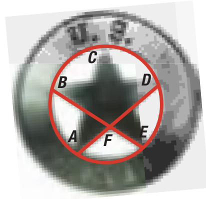
10-7 Special Segments in a Circle

Main Ideas

- Find measures of segments that intersect in the interior of a circle.
- Find measures of segments that intersect in the exterior of a circle.

GET READY for the Lesson

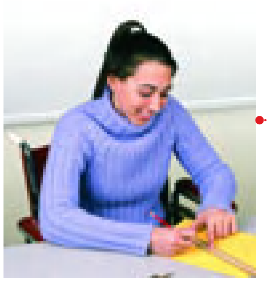
The U.S. Marshals Service is the nation's oldest federal law enforcement agency, serving the country since 1789. Appointed by the President, there are 94 U.S. Marshals, one for each federal court district in the country. The "Eagle Top" badge, introduced in 1941, was the first uniform U.S. Marshals badge.



Segments Intersecting Inside a Circle In Lesson 10-2, you learned how to find lengths of parts of a chord that is intersected by the perpendicular diameter. But how do you find lengths for other intersecting chords?

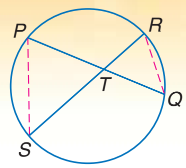
GEOMETRY LAB

Intersecting Chords



MAKE A MODEL

- Draw a circle and two intersecting chords.
- Name the chords $\overline{PQ}$ and $\overline{RS}$ intersecting at T.
- Draw $\overline{PS}$ and $\overline{RQ}$.



ANALYZE

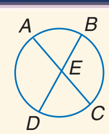
1. Name pairs of congruent angles. Explain your reasoning.
2. How are $\triangle PTS$ and $\triangle RTQ$ related? Why? Cut out both triangles, move them, and verify your conjecture. **similar by AA Similarity**
3. **Make a conjecture** about the relationship of $\overline{PT}$, $\overline{TQ}$, $\overline{RT}$, and $\overline{ST}$. **See margin.**
4. Measure each angle and verify your conjecture. **See students' work.**

The results of the lab suggest a proof for Theorem 10.15.

THEOREM 10.15

If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.

Example: $AE \cdot EC = BE \cdot ED$



You will prove Theorem 10.15 in Exercise 16.

1. $\angle PTS \cong \angle RTQ$ (Vertical ∡ are ≅.); $\angle P \cong \angle R$ (∡ intercepting same arc are ≅.); $\angle S \cong \angle Q$ (∡ intercepting same arc are ≅.)

10-7 Lesson Notes

1 Focus

Vertical Alignment

Before Lesson 10-7
Make conjectures about circles.

Lesson 10-7
Use numeric patterns to make generalizations about angle relationships in circles.
Formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models.

After Lesson 10-7
Find areas of circles.

2 Teach

Scaffolding Questions Have students observe the picture and read the adjacent caption in *Get Ready for the Lesson.*

Ask:

- Name the segments that are defined by the intersection of $\overline{AD}$ and $\overline{EB}$ in the figure. $\overline{AF}$, $\overline{FD}$, $\overline{EF}$, and $\overline{FB}$

(continued on the next page)

Additional Answer

3. $\frac{PT}{RT} = \frac{ST}{TQ}$ or $PT \cdot TQ = RT \cdot ST$

Lesson 10-7 Resources

Chapter 10 Resource Masters
Lesson Reading Guide, p. 49 BL OL
Study Guide and Intervention, pp. 50–51 BL OL
Skills Practice, p. 52 BL OL
Practice, p. 53 OL AL
Word Problem Practice, p. 54 BL OL AL
Enrichment, p. 55 OL AL

Transparencies
5-Minute Check Transparency 10-7

Additional Print Resources
Noteables™ Interactive Study Notebook with Foldables™
Teaching Geometry with Manipulatives

Technology
geometryonline.com
Interactive Classroom CD-ROM
AssignmentWorks CD-ROM
Graphing Calculator Easy Files

- Name the inscribed angles in the figure. ∠A, ∠B, ∠C, ∠D, and ∠E
- Is △DFB inscribed in the circle? Why or why not? No; because F is not a point on the circle.

Segments Intersecting Inside a Circle

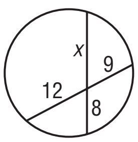
Examples 1 and 2 show how to use the theorem presented to find the measure of line segments inside a circle.

Formative Assessment

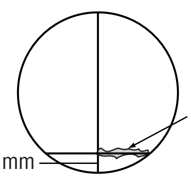
Use the Check Your Progress exercises after each example to determine students' understanding of concepts.

ADDITIONAL EXAMPLES

1 Find x. 13.5



2 **BIOLOGY** Biologists often examine organisms under microscopes. The circle represents the field of view under the microscope with a diameter of 2 mm. Determine the length of the organism if it is located 0.25 mm from the bottom of the field of view. Round to the nearest hundredth. 0.66 mm



Additional Examples also in:

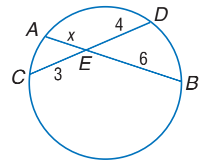
- Noteables™ Interactive Study Notebook with Foldables™
- Interactive Classroom PowerPoint® Presentations

Focus on Mathematical Content

Remind students that a diameter can be drawn to bisect any chord of a circle.

EXAMPLE Intersection of Two Chords

1 Find x.



$AE \cdot EB = CE \cdot ED$

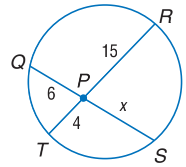
$x \cdot 6 = 3 \cdot 4$ Substitution

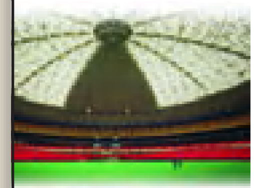
$6x = 12$ Multiply.

$x = 2$ Divide each side by 6.

Check Your Progress

1. Find x. **10**





Real-World Link

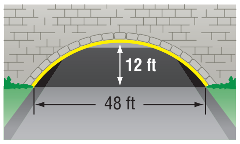
The Astrodome in Houston was the first ballpark to be built with a roof over the playing field. It originally had real grass and clear panels to allow sunlight in. This setup made it difficult to see the ball in the air, so they painted the ceiling and replaced the grass with carpet, which came to be known as "astro-turf."

Source: ballparks.com

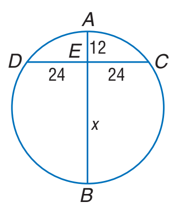
Intersecting chords can also be used to measure arcs.

Real-World EXAMPLE

TUNNELS **Tunnels are constructed to allow roadways to pass through mountains. What is the radius of the circle containing the arc if the opening is not a semicircle?**



Draw a model using a circle. Let x represent the unknown measure of the segment of diameter $\overline{AB}$. Use the products of the lengths of the intersecting chords to find the length of the diameter.



$AE \cdot EB = DE \cdot EC$ Segment products

$12x = 24 \cdot 24$ Substitution

$x = 48$ Divide each side by 12.

$AB = AE + EB$ Segment Addition Postulate

$AB = 12 + 48$ or 60 Substitution and addition

Since the diameter is 60, $r = 30$.

Check Your Progress

2. **ASTRODOME** The highest point, or apex, of the Astrodome is 208 feet high, and the diameter of the circle containing the arc is 710 feet. How long is the stadium from one side to the other? **about 323 ft**

Personal Tutor at geometryonline.com

Geometry Lab

Materials: compass, straightedge

Tell students to draw chords that are not congruent and that do not intersect at the center of the circle. Students can cut out the triangles and move them to verify that the triangles are similar, and they can determine the scale factor by which the triangles are related.

Segments Intersecting Outside a Circle Nonparallel chords of a circle that do not intersect inside the circle can be extended to form secants that intersect in the exterior of a circle. The special relationship among secant segments excludes the chord.

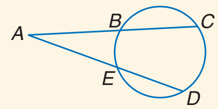
Study Tip

Helping You Remember

To remember this concept, the wording of Theorem 10.16 can be simplified by saying that each side of the equation is the product of the exterior part and the whole segment.

THEOREM 10.16

If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

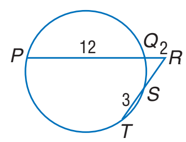


Example: $AB \cdot AC = AE \cdot AD$

You will prove Theorem 10.16 in Exercise 25.

EXAMPLE Intersection of Two Secants

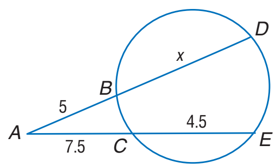
1 Find RS if $PQ = 12$, $QR = 2$, and $TS = 3$.



Let $RS = x$.

$QR \cdot PR = RS \cdot RT$	Secant Segment Products
$2 \cdot (12 + 2) = x \cdot (x + 3)$	Substitution
$28 = x^2 + 3x$	Distributive Property
$0 = x^2 + 3x - 28$	Subtract 28 from each side.
$0 = (x + 7)(x - 4)$	Factor.
$x + 7 = 0 \quad x - 4 = 0$	
$x = -7 \quad x = 4$	Disregard negative value.

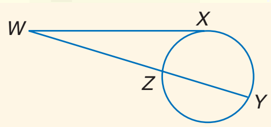
3. Find x. 13



The same secant segment product can be used with a secant segment and a tangent. In this case, the tangent is both the exterior part and the whole segment. This is stated in Theorem 10.17.

THEOREM 10.17

If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



Example: $WX \cdot WX = WZ \cdot WY$

You will prove Theorem 10.17 in Exercise 26.

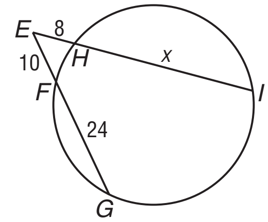
Extra Examples at geometryonline.com

Secant Theorems

In **Examples 3 and 4** students use properties and theorems to find lengths of external secant segments.

ADDITIONAL EXAMPLE

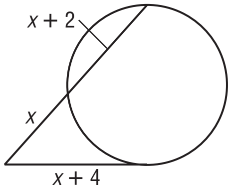
3 Find x if $EF = 10$, $EH = 8$, and $FG = 24$. 34.5



Lesson 10-7 Special Segments in a Circle 609

ADDITIONAL EXAMPLE

4 Find x. Assume that segments that appear to be tangent are tangent. 8



3 Practice

Formative Assessment

Use Exercises 1–4 to check for understanding.

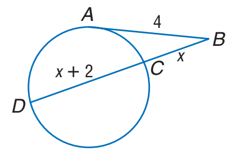
Then use the chart at the bottom of this page to customize assignments for your students.

Odd/Even Assignments

Exercises 5–15 are structured so that students practice the same concepts whether they are assigned odd or even problems.

EXAMPLE 4 Intersection of a Secant and a Tangent

Find x. Assume that segments that appear to be tangent are tangent.



$(AB)^2 = BC \cdot BD$

$4^2 = x(x + x + 2)$

$16 = x(2x + 2)$

$16 = 2x^2 + 2x$

$0 = 2x^2 + 2x - 16$

$0 = x^2 + x - 8$

This expression is not factorable. Use the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

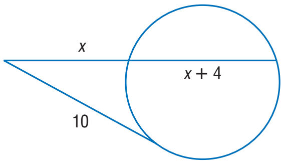
$= \frac{-1 \pm \sqrt{1^2 - 4(1)(-8)}}{2(1)}$ $a = 1, b = 1, c = -8$

$= \frac{-1 + \sqrt{33}}{2}$ or $x = \frac{-1 - \sqrt{33}}{2}$ Disregard the negative solution.

≈ 2.37 Use a calculator.

CHECK Your Progress

4. Find x. $-1 + \sqrt{51} \approx 6.14$

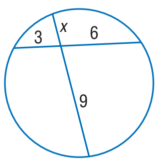


★ indicates multi-step problem

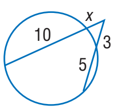
CHECK Your Understanding

Examples 1, 3, 4 (pp. 608, 609, 610)

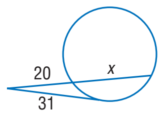
Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

1. 

2

2. 

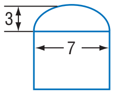
2

3. 

28.05

Example 2 (p. 608)

★**4. HISTORY** The Roman Coliseum has many "entrances" in the shape of a door with an arched top. The ratio of the arch width to the arch height is 7:3. Find the ratio of the arch width to the radius of the circle that contains the arch. 7:3.5



DIFFERENTIATED HOMEWORK OPTIONS

Level	Assignment	Two-Day Option	
BL Basic	5–15, 27–29, 31–43	5–15 odd, 32, 33	6–14 even, 27–29, 31, 34–43
OL Core	5–25 odd, 27–29, 31–43	5–15, 32, 33	16–29, 31, 34–43
AL Advanced /Pre-AP	16–43		

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Exercises

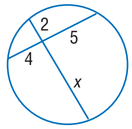
HOMEWORK HELP	
For Exercises	See Examples
5–7	1
8–9	2
10–12	3
13–15	4

Exercise Levels
A: 5–15
B: 16–28
C: 29–33

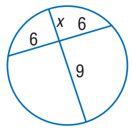
Real-World Career
Construction Worker
Construction workers must know how to measure and fit shapes together to make a sound building that will last for years to come. These workers also must master using machines to cut wood and metal to certain specifications that are based on geometry.

For more information, go to geometryonline.com.

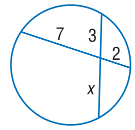
Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

5. 

10

6. 

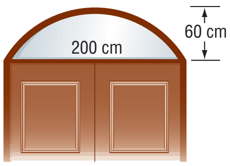
4

7. 

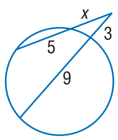
4.7

★**8. KNOBS** If you remove a knob from a kitchen appliance, you will notice that the hole is not completely round. Suppose the flat edge is 4 millimeters long and the distance from the curved edge to the flat edge is about 4.25 millimeters. Find the radius of the circle containing the hole. **about 2.6 mm**

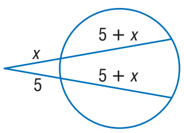
★**9. ARCHITECTURE** An arch over a courtroom door is 60 centimeters high and 200 centimeters wide. Find the radius of the circle containing the arc of the arch. **113.3 cm**



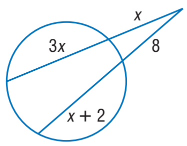
Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

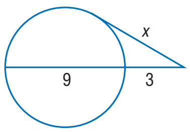
10. 

4

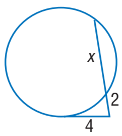
11. 

5

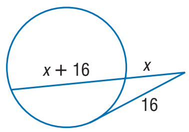
12. **5.6** 

13. 

6

14. 

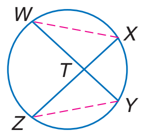
6

15. **8** 

16. PROOF Copy and complete the proof of Theorem 10.15.

Given: $\overline{WY}$ and $\overline{ZX}$ intersect at T.

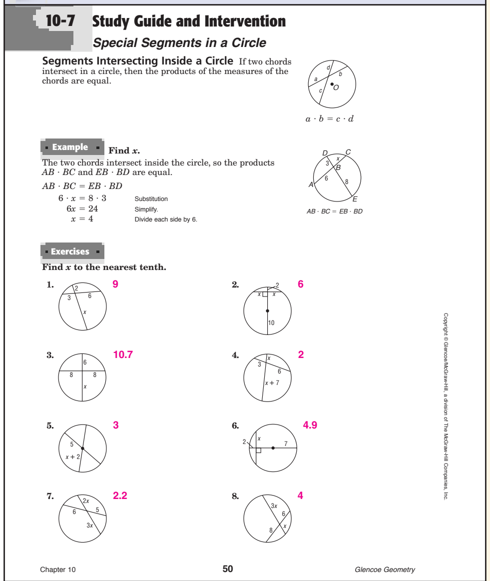
Prove: $WT \cdot TY = ZT \cdot TX$



Statements	Reasons
a. $\overline{WY}$ and $\overline{ZX}$ intersect at T	a. Given
b. $\angle W \cong \angle Z$, $\angle X \cong \angle Y$	b. ?
c. ? $\triangle WXT \sim \triangle ZYT$	c. AA Similarity
d. $\frac{WT}{ZT} = \frac{TX}{TY}$	d. ? **Def. of ~ △s**
e. ? $WT \cdot TY = ZT \cdot TX$	e. Cross products

16. b. Inscribed ∠s that intercept the same arc are ≅.

Study Guide and Intervention
CRM pp. 50–51 BL OL ELL


10-7 Study Guide and Intervention
Special Segments in a Circle

Segments Intersecting Inside a Circle If two chords intersect in a circle, then the products of the measures of the chords are equal.

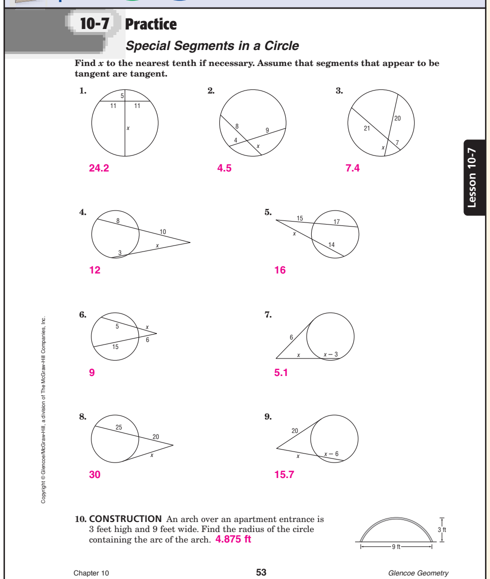
$a \cdot b = c \cdot d$

Example Find x.
The two chords intersect inside the circle, so the products $AB \cdot BC$ and $EB \cdot BD$ are equal.
$AB \cdot BC = EB \cdot BD$
$6 \cdot x = 8 \cdot 3$ Substitution
$6x = 24$ Simplify.
$x = 4$ Divide each side by 6.

Exercises
Find x to the nearest tenth.
1. 9
2. 6
3. 10.7
4. 2
5. 3
6. 4.9
7. 2.2
8. 4

Chapter 10 50 Glencoe Geometry

Practice
CRM p. 53 OL AL

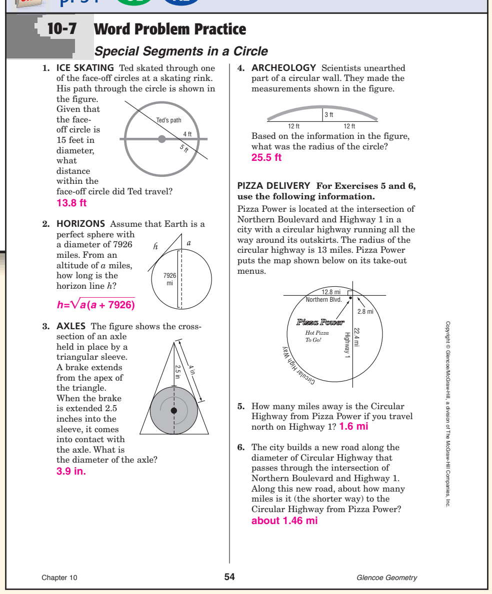

10-7 Practice
Special Segments in a Circle

Find x to the nearest tenth if necessary. Assume that segments that appear to be tangent are tangent.
1. 24.2
2. 4.5
3. 7.4
4. 12
5. 16
6. 9
7. 5.1
8. 30
9. 15.7

10. CONSTRUCTION An arch over an apartment entrance is 3 feet high and 9 feet wide. Find the radius of the circle containing the arc of the arch. 4.875 ft

Chapter 10 53 Glencoe Geometry

Word Problem Practice
CRM p. 54 OL AL


10-7 Word Problem Practice
Special Segments in a Circle

1. ICE SKATING Ted skated through one of the face-off circles at a skating rink. His path through the circle is shown in the figure. Given that the face-off circle is 15 feet in diameter, what distance within the face-off circle did Ted travel? 13.8 ft

2. HORIZONS Assume that Earth is a perfect sphere with a diameter of 7926 miles. From an altitude of a miles, how long is the horizon line h? $h = \sqrt{a(a + 7926)}$

3. AXLES The figure shows the cross-section of an axle held in place by a triangular sleeve. A brake extends from the apex of the triangle. When the brake is extended 2.5 inches into the sleeve, it comes into contact with the axle. What is the diameter of the axle? 3.9 in.

4. ARCHEOLOGY Scientists unearthed part of a circular wall. They made the measurements shown in the figure. Based on the information in the figure, what was the radius of the circle? 25.5 ft

PIZZA DELIVERY For Exercises 5 and 6, use the following information.
Pizza Power is located at the intersection of Northern Boulevard and Highway 1 in a city with a circular highway running all the way around its outskirts. The radius of the circular highway is 13 miles. Pizza Power puts the map shown below on its take-out menus.

5. How many miles away is the Circular Highway from Pizza Power if you travel north on Highway 1? 1.6 mi

6. The city builds a new road along the diameter of Circular Highway that passes through the intersection of Northern Boulevard and Highway 1. Along this new road, about how many miles is it (the shorter way) to the Circular Highway from Pizza Power? about 1.46 mi

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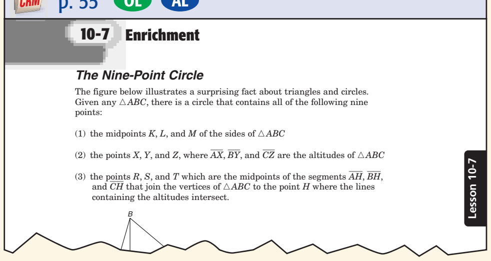
BL = Below Grade Level
OL = On Grade Level
AL = Above Grade Level
ELL = English Language Learner

Additional pages not shown:
CRM **Lesson Reading Guide**, p. 49 BL OL ELL
CRM **Skills Practice**, p. 52 BL OL

Enrichment
CRM p. 55 OL AL


10-7 Enrichment
The Nine-Point Circle

The figure below illustrates a surprising fact about triangles and circles. Given any △ABC, there is a circle that contains all of the following nine points:
(1) the midpoints K, L, and M of the sides of △ABC
(2) the points X, Y, and Z, where $\overline{AX}$, $\overline{BY}$, and $\overline{CZ}$ are the altitudes of △ABC
(3) the points R, S, and T which are the midpoints of the segments $\overline{AH}$, $\overline{BH}$, and $\overline{CH}$ that join the vertices of △ABC to the point H where the lines containing the altitudes intersect.

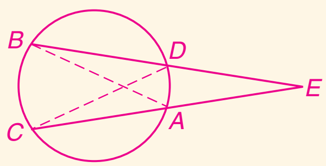
Exercise Alert!

Find the Error Point out to students that in this lesson they do not use the entire length of the interior secant segment in any of the relationships. Explain that in each case with exterior secant segments, they do use the entire length of the exterior secant segment.

Additional Answers

25. Given: $\overline{EC}$ and $\overline{EB}$ are secant segments.

Prove: $EA \cdot EC = ED \cdot EB$

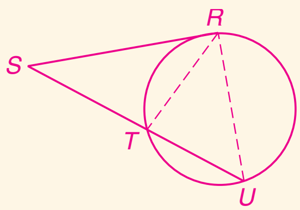


Proof:

$\overline{EC}$ and $\overline{EB}$ are secant segments. By the Reflexive Prop., $\angle DEC \cong \angle AEB$. Inscribed angles that intercept the same arc are congruent. So, $\angle ECD \cong \angle EAB$. By AA Similarity, $\triangle ABE \sim \triangle DCE$. By the definition of similar triangles, $\frac{EA}{ED} = \frac{EB}{EC}$. Since the cross products of a proportion are equal, $EA \cdot EC = ED \cdot EB$.

26. Given: tangent $\overline{RS}$ and secant $\overline{SU}$

Prove: $(SR)^2 = ST \cdot SU$

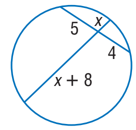


Proof:

Statements (Reasons)

1. tangent $\overline{SR}$ and secant $\overline{SU}$ (Given)
2. $m\angle RUT = \frac{1}{2}m\widehat{RT}$ (The measure of an inscribed $\angle$ = half the measure of its intercepted arc.)
3. $m\angle SRT = \frac{1}{2}m\widehat{RT}$ (The measure of an $\angle$ formed by a secant and a tangent = half the measure of its intercepted arc.)
4. $m\angle RUT = m\angle SRT$ (Substitution)
5. $\angle RUT \cong \angle SRT$ (Def. of $\cong \angle$s)
6. $\angle S \cong \angle S$ (Reflexive Prop.)
7. $\triangle SUR \sim \triangle SRT$ (AA Similarity)
8. $\frac{SR}{ST} = \frac{SU}{SR}$ (Def. of $\sim \triangle$s)
9. $(RS)^2 = ST \cdot SU$ (Cross Products)

Find each variable to the nearest tenth.

17. 

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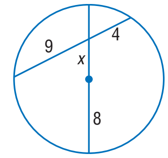
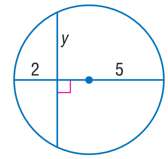
18. 

5.3

19. 

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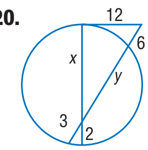
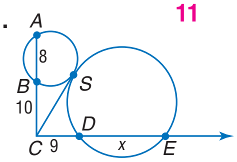
20. 

$y = 15$, $x = 22.5$

21. 

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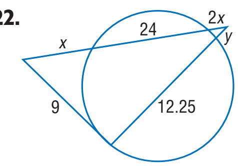
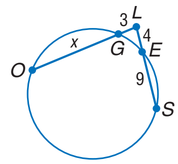
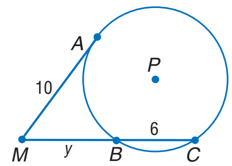
22. 

$x = 3$, $y = 8.6$

23. 

14.3

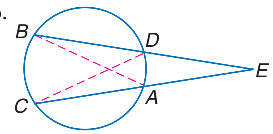
24. 

7.4

★**25. PROOF** Write a paragraph proof of Theorem 10.16.

Given: secants $\overline{EC}$ and $\overline{EB}$

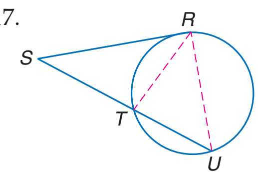
Prove: $EA \cdot EC = ED \cdot EB$ **See margin.**



★**26. PROOF** Write a two-column proof of Theorem 10.17.

Given: tangent $\overline{SR}$, secant $\overline{SU}$

Prove: $(SR)^2 = ST \cdot SU$ **See margin.**



EXTRA PRACTICE See pages 821, 837.

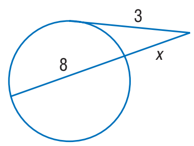
Self-Check Quiz at geometryonline.com

H.O.T. Problems

27. REASONING Explain how the products for secant segments are similar to the products for a tangent and a secant segment.

27. Sample answer: The product equation for secant segments equates the product of the exterior segment measure and the whole segment measure for each secant. In the case of secant-tangent, the product involving the tangent segment becomes (measure of tangent segment)^2 because the exterior segment and the whole segment are the same segment.

28. FIND THE ERROR Becky and Latisha are writing products to find x. Who is correct? Explain your reasoning.



Becky

$3^2 = x \cdot 8$

$9 = 8x$

$\frac{9}{8} = x$

Latisha

$3^2 = x(x + 8)$

$9 = x^2 + 8x$

$0 = x^2 + 8x - 9$

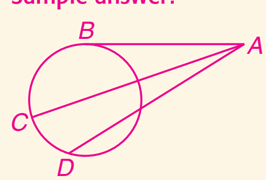
$0 = (x + 9)(x - 1)$

$x = 1$

28. Latisha; the length of the tangent segment squared equals the product of the exterior secant segment and the entire secant, not the interior secant segment.

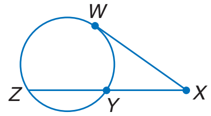
29. OPEN ENDED Draw a circle with two secant segments and one tangent segment that intersect at the same point. Give a real-life object that could be modeled by this drawing. **See margin.**

29. Sample answer:



This could represent rays of light traveling through a magnifying glass and being sent to one point.

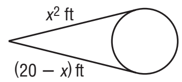
30. **CHALLENGE** In the figure, Y is the midpoint of $\overline{XZ}$. Find WX. Explain how you know this. **See margin.**



31. ***Writing in Math*** Use the figure on page 607 to explain how the lengths of intersecting chords are related. Describe the segments that are formed by the intersecting segments, $\overline{AD}$ and $\overline{EF}$, and the relationship among these segments. **See margin.**

STANDARDIZED TEST PRACTICE

32. Find two possible values for x from the information in the figure. **D**



A $-4, -5$ **C** $4, 5$

B $-4, 5$ **D** $4, -5$

33. **REVIEW** In the system of equations $4x + 3y = 6$ and $-5x + 2y = 13$, which expression can be substituted for x in the equation $4x + 3y = 6$? **H**

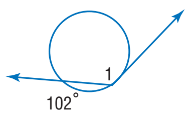
F $\frac{3}{2} - \frac{3}{4}y$ **H** $-\frac{13}{5} + \frac{2}{5}y$

G $6 - 3y$ **J** $13 - 2y$

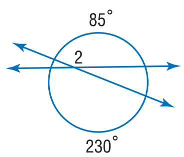
Spiral Review

Find the measure of each numbered angle. Assume that segments that appear tangent are tangent. (Lesson 10-6)

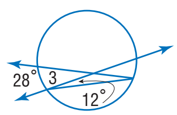
34. **129**



35. **157.5**

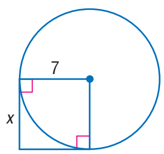


36. **26**

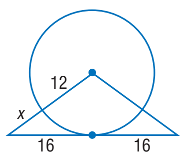


Find x. Assume that segments that appear to be tangent are tangent. (Lesson 10-5)

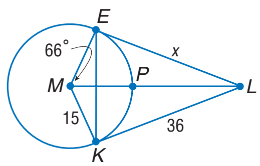
37. **7**



38. **8**



39. **36**



40. **INDIRECT MEASUREMENT** Joseph is measuring the width of a stream to build a bridge over it. He picks out a rock across the stream as landmark A and places a stone on his side as point B. Then he measures 5 feet at a right angle from $\overline{AB}$ and marks this C. From C, he sights a line to point A on the other side of the stream and measures the angle to be about 67°. How far is it across the stream rounded to the nearest whole foot? (Lesson 8-5) **12 ft**

GET READY for the Next Lesson

PREREQUISITE SKILL **Find the distance between each pair of points.** (Lesson 1-3)

41. $C(-2, 7), D(10, 12)$ **13** **42.** $E(1, 7), F(3, 4)$ $\sqrt{13}$ **43.** $G(9, -4), H(15, -2)$ $\sqrt{40}$

4 Assess

Yesterday's News Ask students to describe how the lesson on secants, tangents, and angles helped them better understand the lesson on special segments in a circle.

Foldables™ Follow-Up

Remind students to use the seventh flap in their Foldables to record notes on what they have learned about special segments in a circle.

Additional Answers

30. $WX = \sqrt{2} \cdot XY$ because:

$ZY = XY$

$(WX)^2 = XY \cdot XZ$

$(WX)^2 = XY(XY + ZY)$

$(WX)^2 = XY(2XY)$

$(WX)^2 = 2(XY)^2$

$WX = \sqrt{2(XY)^2}$

$WX = \sqrt{2} \cdot XY$

31. Sample answer: The product of the parts on one intersecting chord equals the product of the parts of the other chord. Answers should include the following: $\overline{AF}$, $\overline{FD}$, $\overline{EF}$, $\overline{FB}$, and $AF \cdot FD = EF \cdot FB$.

Pre-AP Activity Use before the Exercises.

In the figure, decide whether a polygon is inscribed in a circle, circumscribed about a circle, or neither of these. The polygon is inscribed in the circle.