF}$ **44**

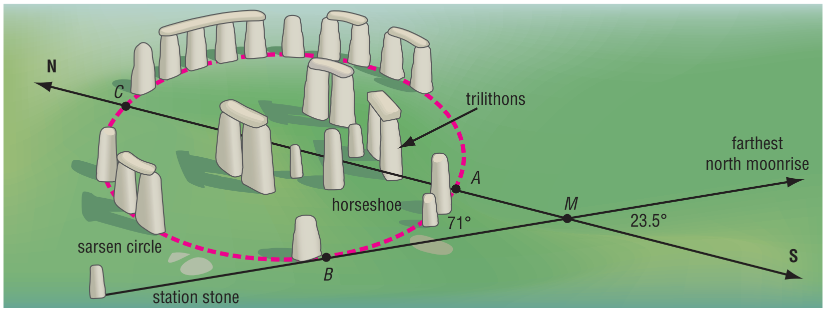
34. $m\angle EDB$ **15**

Real-World Link

Stonehenge is located in southern England near Salisbury. In its final form, Stonehenge included 30 upright stones about 18 feet tall by 7 feet thick.

Source: *World Book Encyclopedia*

LANDMARKS **For Exercises 35–37, use the following information.**
Stonehenge is a British landmark made of huge stones arranged in a circular pattern that reflects the movements of Earth and the moon. The diagram shows that the angle formed by the north/south axis and the line aligned from the station stone to the northmost moonrise position measures 23.5°.



35. Find $m\widehat{BC}$. **118**

36. Is $\widehat{ABC}$ a semicircle? Explain. **No, its measure is 189.**

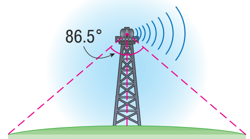
37. If the circle measures about 100 feet across, approximately how far would you walk around the circle from point B to point C? **about 103 ft**

40c. Statements (Reasons)

1. $\overleftrightarrow{HI}$ and $\overleftrightarrow{HJ}$ are tangents to the circle. (Given)
2. $m\angle IJK = \frac{1}{2}m\widehat{IXJ}$, $m\angle HIJ = \frac{1}{2}m\widehat{IJ}$ (The measure of a secant-tangent ∠ = $\frac{1}{2}$ the measure of its intercepted arc.)
3. $m\angle IJK = m\angle HIJ + m\angle IHJ$ (Ext. △ Th.)
4. $\frac{1}{2}m\widehat{IXJ} = \frac{1}{2}m\widehat{IJ} + m\angle IHJ$ (Substitution)
5. $\frac{1}{2}m\widehat{IXJ} - \frac{1}{2}m\widehat{IJ} = m\angle IHJ$ (Subtr. Prop.)
6. $\frac{1}{2}(m\widehat{IXJ} - m\widehat{IJ}) = m\angle IHJ$ (Distrib. Prop.)

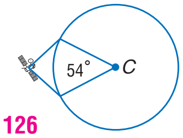
604 Chapter 10 Circles

38. TELECOMMUNICATIONS The signal from a telecommunication tower follows a ray that has its endpoint on the tower and is tangent to Earth. Suppose a tower is located at sea level, as shown in the figure. Determine the measure of the arc intercepted by the two tangents. **93.5**



Note: Art not drawn to scale

39. SATELLITES A satellite is orbiting so that it maintains a constant altitude above the equator. The camera on the satellite can detect an arc of 6000 kilometers on Earth's surface. This arc measures 54°. What is the measure of the angle of view of the camera located on the satellite? **126**

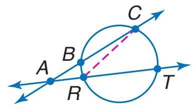


40. PROOF Write a two-column proof of Theorem 10.14. Consider each case.

a. Case 1: two secants **40a–c. See margin.**

Given: $\overleftrightarrow{AC}$ and $\overleftrightarrow{AT}$ are secants to the circle.

Prove: $m\angle CAT = \frac{1}{2}(m\widehat{CT} - m\widehat{BR})$

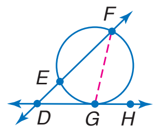


b. Case 2: secant and a tangent

Given: $\overleftrightarrow{DG}$ is a tangent to the circle.

$\overleftrightarrow{DF}$ is a secant to the circle.

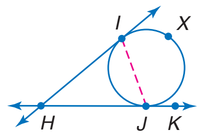
Prove: $m\angle FDG = \frac{1}{2}(m\widehat{FG} - m\widehat{GE})$



c. Case 3: two tangents

Given: $\overleftrightarrow{HI}$ and $\overleftrightarrow{HJ}$ are tangents to the circle.

Prove: $m\angle IHJ = \frac{1}{2}(m\widehat{IXJ} - m\widehat{IJ})$



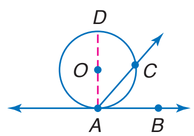
41. PROOF Write a paragraph proof of Theorem 10.13.

a. Given: $\overleftrightarrow{AB}$ is a tangent of $\odot O$. **41a–b. See margin.**

$\overrightarrow{AC}$ is a secant of $\odot O$.

$\angle CAB$ is acute.

Prove: $m\angle CAB = \frac{1}{2}m\widehat{CA}$



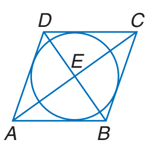
b. Prove Theorem 10.13 if the angle in part **a** is obtuse.

EXTRA PRACTICE
See pages 821, 837.
Self-Check Quiz at
geometryonline.com

H.O.T. Problems

42. OPEN ENDED Draw a circle and one of its diameters. Call the diameter $\overline{AC}$. Draw a line tangent to the circle at A. What type of angle is formed by the tangent and the diameter? Explain. **See margin.**

43. CHALLENGE Circle E is inscribed in rhombus $ABCD$. The diagonals of the rhombus are 10 centimeters and 24 centimeters long. To the nearest tenth centimeter, how long is the radius of circle E? (*Hint:* Draw an altitude from E.) **4.6 cm**

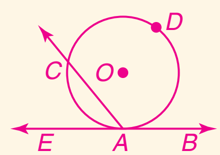


Additional Answers

41a. Proof: $\angle DAB$ is a right $\angle$ with measure 90, and $\widehat{DCA}$ is a semicircle with measure 180, since if a line is tangent to a $\odot$, it is $\perp$ to the radius at the point of tangency. Since $\angle CAB$ is acute, C is in the interior of $\angle DAB$, so by the Angle and Arc Addition Postulates, $m\angle DAB = m\angle DAC + m\angle CAB$ and $m\widehat{DCA} = m\widehat{DC} + m\widehat{CA}$. By substitution, $90 = m\angle DAC + m\angle CAB$ and $180 = m\widehat{DC} + m\widehat{CA}$. So, $90 = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$ by Mult. Prop., and $m\angle DAC + m\angle CAB = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$ by substitution. $m\angle DAC = \frac{1}{2}m\widehat{DC}$ since $\angle DAC$ is inscribed, so substitution yields $\frac{1}{2}m\widehat{DC} + m\angle CAB = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$. By Subtraction Prop., $m\angle CAB = \frac{1}{2}m\widehat{CA}$.

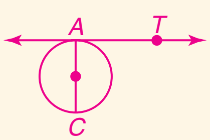
41b. Given: $\overleftrightarrow{AB}$ is a tangent to $\odot O$. $\overrightarrow{AC}$ is a secant to $\odot O$. $\angle CAB$ is obtuse.

Prove: $m\angle CAB = \frac{1}{2}m\widehat{CDA}$



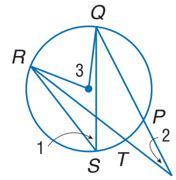
Proof: $\angle CAB$ and $\angle CAE$ form a linear pair, so $m\angle CAB + m\angle CAE = 180$. Since $\angle CAB$ is obtuse, $\angle CAE$ is acute and Case 1 applies, so $m\angle CAE = \frac{1}{2}m\widehat{CA}$. $m\widehat{CA} + m\widehat{CDA} = 360$, so $\frac{1}{2}m\widehat{CA} + \frac{1}{2}m\widehat{CDA} = 180$ by Mult. Prop., and $m\angle CAE + \frac{1}{2}m\widehat{CDA} = 180$ by substitution. By the Transitive Prop., $m\angle CAB + m\angle CAE = m\angle CAE + \frac{1}{2}m\widehat{CDA}$, so by Subtraction Prop., $m\angle CAB = \frac{1}{2}m\widehat{CDA}$.

42. Sample answer:



Angle TAC is a right angle. There are several reasons. (1) If the point of tangency is the endpoint of a diameter, then the tangent is perpendicular to the tangent at that point. (2) The arc intercepted by the secant (diameter) and the tangent is a semicircle. Thus the measure of the angle is half of 180 or 90.

44. **CHALLENGE** In the figure, $\angle 3$ is a central angle. List the numbered angles in order from greatest measure to least measure. Explain your reasoning. **See margin.**



45. ***Writing in Math*** Refer to the information on page 599 to explain how you would calculate the angle representing how the light deviates from its original path. Include in your description the types of segments represented in the figure on page 599. **See margin.**

STANDARDIZED TEST PRACTICE

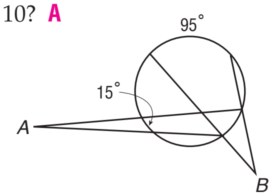
46. What is the measure of $\angle B$ if $m\angle A = 10$? **A**

A 30

B 35

C 47.5

D 90



47. **REVIEW** Larry's Fish Food comes in tubes that have a radius of 2 centimeters and a height of 7 centimeters. Odell bought a full tube, but he thinks he's used about $\frac{1}{4}$ of it. About how much is left? **G**

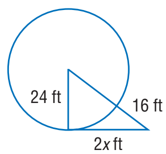
F 88 cm^3 H 53 cm^3

G 66 cm^3 J 41 cm^3

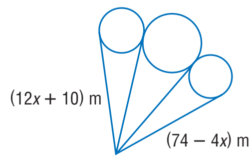
Spiral Review

Find x. Assume that segments that appear to be tangent are tangent. (Lesson 10-5)

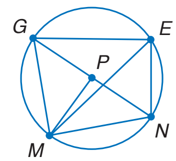
48. **16**



48. **4**



In $\odot P$, $m\widehat{EN} = 66$ and $m\angle GPM = 89$. Find each measure. (Lesson 10-4)



50. $m\angle EGN$ **33** **51.** $m\angle GME$ **57** **52.** $m\angle GNM$ **44.5**

RAMPS **For Exercises 53 and 54, use the following information.**
The Americans with Disabilities Act requires that wheelchair ramps have at least a 12-inch run for each rise of 1 inch. (Lesson 3-3)

53. Determine the slope represented by this requirement. $\frac{1}{12}$

54. The maximum length the law allows for a ramp is 30 feet. How many inches tall is the highest point of this ramp? **30 in.**

Get Ready for the Next Lesson

PREREQUISITE SKILL **Use the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$, to solve each equation to the nearest tenth.**

55. $x^2 + 6x - 40 = 0$ **4, −10** **56.** $2x^2 + 7x - 30 = 0$ **−6, 2.5** **57.** $3x^2 - 24x + 45 = 0$ **3, 5**

4 Assess

Name the Math Select examples and ask students to call out the names of the segments in the figure.

Formative Assessment

Check for student understanding of concepts in Lessons 10-5 and 10-6.

CRM Quiz 3, p. 66

Foldables™ Follow-Up

Remind students to use the sixth flap in their Foldables to record notes on what they have learned about secants, tangents, and angle measures. Students should include sketches like those in the Concept Summary on p. 620.

Additional Answers

44. $\angle 3, \angle 1, \angle 2$; $m\angle 3 = m\widehat{RQ}$, $m\angle 1 = \frac{1}{2}m\widehat{RQ}$ so $m\angle 3 > m\angle 1$, $m\angle 2 = \frac{1}{2}(m\widehat{RQ} - m\widehat{TP}) = \frac{1}{2}m\widehat{RQ} - \frac{1}{2}m\widehat{TP}$, which is less than $\frac{1}{2}m\widehat{RQ}$, so $m\angle 2 < m\angle 1$.

45. Sample answer: Each raindrop refracts light from the Sun and sends the beam to Earth. The raindrop is actually spherical, but the angle of the light is an inscribed angle from the bent rays. $\angle C$ is an inscribed angle and $\angle F$ is a secant-secant angle. The measure of $\angle F$ can be calculated by finding the positive difference between $m\widehat{BD}$ and the measure of the small intercepted arc containing point C.

Pre-AP Activity

Use as an Extension

If two chords in the same circle cut two arcs of 75 degrees, what do you know about the chords? If the arcs have an endpoint on each chord, then the chords are parallel. If the arcs have both endpoints on the same chord, then the chords are congruent.