10-8 Lesson Notes

1 Focus

Vertical Alignment

Before Lesson 10-8
Use numeric and geometric patterns to make generalizations about geometric properties including angle relationships in circles.

Lesson 10-8
Make conjectures about circles and determine the validity of the conjectures, choosing from a variety of approaches such as coordinates.
Use two-dimensional coordinate systems to represent figures.

After Lesson 10-8
Extend similarity properties and transformations to explore and justify conjectures about geometric figures.

2 Teach

Scaffolding Questions

Have students observe the picture and read the adjacent caption in *Get Ready for the Lesson.*

Ask:

- Refer to a dictionary and find the definition of concentric. a common middle point

(continued on the next page)

10-8 Equations of Circles

Main Ideas

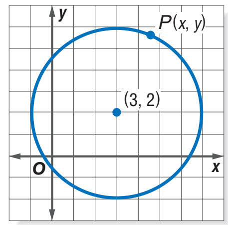
- Write the equation of a circle.
- Graph a circle on the coordinate plane.

GET READY for the Lesson

When a rock enters the water, ripples move out from the center forming concentric circles. If the rock is assigned coordinates, each ripple can be modeled by an equation of a circle.

Equation of a Circle The fact that a circle is the *locus* of points in a plane equidistant from a given point creates an equation for any circle.

Suppose the center is at (3, 2) and the radius is 4. The radius is the distance from the center. Let $P(x, y)$ be the endpoint of any radius.



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

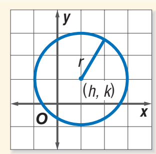
$4 = \sqrt{(x - 3)^2 + (y - 2)^2}$ $d = 4, (x_1, y_1) = (3, 2)$

$16 = (x - 3)^2 + (y - 2)^2$ Square each side.

Applying this same procedure to an unknown center (h, k) and radius r yields a general equation for any circle.

KEY CONCEPT — *Standard Equation of a Circle*

An equation for a circle with center at (h, k) and radius of r units is $(x - h)^2 + (y - k)^2 = r^2$.



Study Tip

Equations of Circles
Note that the equation of a circle is kept in the form shown above. The terms being squared are not expanded.

EXAMPLE Equation of a Circle

1 Write an equation for the circle with center at (−2, 4), $d = 4$.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$[x - (-2)]^2 + [y - 4]^2 = 2^2$ $(h, k) = (-2, 4)$, If $d = 4$, $r = 2$.

$(x + 2)^2 + (y - 4)^2 = 4$ Simplify.

CHECK Your Progress

Write an equation for each circle described below.

1A. center at (3, −2), $d = 10$ **1B.** center at origin, $r = 6$

1A. $(x - 3)^2 + (y + 2)^2 = 25$
1B. $x^2 + y^2 = 36$

Lesson 10-8 Resources

Chapter 10 Resource Masters

- Lesson Reading Guide, p. 56 BL OL
- Study Guide and Intervention, pp. 57–58 BL OL
- Skills Practice, p. 59 BL OL
- Practice, p. 60 OL AL
- Word Problem Practice, p. 61 BL OL AL
- Enrichment, p. 62 OL AL
- Quiz 4, p. 66

Transparencies

5-Minute Check Transparency 10-8

Additional Print Resources

Noteables™ Interactive Study Notebook with Foldables™

Technology

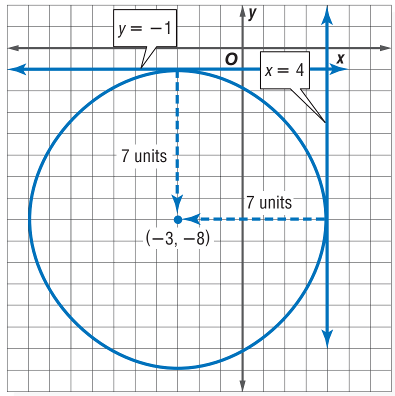
geometryonline.com
Interactive Classroom CD-ROM
AssignmentWorks CD-ROM
Graphing Calculator Easy Files

Other information about a circle can be used to find the equation of the circle.

EXAMPLE **Use Characteristics of Circles**

A circle with a diameter of 14 has its center in the third quadrant. The lines $y = -1$ and $x = 4$ are tangent to the circle. Write an equation of the circle.

Sketch a drawing of the two tangent lines.



Since $d = 14$, $r = 7$. The line $x = 4$ is perpendicular to a radius. Since $x = 4$ is a vertical line, the radius lies on a horizontal line. Count 7 units to the left from $x = 4$. Find the value of h.

$$h = 4 - 7 \text{ or } -3$$

Likewise, the radius perpendicular to the line $y = -1$ lies on a vertical line. The value of k is 7 units down from -1.

$$k = -1 - 7 \text{ or } -8$$

The center is at $(-3, -8)$, and the radius is 7. An equation for the circle is $(x + 3)^2 + (y + 8)^2 = 49$.

Check Your Progress

2. A circle with center at (5, 4) has a radius with endpoint at (−3, 4). Write an equation of the circle. $(x - 5)^2 + (y - 4)^2 = 64$

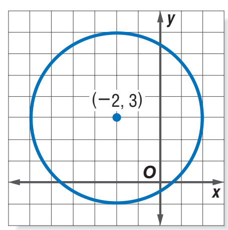
Personal Tutor at geometryonline.com

Graph Circles You can analyze the equation of a circle to find information that will help you graph the circle on a coordinate plane.

EXAMPLE **Graph a Circle**

Graph $(x + 2)^2 + (y - 3)^2 = 16$.

Compare each expression in the equation to the standard form.



$(x - h)^2 = (x + 2)^2$ | $(y - k)^2 = (y - 3)^2$

$x - h = x + 2$ | $y - k = y - 3$

$-h = 2$ | $-k = -3$

$h = -2$ | $k = 3$

$r^2 = 16$, so $r = 4$.

The center is at $(-2, 3)$, and the radius is 4.

Graph the center. Use a compass set at a width of 4 grid squares to draw the circle.

Check Your Progress **3A–3B. See margin.**

3A. $(x - 4)^2 + (y + 1)^2 = 9$ **3B.** $x^2 + y^2 = 25$

Study Tip

Graphing Calculator

To use the center and radius to graph a circle, select a suitable window that contains the center of the circle. For a TI-83/84 Plus, press ZOOM 5.

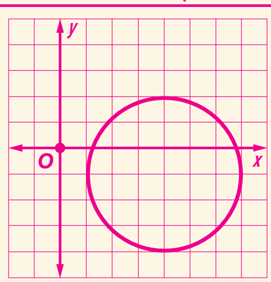
Then use **9: Circle (** on the **Draw** menu. Put in the coordinates of the center and then the radius so that the screen shows "Circle (−2, 3, 4)."

Then press ENTER.

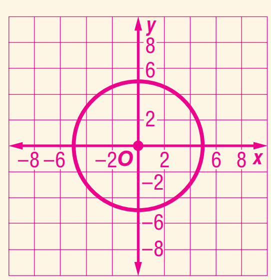
Extra Examples at geometryonline.com

Additional Answers (Check Your Progress)

3A.



3B.



- In order to cause water ripples to form concentric circles, what has to happen? Something must break the surface tension of the water, creating the force that causes the ripples, like the rock in the example.
- If the rock is thrown with a greater force, would you see fewer circles or more circles? more circles

Equation of a Circle

Examples 1 and 2 show how to use information given about a circle to find the equation of a circle.

Formative Assessment

Use the Check Your Progress exercises after each example to determine students' understanding of concepts.

ADDITIONAL EXAMPLES

1. Write an equation for the circle with center at (3, −3), $d = 12$. $(x - 3)^2 + (y + 3)^2 = 36$
2. A circle with diameter 10 has its center in the first quadrant. The lines $y = -3$ and $x = -1$ are tangent to the circle. Write an equation of the circle. $(x - 4)^2 + (y - 2)^2 = 25$

Additional Examples also in:

- Noteables™ Interactive Study Notebook with Foldables™
- Interactive Classroom PowerPoint® Presentations

Focus on Mathematical Content

In **Example 2,** students should note that the two tangent lines have slopes that indicate they are perpendicular to each other.

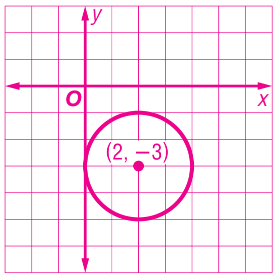
Students should remember that a radius is the shortest distance from the tangent to the center of the circle.

Graph Circles

Examples 3 and 4 show how to analyze the equation of a circle that will help you graph the circle on a coordinate plane.

ADDITIONAL EXAMPLES

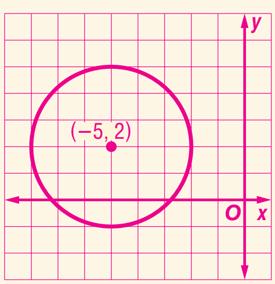
3 Graph $(x - 2)^2 + (y + 3)^2 = 4$.



4 **ELECTRICITY** Strategically located substations are extremely important in the transmission and distribution of a power company's electric supply. Suppose three substations are modeled by the points $D(3, 6)$, $E(-1, 0)$, and $F(3, -4)$. Determine the location of a town equidistant from all three substations, and write an equation for the circle.
$(4, 1)$; $(x - 4)^2 + (y - 1)^2 = 26$

Additional Answers

1. $x^2 + y^2 = 1600$
2. $(x + 3)^2 + (y - 5)^2 = 100$
3. $x^2 + y^2 = 7$
4. $(x + 2)^2 + (y - 11)^2 = 32$
5. $(x + 11)^2 + (y - 2)^2 = 32$
6.



If you know three points on the circle, you can find the center and radius of the circle and write its equation.

Study Tip

Locus
The center of the circle is the locus of points equidistant from the three given points. This is a **compound locus** because the point satisfies more than one condition.

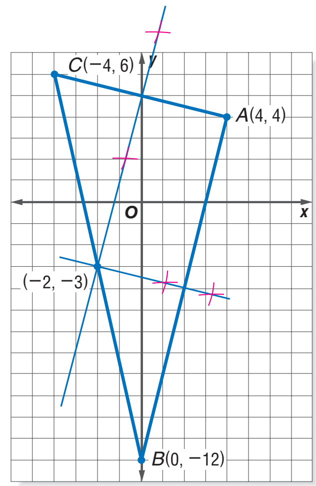
Real-World EXAMPLE

5 **CELL PHONES** **Cell phones work by the transfer of phone signals from one tower to another via satellite. Cell phone companies try to locate towers so that they service multiple communities. Suppose three large metropolitan areas are modeled by the points $A(4, 4)$, $B(0, -12)$, and $C(-4, 6)$, and each unit equals 100 miles. Determine the location of a tower equidistant from all three cities, and write an equation for the circle.**

Explore You are given three points that lie on a circle.

Plan Graph $\triangle ABC$. Construct the perpendicular bisectors of two sides to locate the center, which is the location of the tower. Find the length of a radius. Use the center and radius to write an equation.

Solve Graph $\triangle ABC$ and construct the perpendicular bisectors of two sides. The center appears to be at $(-2, -3)$. This is the location of the tower.



Find r by using the Distance Formula with the center and any of the three points.

$$r = \sqrt{[-2 - 4]^2 + [-3 - 4]^2}$$
$$= \sqrt{85}$$

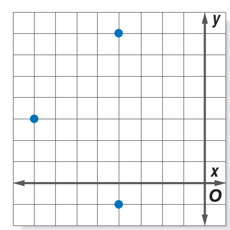
Write an equation.

$$[x - (-2)]^2 + [y - (-3)]^2 = (\sqrt{85})^2$$
$$(x + 2)^2 + (y + 3)^2 = 85$$

Check You can verify the location of the center by finding the equations of the two bisectors and solving a system of equations. You can verify the radius by finding the distance between the center and another of the three points on the circle.

Check Your Progress

4. Three tornado sirens are placed strategically on a circle around a town so that they can be heard by all of the people. Write the equation of the circle on which they are placed. $(x + 4)^2 + (y - 3)^2 = 16$



Differentiated Instruction

Logical/Mathematical Learners Explain that students will rely heavily on their geometric knowledge and reasoning skills to solve the problems in this lesson. Allow students to explain how to explore and collaborate as they work through examples and exercises. Students need to recall definitions, concepts, and theorems to help explain why they use certain methods to solve problems.

★ indicates multi-step problem

CHECK Your Understanding

Example 1 (p. 614)

1. WEATHER Meteorologists track severe storms using Doppler radar. A polar grid is used to measure distances as the storms progress. If the center of the radar screen is the origin and each ring is 10 miles farther from the center, what is the equation of the fourth ring? **See margin.**

Example 2 (p. 615)

Write an equation for each circle described below.

2. center at $(-3, 5)$, $r = 10$ **2–5. See margin.**
3. center at origin, $r = \sqrt{7}$
4. diameter with endpoints at $(2, 7)$ and $(-6, 15)$
5. diameter with endpoints at $(-7, -2)$ and $(-15, 6)$

Example 3 (p. 615)

Graph each equation. **6–7. See margin.**

6. $(x + 5)^2 + (y - 2)^2 = 9$ **7.** $(x - 3)^2 + y^2 = 16$

Example 4 (p. 616)

8. Write an equation of a circle that contains $M(-2, -2)$, $N(2, -2)$, and $Q(2, 2)$. Then graph the circle. **See margin.**

Exercises

HOMEWORK HELP

For Exercises	See Examples
9–16	1
17–21	2
22–27	3
28–29	4

Exercise Levels
A: 9–29
B: 30–40
C: 41–44

Write an equation for each circle described below. **9–19. See margin.**

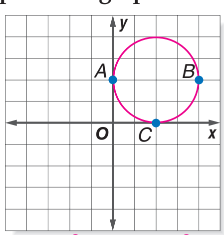
9. center at origin, $r = 3$
10. center at $(-2, -8)$, $r = 5$
11. center at $(1, -4)$, $r = \sqrt{17}$
12. center at $(0, 0)$, $d = 12$
13. center at $(5, 10)$, $r = 7$
14. center at $(0, 5)$, $d = 20$
15. center at $(-8, 8)$, $d = 16$
16. center at $(-3, -10)$, $d = 24$
17. a circle with center at $(-3, 6)$ and a radius with endpoint at $(0, 6)$.
18. a circle whose diameter has endpoints at $(2, 2)$ and $(-2, 2)$
19. a circle with center at $(-2, 1)$ and a radius with endpoint at $(1, 0)$
20. a circle with $d = 12$ and a center translated 18 units left and 7 units down from the origin **$(x + 18)^2 + (y + 7)^2 = 36$**
21. a circle with its center in quadrant I, radius of 5 units, and tangents $x = 2$ and $y = 3$ **$(x - 7)^2 + (y - 8)^2 = 25$**

Graph each equation. **22–27. See Ch. 10 Answer Appendix.**

22. $x^2 + y^2 = 25$ **23.** $x^2 + y^2 = 36$ **24.** $x^2 + y^2 - 1 = 0$
25. $x^2 + y^2 - 49 = 0$ **26.** $(x - 2)^2 + (y - 1)^2 = 4$ **27.** $(x + 1)^2 + (y + 2)^2 = 9$

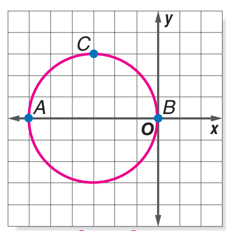
Write an equation of the circle containing each set of points. Copy and complete the graph of the circle.

28.



$(x - 2)^2 + (y - 2)^2 = 4$

29.



$(x + 3)^2 + y^2 = 9$

DIFFERENTIATED HOMEWORK OPTIONS

Level	Assignment	Two-Day Option	
BL Basic	9–29, 41, 43, 44–56	9–29 odd, 45, 46	10–28 even, 41, 43, 44, 47–56
OL Core	9–29 odd, 30–41, 43–56	9–29, 45, 46	30–41, 43, 44, 47–56
AL Advanced /Pre-AP	30–56		

3 Practice

Formative Assessment

Use Exercises 1–8 to check for understanding.

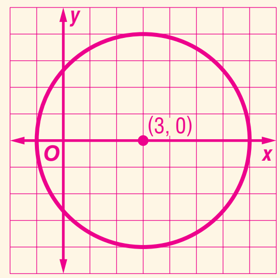
Then use the chart at the bottom of this page to customize assignments for your students.

Odd/Even Assignments

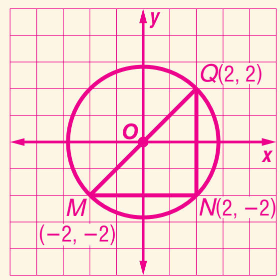
Exercises 9–29 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Additional Answers

7.



8. $x^2 + y^2 = 8$;



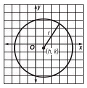
9. $x^2 + y^2 = 9$
10. $(x + 2)^2 + (y + 8)^2 = 25$
11. $(x - 1)^2 + (y + 4)^2 = 17$
12. $x^2 + y^2 = 36$
13. $(x - 5)^2 + (y - 10)^2 = 49$
14. $x^2 + (y - 5)^2 = 100$
15. $(x + 8)^2 + (y - 8)^2 = 64$
16. $(x + 3)^2 + (y + 10)^2 = 144$
17. $(x + 3)^2 + (y - 6)^2 = 9$
18. $x^2 + (y - 2)^2 = 4$
19. $(x + 2)^2 + (y - 1)^2 = 10$

Study Guide and Intervention

CRM pp. 57–58 BL OL ELL

10-8 Study Guide and Intervention
Equations of Circles

Equation of a Circle A **circle** is the locus of points in a plane equidistant from a given point. You can use this definition to write an equation of a circle.



Standard Equation of a Circle	An equation for a circle with center at (h, k) and a radius of r units is $(x - h)^2 + (y - k)^2 = r^2$.

Example **Write an equation for a circle with center (−1, 3) and radius 6.**

Use the formula $(x - h)^2 + (y - k)^2 = r^2$ with $h = -1$, $k = 3$, and $r = 6$.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$(x - (-1))^2 + (y - 3)^2 = 6^2$ Substitution

$(x + 1)^2 + (y - 3)^2 = 36$ Simplify.

Exercises

Write an equation for each circle.

1. center at $(0, 0)$, $r = 8$
   $x^2 + y^2 = 64$
2. center at $(-2, 3)$, $r = 5$
   $(x + 2)^2 + (y - 3)^2 = 25$
3. center at $(2, -4)$, $r = 1$
   $(x - 2)^2 + (y + 4)^2 = 1$
4. center at $(-1, -4)$, $r = 2$
   $(x + 1)^2 + (y + 4)^2 = 4$
5. center at $(-2, -6)$, diameter $= 8$
   $(x + 2)^2 + (y + 6)^2 = 16$
6. center at $\left(-\frac{1}{2}, \frac{1}{4}\right)$, $r = \sqrt{3}$
   $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = 3$
7. center at the origin, diameter $= 4$
   $x^2 + y^2 = 4$
8. center at $\left(1, -\frac{5}{8}\right)$, $r = \sqrt{5}$
   $(x - 1)^2 + \left(y + \frac{5}{8}\right)^2 = 5$
9. Find the center and radius of a circle with equation $x^2 + y^2 = 20$.
   center (0, 0); radius $2\sqrt{5}$
10. Find the center and radius of a circle with equation $(x + 4)^2 + (y + 3)^2 = 16$.
   center (−4, −3); radius 4

Lesson 10-8

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Practice

CRM p. 60 OL AL

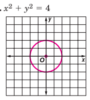
10-8 Practice
Equations of Circles

Write an equation for each circle.

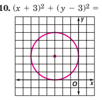
1. center at origin, $r = 7$
   $x^2 + y^2 = 49$
2. center at $(0, 0)$, $d = 18$
   $x^2 + y^2 = 81$
3. center at $(-7, 11)$, $r = 8$
   $(x + 7)^2 + (y - 11)^2 = 64$
4. center at $(12, -9)$, $d = 22$
   $(x - 12)^2 + (y + 9)^2 = 121$
5. center at $(-6, -4)$, $r = \sqrt{5}$
   $(x + 6)^2 + (y + 4)^2 = 5$
6. center at $(3, 0)$, $d = 28$
   $(x - 3)^2 + y^2 = 196$
7. a circle with center at $(-5, 3)$ and a radius with endpoint $(2, 3)$
   $(x + 5)^2 + (y - 3)^2 = 49$
8. a circle whose diameter has endpoints $(4, 6)$ and $(-2, 6)$
   $(x - 1)^2 + (y - 6)^2 = 9$

Graph each equation.

9. $x^2 + y^2 = 4$



10. $(x + 3)^2 + (y - 3)^2 = 9$



11. **EARTHQUAKES** When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Seismograph stations monitor seismic activity and record the intensity and duration of earthquakes. Suppose a station determines that the epicenter of an earthquake is located about 50 kilometers from the station. If the station is located at the origin, write an equation for the circle that represents a possible epicenter of the earthquake. $x^2 + y^2 = 2500$

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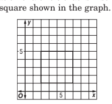
Word Problem Practice

CRM p. 61 OL AL

10-8 Word Problem Practice
Equations of Circles

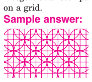
1. **DESIGN** Arthur wants to write the equation of a circle that is inscribed in the square shown in the graph.



   What is the equation of the desired circle?
   $(x - 4)^2 + (y - 3)^2 = 4$

2. **DRAFTING** The design for a park is drawn on a coordinate graph. The perimeter of the park is modeled by the equation $(x - 3)^2 + (x - 7)^2 = 225$. Each unit on the graph represents 10 feet. What is the radius of the actual park?
   150 ft

3. **WALLPAPER** The design of a piece of wallpaper consists of circles that can be modeled by the equation $(x - a)^2 + (y - b)^2 = 4$, for all even integers b. Sketch part of the wallpaper on a grid.
   Sample answer:



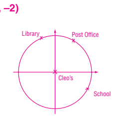
4. **SECURITY RING** A circular safety ring surrounds a top-secret laboratory. On one map of the laboratory grounds, the safety ring is given by the equation $x^2 + y^2 - 20x + 14y = 175$. Each unit on the map represents 1 mile. What is the radius of the safety ring?
   18 mi

DISTANCE For Exercises 5-7, use the following information.

Cleo lives the same distance from the library, the post office, and her school. The table below gives the coordinates of these places on a map with a coordinate grid where one unit represents one yard.

Location	Coordinates
Library	(–78, 202)
Post Office	(111, 193)
School	(202, –106)

5. What are the coordinates of Cleo's home? Sketch the circle on a map locating all three places and Cleo's home.
   (7, –2)



6. How far is Cleo's house from the places mentioned?
   221 yd

7. Write an equation for the circle that passes through the library, post office, and school.
   $(x - 7)^2 + (y + 2)^2 = 221^2$

Lesson 10-8

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30. Find the radius of a circle that has equation $(x - 2)^2 + (y - 3)^2 = r^2$ and contains (2, 5). **2**

31. Find the radius of a circle that has equation $(x - 5)^2 + (y - 3)^2 = r^2$ and contains (5, 1). **2**

32. **COORDINATE GEOMETRY** Refer to the Check part of Example 4. Verify the coordinates of the center by solving a system of equations that represent the perpendicular bisectors. **See Ch. 10 Answer Appendix.**

MODEL ROCKETS For Exercises 33–35, use the following information.

Different sized engines will launch model rockets to different altitudes. The higher the rocket goes, the larger the circle of possible landing sites becomes. Under normal wind conditions, the landing radius is three times the altitude of the rocket.

33. Write the equation of the landing circle for a rocket that travels 300 feet in the air. **See margin.**

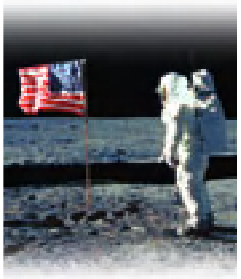
34. What type of circles are modeled by the landing areas for engines that take the rocket to different altitudes? **concentric circles**

35. What would the radius of the landing circle be for a rocket that travels 1000 feet in the air? **3000 ft**

36. The equation of a circle is $(x - 6)^2 + (y + 2)^2 = 36$. Determine whether the line $y = 2x - 2$ is a secant, a tangent, or neither of the circle. Explain. **See margin.**

37. The equation of a circle is $x^2 - 4x + y^2 + 8y = 16$. Find the center and radius of the circle. **(2, −4); $r = 6$**

38. **WEATHER** The geographic center of Tennessee is near Murfreesboro. The closest Doppler weather radar is in Nashville. If Murfreesboro is designated as the origin, then Nashville has coordinates (−58, 55), where each unit is one mile. If the radar has a radius of 80 miles, write an equation for the circle that represents the radar coverage from Nashville. **$(x + 58)^2 + (y - 55)^2 = 6400$**



Real-World Link

The Apollo program was designed to successfully land a man on the moon. The first landing was July 20, 1969. There was a total of six landings on the moon during 1969–1972.

Source: infoplease.com

39. **SPACE TRAVEL** Apollo 8 was the first manned spacecraft to orbit the Moon at an average altitude of 185 kilometers above the Moon's surface. Write an equation to model a single circular orbit of the command module if the radius of the Moon is 1740 kilometers. Let the center of the Moon be at the origin. **$x^2 + y^2 = 3{,}705{,}625$**

40. **RESEARCH** Use the Internet or other materials to find the closest Doppler radar to your home. Write an equation of the circle for the radar coverage if your home is the center. **See students' work.**

EXTRA PRACTICE

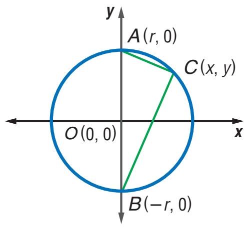
See pages 821, 837.

Self-Check Quiz at geometryonline.com

H.O.T. Problems

41. **OPEN ENDED** Draw an obtuse triangle on a coordinate plane and construct the circle that circumscribes it. **See margin.**

42. **CHALLENGE** Write a coordinate proof to show that if an inscribed angle intercepts the diameter of a circle, as shown the angle is a right angle. **See Ch. 10 Answer Appendix.**



43. **REASONING** Explain how the definition of a circle leads to its equation. **See margin.**

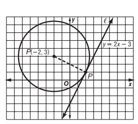
Enrichment

CRM p. 62 OL AL

10-8 Enrichment

Equations of Circles and Tangents

Recall that the circle whose radius is r and whose center has coordinates (h, k) is the graph of $(x - h)^2 + (y - k)^2 = r^2$. You can use this idea and what you know about circles and tangents to find an equation of the circle that has a given center and is tangent to a given line.



Use the following steps to find an equation for the circle that has center $C(-2, 3)$ and is tangent to the graph $y = 2x - 3$. Refer to the figure.

BL = *Below Grade Level*

OL = *On Grade Level*

AL = *Above Grade Level*

ELL = *English Language Learner*

Additional pages not shown:

CRM *Lesson Reading Guide*, p. 56 BL OL ELL

CRM *Skills Practice*, p. 59 BL OL

44. *Writing in Math* Refer to the information on page 614 to describe the kinds of equations used to describe the ripples of a splash. Include the general form of the equation of a circle in your answer. Then produce the equations of five ripples if each ripple is 3 inches farther from the center. **See margin.**

STANDARDIZED TEST PRACTICE

45. Which of the following is an equation of a circle with center at $(-2, 7)$ and a diameter of 18? **B**

A $x^2 + y^2 - 4x + 14y + 53 = 324$

B $x^2 + y^2 + 4x - 14y + 53 = 81$

C $x^2 + y^2 - 4x + 14y + 53 = 18$

D $x^2 + y^2 + 4x - 14y + 53 = 3$

46. **REVIEW** A rectangle has an area of 180 square feet and a perimeter of 54 feet. What are the dimensions of the rectangle? **H**

F 13 ft and 13 ft

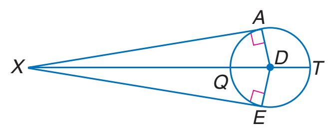
G 13 ft and 14 ft

H 15 ft and 12 ft

J 16 ft and 9 ft

Spiral Review

Find each measure if $EX = 24$ and $DE = 7$. (Lesson 10-7)



47. AX **24**

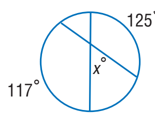
48. DX **25**

49. QX **18**

50. TX **32**

Find x. (Lesson 10-6)

51.



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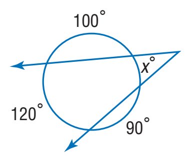
52.



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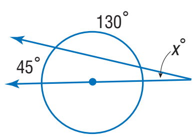
53.



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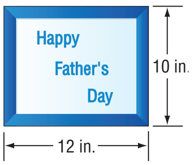
Use the following information for Exercises 54 and 55.
Triangle ABC has vertices $A(-3, 2)$, $B(4, -1)$, and $C(0, -4)$.

54. What are the coordinates of the image after moving $\triangle ABC$ 3 units left and 4 units up? (Lesson 9-2) **(−6, 6), (1, 3), (−3, 0)**

55. What are the coordinates of the image of $\triangle ABC$ after a reflection in the y-axis? (Lesson 9-1) **(3, 2), (−4, −1), (0, −4)**

56. **CRAFTS** For a Father's Day present, a kindergarten class is making foam plaques. The edge of each plaque is covered with felt ribbon all the way around with 1 inch overlap. There are 25 children in the class. How much ribbon does the teacher need for all 25 children to complete this craft? (Lesson 1-6) **1125 in. or 31.25 yd**



Pre-AP Activity

Use as an Extension.

What is the relationship between concentric circles with the same radius? Explain. They are the same circle. A center and a radius are all it takes to define a circle. Two circles that share a center and have the same radius are identical.

4 Assess

Name the Math Let students take turns saying an equation of a circle. Then they should name the centers of the circles, and state the lengths of the radii.

Formative Assessment

Check for student understanding of concepts in Lessons 10-7 and 10-8.

CRM Quiz 4, p. 66

Foldables™ Follow-Up

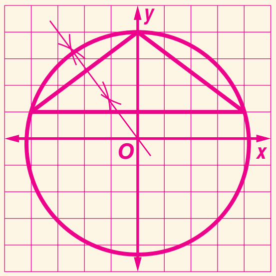
Remind students to use the eighth flap in their Foldables to record notes on what they have learned about equations of circles.

Additional Answers

33. $x^2 + y^2 = 810{,}000$

36. Secant; the line intersects the circle at $(0, -2)$ and $(2.4, 2.8)$.

41. Sample answer:



43. A circle is the locus of all points in a plane (coordinate plane) a given distance (the radius) from a given point (the center). The equation of a circle is written from knowing the location of the given point and the radius.

44. Sample answer: Equations of concentric circles; answers should include the following: $(x - h)^2 + (y - k)^2 = r^2$; $x^2 + y^2 = 9$; $x^2 + y^2 = 36$; $x^2 + y^2 = 81$; $x^2 + y^2 = 144$; $x^2 + y^2 = 225$

CHAPTER 10 Study Guide and Review

Dinah Zike's Foldables™

FOLDABLES Study Organizer

Have students look through the chapter to make sure they have included the key concepts under the proper lesson tabs in their Foldables.

Suggest that students keep their Foldables handy while completing the Study Guide and Review pages. Point out that their Foldables can serve as a quick review tool for studying for the Chapter Test.

Formative Assessment

Key Vocabulary The page references after each word denote where that term was first introduced. If students have difficulty completing Exercises 1–12, remind them that they can use these page references to refresh their memories about the vocabulary terms.

Math Online geometryonline.com

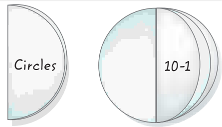
Vocabulary PuzzleMaker improves students' mathematics vocabulary using four puzzle formats-crossword, scramble, word search using a word list, and word search using clues. Students can work online or from a printed worksheet.

CHAPTER 10 Study Guide and Review

Download Vocabulary Review from geometryonline.com

FOLDABLES GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Circles and Circumference (Lesson 10-1)

- Circumference: $C = \pi d$ or $C = 2\pi r$

Angles, Arcs, Chords, and Inscribed Angles (Lessons 10-2 to 10-4)

- The sum of the measures of the central angles of a circle is 360. The measure of each arc is related to the measure of its central angle.
- The length of an arc is proportional to the length of the circumference.
- Diameters perpendicular to chords bisect chords and intercepted arcs.
- The measure of the inscribed angle is half the measure of its intercepted arc.

Tangents, Secants, and Angle Measures (Lessons 10-5 and 10-6)

- A line that is tangent to a circle intersects the circle in exactly one point and is perpendicular to a radius.
- Two segments tangent to a circle from the same exterior point are congruent.
- The measure of an angle formed by two secant lines is half the positive difference of its intercepted arcs.
- The measure of an angle formed by a secant and tangent line is half its intercepted arc.

Special Segments and Equation of a Circle (Lessons 10-7 and 10-8)

- The lengths of intersecting chords in a circle can be found by using the products of the measures of the segments.
- The equation of a circle with center (h, k) is $(x - h)^2 + (y - k)^2 = r^2$.

Key Vocabulary

arc (p. 564)	intercepted (p. 578)
center (p. 554)	major arc (p. 564)
central angle (p. 563)	minor arc (p. 564)
chord (p. 554)	pi (π) (p. 556)
circle (p. 554)	point of tangency (p. 588)
circumference (p. 556)	radius (p. 554)
circumscribed (p. 571)	secant (p. 599)
diameter (p. 554)	semicircle (p. 564)
inscribed (p. 571)	tangent (p. 588)

Vocabulary Check

Choose the term that best matches each phrase. Choose from the list above.

1. a line that intersects a circle in exactly one point **tangent**
2. a polygon with all of its vertices on the circle **inscribed**
3. an angle with a vertex that is at the center of the circle **central angle**
4. a segment that has its endpoints on the circle **chord**
5. a line that intersects a circle in exactly two points **secant**
6. the distance around a circle **circumference**
7. a chord that passes through the center of a circle **diameter**
8. an irrational number that is the ratio of $\frac{C}{d}$ **pi**
9. an arc that measures greater than 180
10. a point where a circle meets a tangent
11. locus of all points in a plane equidistant from a given point **circle**
12. a central angle separates the circle into two of these **arcs**

9. major arc
10. point of tangency

Vocabulary Review at geometryonline.com

Summative Assessment

CRM Vocabulary Test, p. 68

Lesson-by-Lesson Review

10–1 Circles and Circumference (pp. 554–561)

The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary. 14. 10.82 yd; 21.65 yd

13. $d = 15$ in., $r = \underline{?}$, $C = \underline{?}$ 7.5 in.; 47.12 in.

14. $C = 68$ yd, $r = \underline{?}$, $d = \underline{?}$

15. $r = 11$ mm, $d = \underline{?}$, $C = \underline{?}$
22 mm; 69.12 mm

16. BICYCLES If the circumference of the bicycle tire is 81.7 inches, how long is one spoke? 13 in.

Example 1 Find r to the nearest hundredth if $C = 76.2$ feet.

$C = 2\pi r$ Circumference formula

$76.2 = 2\pi r$ Substitution

$\frac{76.2}{2\pi} = r$ Divide each side by 2π.

$12.13 \approx r$ Use a calculator.

10–2 Measuring Angles and Arcs (pp. 563–569)

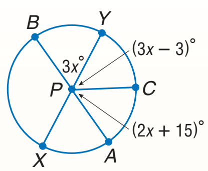
Find each measure.

17. $m\widehat{YC}$ 60

18. $m\widehat{BC}$ 123

19. $m\widehat{BX}$ 117

20. $m\widehat{BCA}$ 180

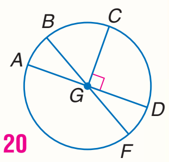


In ⊙G, $m\angle AGB = 30$ and $\overline{CG} \perp \overline{GD}$. Find each measure.

21. $m\widehat{AB}$ 30 **22.** $m\widehat{BC}$ 60

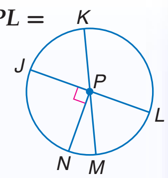
23. $m\widehat{FD}$ 30 **24.** $m\widehat{CDF}$ 120

25. $m\widehat{BCD}$ 150 **26.** $m\widehat{FAB}$ 180



27. CLOCKS If a clock has a diameter of 6 inches, what is the distance along the edge of the clock from the minute hand to the hour hand at 5:00?

Example 2 In ⊙P, $m\angle MPL = 65$ and $\overline{NP} \perp \overline{PL}$.



a. Find $m\widehat{NM}$.

$\widehat{NM}$ is a minor arc, so $m\widehat{NM} = m\angle NPM$. $\angle JPN$ is a right angle and $m\angle MPL = 65$, so $m\angle NPM = 25$.
$m\widehat{NM} = 25$

b. Find $m\widehat{NJK}$.

$\widehat{NJK}$ is composed of adjacent arcs $\widehat{NJ}$ and $\widehat{JK}$. $\angle MPL \cong \angle JPK$, so $m\angle JPK = 65$.

$m\widehat{NJ} = m\angle NPJ$ or 90 $\angle NPJ$ is a right angle.

$m\widehat{NJK} = m\widehat{NJ} + m\widehat{JK}$ Arc Addition Postulate

$m\widehat{NJK} = 90 + 65$ or 155 Substitution

27. $\frac{5}{2}\pi$ in. ≈ 7.9 in.

Chapter 10 Study Guide and Review

Lesson-by-Lesson Review

Intervention If the given examples are not sufficient to review the topics covered by the questions, remind students that the page references tell them where to review that topic in their textbooks.

Two-Day Option Have students complete the Lesson-by-Lesson Review on pp. 620–624. Then you can use ExamView® Assessment Suite to customize another review worksheet that practices all the objectives of this chapter or only the objectives on which your students need more help.

ExamView Assessment Suite For more information on ExamView® Assessment Suite, see p. 552C.

Differentiated Instruction

Super DVD: MindJogger Plus
Use this DVD as an alternative format of review for the test. For more information on this game show format, see p. 552D.

Additional Answers

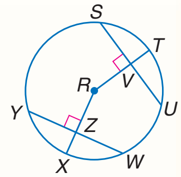
37. $m\angle 1 = m\angle 3 = 39$, $m\angle 2 = 51$

38. $m\angle 1 = m\angle 3 = 30$, $m\angle 2 = 60$

39. $m\angle 2 = 57$, $m\angle 3 = m\angle 1 = 33$

10-3 Arcs and Chords (pp. 570–577)

In $\odot R$, $SU = 20$, $YW = 20$, and $m\widehat{YX} = 45$. Find each measure.



28. SV **10**

29. WZ **10**

30. UV **10**

31. $m\widehat{YW}$ **90**

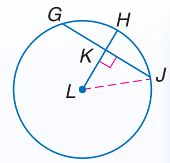
32. $m\widehat{ST}$ **45**

33. $m\widehat{SU}$ **90**

34. **ART** Leonardo DaVinci saw the ideal proportions for man as being measured in relation to two geometric shapes: the circle and the square. The square inscribed in a circle and the circle inscribed in a square are useful to artists, architects, engineers, and designers. Find the measure of each arc of the circle circumscribed about the square.

Each arc measures 90°.

Example 4 Circle L has a radius of 32. Find LK if $GJ = 40$.



Draw radius $\overline{LJ}$. $LJ = 32$ and $\triangle LKJ$ is a right triangle.

Since $\overline{LH} \perp \overline{GJ}$, $\overline{LH}$ bisects $\overline{GJ}$.

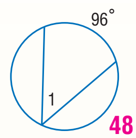
$KJ = \frac{1}{2}(GJ)$	Definition of segment bisector
$= \frac{1}{2}(40)$ or 20	$GJ = 40$

Use the Pythagorean Theorem to find LK.

$(LK)^2 + (KJ)^2 = (LJ)^2$	Pythagorean Theorem
$(LK)^2 + 20^2 = 32^2$	$KJ = 20$, $LJ = 32$
$(LK)^2 + 400 = 1024$	Simplify.
$(LK)^2 = 624$	Subtract.
$LK = \sqrt{624}$ or about 24.98	

10-4 Inscribed Angles (pp. 578–586)

Find the measure of each numbered angle.

35. 

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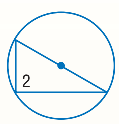
36. 

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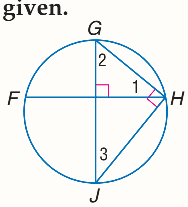
Find the measure of each numbered angle for each situation given.



37. $m\widehat{GH} = 78$

38. $m\angle 2 = 2x$, $m\angle 3 = x$

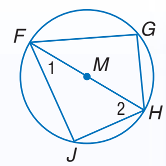
39. $m\widehat{JH} = 114$

37–39. See margin.

40. **ICE SKATING** Sara skates on a circular ice rink and inscribes quadrilateral $ABCD$ in the circle. If $m\angle A = 120$ and $m\angle B = 66$, find $m\angle C$ and $m\angle D$.

$m\angle C = 60$, $m\angle D = 114$

Example 5 Triangle FGH and FHJ are inscribed in $\odot M$ with $\widehat{FG} \cong \widehat{FJ}$. Find x if $m\angle 1 = 6x - 5$, and $m\angle 2 = 7x + 4$.



$\angle FJH$ is a right angle because $\widehat{FJH}$ is a semicircle.

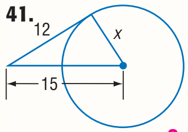
$m\angle 1 + m\angle 2 + m\angle FJH = 180$	Angle Sum Th.
$(6x - 5) + (7x + 4) + 90 = 180$	Substitution
$13x + 89 = 180$	Simplify.
$x = 7$	Solve for x.

Mixed Problem Solving
For mixed problem-solving practice, see page 835.

10–5 Tangents (pp. 588–596)

Find x. Assume that segments that appear to be tangent are tangent.

41.



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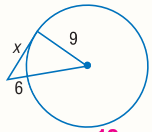
42.



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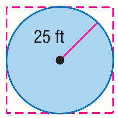
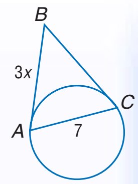
43. SPRINKLER A sprinkler waters a circular section of lawn that is surrounded by a fenced-in square field. If the spray extends to a distance of 25 feet, what is the total length of the fence around the field? **200 ft**



Example 6 Given that the perimeter of $\triangle ABC = 25$, find x. Assume that segments that appear to be tangent to circles are tangent.



In the figure, $\overline{AB}$ and $\overline{BC}$ are drawn from the same exterior point and are tangent to $\odot Q$. So, $\overline{AB} \cong \overline{BC}$.

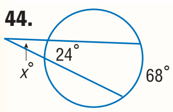
The perimeter of the triangle, $AB + BC + AC$, is 25.

$AB + BC + AC = 25$	Definition of perimeter
$3x + 3x + 7 = 25$	$AB = BC = 3x$, $AC = 7$
$6x + 7 = 25$	Simplify.
$6x = 18$	Subtract 7 from each side.
$x = 3$	Divide each side by 6.

10–6 Secants, Tangents, and Angle Measures (pp. 599–606)

Find x.

44.



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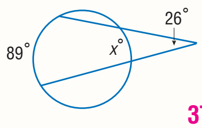
45.



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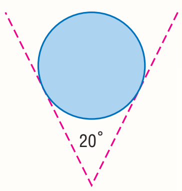
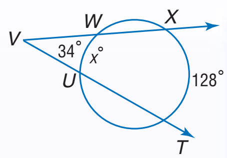
46. JEWELRY Mary has a circular pendant hanging from a chain around her neck. The chain is tangent to the pendant and then forms an angle of 20° below the pendant. Find the measure of the arc at the bottom of the pendant. **160**



Example 7 Find x.



$m\angle V = \frac{1}{2}(m\widehat{XT} - m\widehat{WU})$	
$34 = \frac{1}{2}(128 - x)$	Substitution
$-30 = -\frac{1}{2}x$	Simplify.
$x = 60$	Multiply each side by -2.

Chapter 10 Study Guide and Review 623

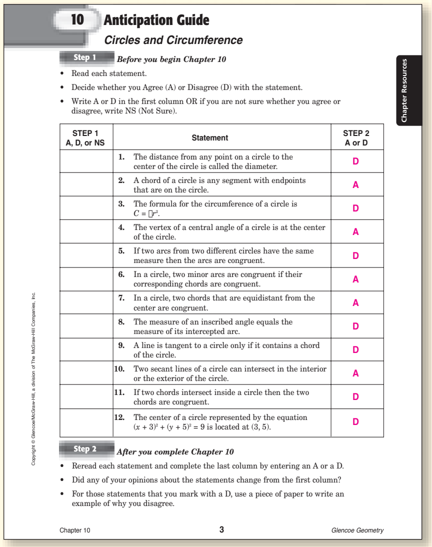
Problem-Solving Review

For additional practice in problem solving for Chapter 10, see Mixed Problem-Solving and Proof Appendix, p. 835 in the Student Handbook section.

Anticipation Guide

Have students complete the Chapter 10 Anticipation Guide and discuss how their responses have changed now that they have completed Chapter 10.

CRM Anticipation Guide, p. 3


10 Anticipation Guide

Circles and Circumference

Chapter Resources

Step 1 ***Before you begin Chapter 10***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. The distance from any point on a circle to the center of the circle is called the diameter.	D
	2. A chord of a circle is any segment with endpoints that are on the circle.	A
	3. The formula for the circumference of a circle is $C = \pi r^2$.	D
	4. The vertex of a central angle of a circle is at the center of the circle.	A
	5. If two arcs from two different circles have the same measure then the arcs are congruent.	D
	6. In a circle, two minor arcs are congruent if their corresponding chords are congruent.	A
	7. In a circle, two chords that are equidistant from the center are congruent.	A
	8. The measure of an inscribed angle equals the measure of its intercepted arc.	D
	9. A line is tangent to a circle only if it contains a chord of the circle.	D
	10. Two secant lines of a circle can intersect in the interior or the exterior of the circle.	A
	11. If two chords intersect inside a circle then the two chords are congruent.	D
	12. The center of a circle represented by the equation $(x + 3)^2 + (y + 5)^2 = 9$ is located at (3, 5).	D

Step 2 ***After you complete Chapter 10***

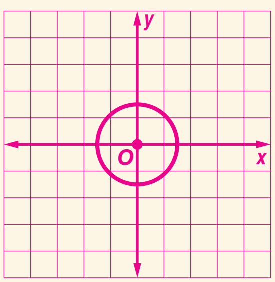
- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

Chapter 10 3 *Glencoe Geometry*

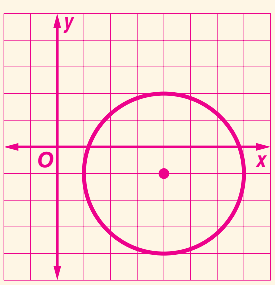
Additional Answers

51. $(x + 4)^2 + (y - 8)^2 = 9$

53.



54.



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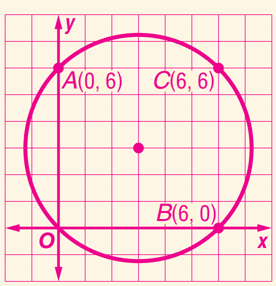
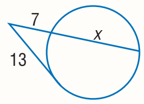


10-7 Special Segments in a Circle (pp. 607–613)

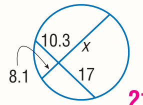
Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

47.



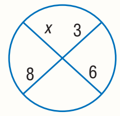
17.1

48.

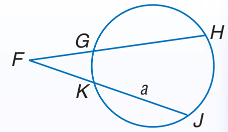


21.6

49. LAMPSHADE The top of a lampshade is a circle with two intersecting chords. Use the figure to find x. **4**



Example 8 Find a, if $FG = 18$, $GH = 42$, and $FK = 15$.



Let $KJ = a$.

$FK \cdot FJ = FG \cdot FH$	Secant Segment Products
$15(a + 15) = 18(18 + 42)$	Substitution
$15a + 225 = 1080$	Distributive Property
$15a = 855$	Subtract.
$a = 57$	Divide each side by 15.

10-8 Equations of Circles (pp. 614–619)

Write an equation for each circle.

50. center at (0, 0), $r = \sqrt{5}$ $x^2 + y^2 = 5$

51. center at (−4, 8), $d = 6$ **See margin.**

52. center at (−1, 4) and is tangent to $x = 1$ $(x + 1)^2 + (y - 4)^2 = 4$

Graph each equation.

53. $x^2 + y^2 = 2.25$ **See margin.**

54. $(x - 4)^2 + (y + 1)^2 = 9$ **See margin.**

For Exercises 55 and 56, use the following information.

A circle graphed on a coordinate plane contains $A(0, 6)$, $B(6, 0)$, $C(6, 6)$.

55. Write an equation of the circle. $(x - 3)^2 + (y - 3)^2 = 18$

56. Graph the circle. **See margin.**

57. PIZZA A pizza parlor is located at the coordinates (7, 3) on a coordinate grid. The pizza parlor's delivery service extends for 15 miles. Write the equation of the circle which represents the outer edge of the pizza delivery service area. $(x - 7)^2 + (y - 3)^2 = 225$

Example 9 Write an equation of a circle with center (−1, 4) and radius 3.

$(x - h)^2 + (y - k)^2 = r^2$	Equation of a circle
$[x - (-1)]^2 + (y - 4)^2 = 3^2$	$h = -1, k = 4, r = 3$
$(x + 1)^2 + (y - 4)^2 = 9$	Simplify.

Example 10

Graph $(x - 2)^2 + (y + 3)^2 = 6.25$.

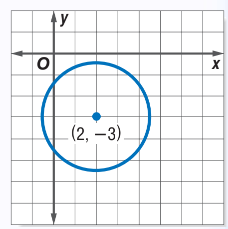
Identify the values of h, k, and r by writing the equation in standard form.

$$(x - 2)^2 + (y + 3)^2 = 6.25$$

$$(x - 2)^2 + [y - (-3)]^2 = 2.5^2$$

$$h = 2, k = -3, \text{ and } r = 2.5$$

Graph the center (2, −3) and use a compass to construct a circle with radius 2.5 units.

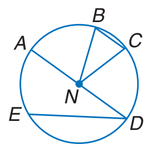


CHAPTER 10 Practice Test

1. Determine the radius of a circle with circumference 25π units. Round to the nearest tenth. **12.5 units**

2. $\overline{NA}$, $\overline{NB}$, $\overline{NC}$, $\overline{ND}$ **4. See margin.**

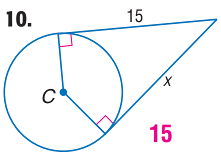
For Questions 2–9, refer to $\odot N$.



2. Name the radii of $\odot N$.
3. If $AD = 24$, find CN. **12**
4. Is $ED > AD$? Explain.
5. If AN is 5 meters long, find the exact circumference of $\odot N$. **10π m**
6. If $m\angle BNC = 20$, find $m\widehat{BC}$. **20**
7. If $\overline{BE} \cong \overline{ED}$ and $m\widehat{ED} = 120$, find $m\widehat{BE}$. **120**
8. If $m\widehat{BC} = 30$ and $\widehat{AB} \cong \widehat{CD}$, find $m\widehat{AB}$. **75**
9. If $\widehat{AE} = 75$, find $m\angle ADE$. **37.5**

Find x. Assume that segments that appear to be tangent are tangent.

10.



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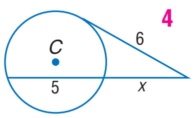
11.



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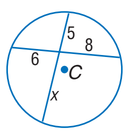
12.



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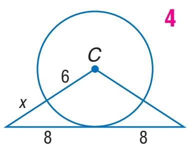
13.



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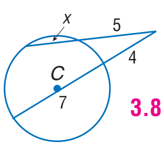

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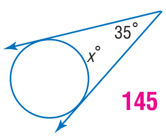
15.



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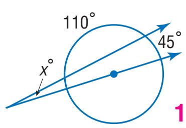
16.



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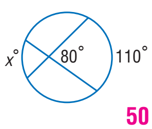
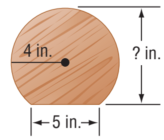
17.

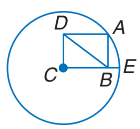


50

18. **AMUSEMENT RIDES** Suppose a Ferris wheel is 50 feet wide. Approximately how far does a rider travel in one rotation of the wheel? **157 ft**
19. Write an equation of a circle with center at $(-2, 5)$ and a diameter of 50. **$(x + 2)^2 + (y - 5)^2 = 625$**
20. **EARTHQUAKES** When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Suppose a seismograph station determines that the epicenter of an earthquake is located 63 miles from the station. If the station is located at the origin, write an equation for the circle that represents a possible epicenter of the earthquake. **$x^2 + y^2 = 3969$**
21. Graph $(x - 1)^2 + (y + 2)^2 = 4$. **See margin.**
22. **PROOF** Write a two-column proof.
    Given: $\odot X$ with diameters $\overline{RS}$ and $\overline{TV}$
    Prove: $\widehat{RT} \cong \widehat{VS}$ **See Ch. 10 Answer Appendix.**
23. **CRAFTS** Takita is making bookends out of circular wood pieces, as shown at the right. What is the height of the cut piece of wood? **about 7.1 in.**



24. **MULTIPLE CHOICE** Circle C has radius r and $ABCD$ is a rectangle. Find DB. **A**



    A r
    B $r\frac{\sqrt{2}}{2}$
    C $r\sqrt{3}$
    D $r\frac{\sqrt{3}}{2}$

25. **CONSTRUCTION** An arch over a doorway is 2 feet high and 7 feet wide. Find the radius of the circle containing the arch. **4.0625 ft**

Chapter Test at geometryonline.com

CHAPTER 10 Practice Test

Summative Assessment

Chapter 10 Resource Masters

Leveled Chapter 10 Tests			
Form	**Type**	**Level**	**Pages**
1	MC	BL	69–70
2A	MC	OL	71–72
2B	MC	OL	73–74
2C	FR	OL	75–76
2D	FR	OL	77–78
3	FR	AL	79–80

MC = multiple-choice questions
FR = free-response questions
BL = below grade level
OL = on grade level
AL = above grade level

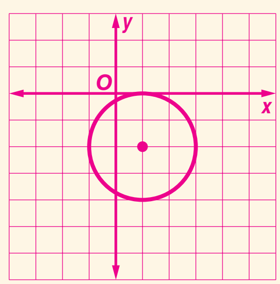
- Vocabulary Test, p. 68
- Extended-Response Test, p. 81

ExamView Assessment Suite — Customize and create multiple versions of your chapter test and their answer keys. All of the questions from the leveled chapter tests in the *Chapter 10 Resource Masters* are also available on ExamView® Assessment Suite.

Additional Answers

4. No, diameters are the longest chords of a circle.

21.



Alternative Assessment

Using Portfolios After completing a chapter with several concepts, students might benefit from going back and categorizing the concepts. Ask students to label two sheets of paper “Chapter 10–Concepts I Knew” and “Chapter 10–Concepts I Learned”. Categorize the concepts from each lesson. Encourage students to use examples and to rephrase the concepts in their own words.

CHAPTER 10 Standardized Test Practice

Formative Assessment

You can use these two pages to benchmark student progress.

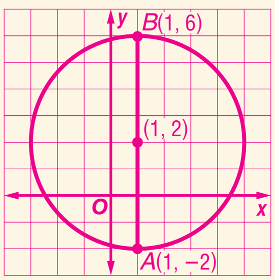
Chapter 10 Resource Masters

- Standardized Test Practice, pp. 82–84

ExamView Assessment Suite Create practice worksheets or tests that align to your state's standards, as well as TIMSS and NAEP tests.

Additional Answers

13a.



13b. The center is a midpoint of the diameter:

$\left(\frac{1+1}{2}, \frac{-2+6}{2}\right)$ or (1, 2). Find the length of the radius r by finding the distance between the center (1, 2) and a point on the circle (1, −2). Use the Distance Formula.

$r = \sqrt{(1-1)^2 + (-2-2)^2}$

$= \sqrt{0^2 + (-4)^2}$

$= \sqrt{16}$ or 4

Use the formula for the circumference of a circle given its radius.

$C = 2\pi r$

$= 2\pi(4)$

$= 8\pi$ or about 25.12

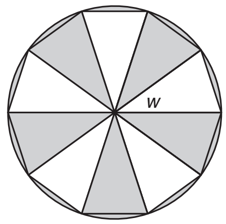
So the circumference of the circle is about 25.12 units.

CHAPTER 10 Standardized Test Practice

Cumulative, Chapters 1–10

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A regular decagon is drawn in a circle as a design on the cover of a school yearbook. Opposite vertices are connected by a line segment.



What is the measure of $\angle w$? **A**

A 45° **C** 60°

B 50° **D** 90°

2. The vertices of $\triangle EFG$ are $E(3, 1)$, $F(4, 5)$, $G(1, -2)$. If $\triangle EFG$ is translated 2 units down and 3 units to the right to create $\triangle MNP$, what are the coordinates of the vertices of $\triangle MNP$? **H**

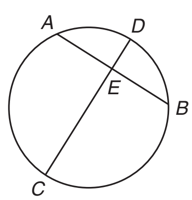
F $M(1, 4)$, $N(3, 8)$, $P(-1, 1)$

G $M(5, 4)$, $N(6, 8)$, $P(3, 1)$

H $M(6, -1)$, $N(7, 3)$, $P(4, -4)$

J $M(6, 3)$, $N(7, 7)$, $P(4, 0)$

3. GRIDDABLE In the circle below, $\overline{AB}$ and $\overline{CD}$ are chords intersecting at E.



If $AE = 8$, $DE = 4$, and $EB = 9$, what is the length of $\overline{EC}$? **18**

4. ALGEBRA Which inequality is equivalent to $7x < 9x - 3(2x + 5)$? **C**

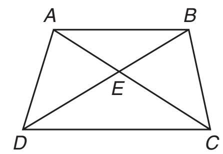
A $4x < 15$

B $4x > -15$

C $4x < -15$

D $4x > 15$

5. Trapezoid $ABCD$ is shown below.



Which pair of triangles can be established as similar to prove that $\frac{AE}{AB} = \frac{EC}{DC}$? **G**

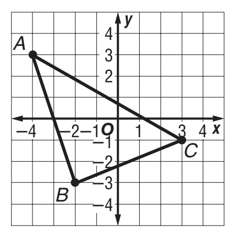
F $\triangle ADB$ and $\triangle BCA$

G $\triangle AEB$ and $\triangle CED$

H $\triangle ADC$ and $\triangle BCD$

J $\triangle AED$ and $\triangle BEC$

6. If $\triangle ABC$ is reflected across the y-axis, what are the coordinates of C'? **C**



A (3, 1)

B (−1, 3)

C (−3, −1)

D (−1, −3)

Standardized Test Practice at geometryonline.com

13c. Use the center and radius of the circle to determine its equation. Since the center is (1, 2), $h = 1$ and $k = 2$.

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 2)^2 = 4^2$

$(x - 1)^2 + (y - 2)^2 = 16$

So, the equation of the circle is

$(x - 1)^2 + (y - 2)^2 = 16$

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

CHAPTER 10 Standardized Test Practice

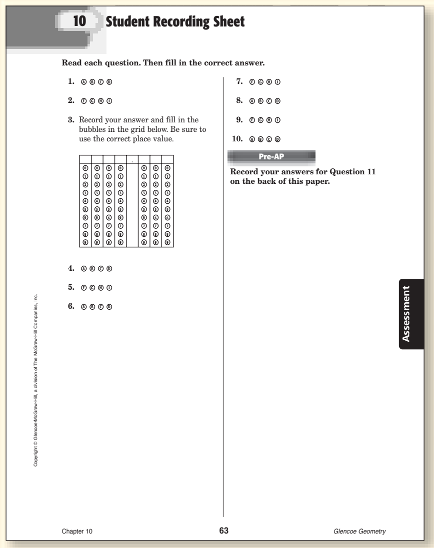
Item Analysis

Questions 3, 7, and 11 are griddable questions. In a griddable question, students arrive at an answer then record it in a special grid, coloring in the appropriate bubble under each digit of the answer.

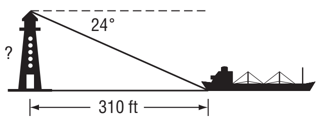
Answer Sheet Practice

Have students simulate taking a standardized test by recording their answers on a practice recording sheet.

CRM Student Recording Sheet, p. 63



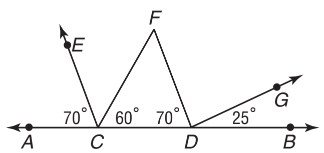
7. GRIDDABLE A passing ship is 310 feet from the base of a lighthouse. The angle of depression from the top of the lighthouse to the ship is 24°.



$\sin 24° \approx 0.41$
$\cos 24° \approx 0.91$
$\tan 24° \approx 0.45$

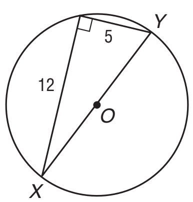
What is the height of the lighthouse in feet to the nearest tenth? **139.5**

8. Which of the following statements is true? **J**



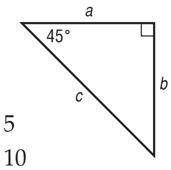
F $\overline{CE} \cong \overline{DF}$
G $\overline{CF} \parallel \overline{DG}$
H $\overline{CF} \cong \overline{DF}$
J $\overline{CE} \parallel \overline{DF}$

9. If $\overline{XY}$ is a diameter of circle O, what is the circumference of circle O? **D**



A 5π **C** 7.5π
B 7π **D** 13π

10. If $a = 5\sqrt{2}$ in the right triangle below, what is the value of c? **J**

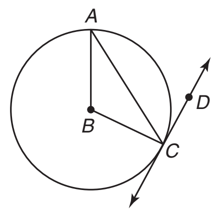


F $5\sqrt{6}$ **H** 5
G $10\sqrt{2}$ **J** 10

TEST-TAKING TIP

Question 10 Question 10 is a multi-step problem. First, identify the figure shown as a 45°-45°-90° right triangle. Next, use the relationship between the sides in this kind of triangle to determine that $c = \sqrt{2}a$. Finally, recall from algebra that $\sqrt{m} \cdot \sqrt{n} = \sqrt{m \cdot n}$.

11. GRIDDABLE $\overleftrightarrow{CD}$ is tangent at point C to a circle, with B at its center. $\overline{BC}$ is a radius. If $m\angle ACB = 35°$, what is $m\angle ACD$? **55**



12. A triangle is dilated so that the ratio between the areas of the triangle and its image is 9 to 8. What is the ratio between the perimeters of the two triangles? **A**

A 3 to 22 **C** 18 to 16
B 4.5 to 4 **D** 81 to 64

Pre-Ap

Record your answers on a sheet of paper. Show your work.

13. The segment with endpoints $A(1, -2)$ and $B(1, 6)$ is the diameter of a circle.

a. Graph the points and draw the circle.
b. What is the circumference of the circle?
c. What is the equation of the circle?

NEED EXTRA HELP?

If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	10-4	9-2	10-7	783	7-3	8-5	9-1	3-5	10-1	8-3	10-5	9-5	10-8

Homework Option

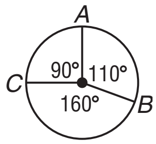
Get Ready for Chapter 11 Assign students the exercises on p. 629 as homework to assess whether they possess the prerequisite skills for the next chapter.

Chapter 10 Standardized Test Practice 627

Page 568, Lesson 10-2

51. Sample answer: Concentric circles have the same center, but different radius measures; congruent circles usually have different centers but the same radius measure.

52. Sample answer: $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{AC}$, $\overset{\frown}{ABC}$, $\overset{\frown}{BCA}$, $\overset{\frown}{CAB}$; $m\overset{\frown}{AB} = 110$, $m\overset{\frown}{BC} = 160$, $m\overset{\frown}{AC} = 90$, $m\overset{\frown}{ABC} = 270$, $m\overset{\frown}{BCA} = 250$, $m\overset{\frown}{CAB} = 200$

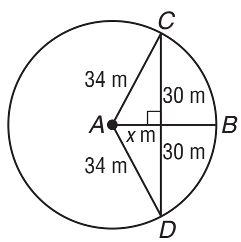


53. No; the radii are not equal, so the proportional part of the circumferences would not be the same. Thus, the arcs would not be congruent.

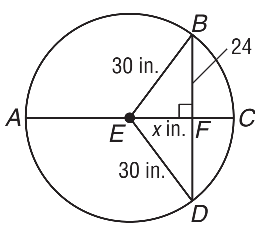
55. Sample answer: The hands of the clock form central angles. The hands form right, acute, and obtuse angles. Some times when the angles formed by the minute and hour hand are congruent are at 1:00 and 11:00, 2:00 and 10:00, 3:00 and 9:00, 4:00 and 8:00, and 5:00 and 7:00. They also form congruent angles at many other times of the day, such as 3:05 and 8:55.

Pages 575—577, Lesson 10-3

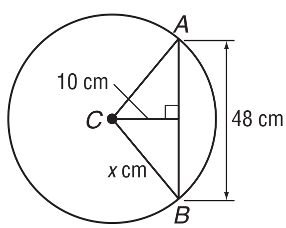
37. 16 m



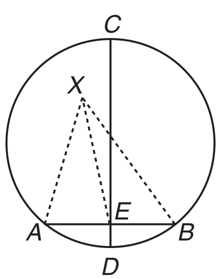
38. 26 cm



39. 18 in.

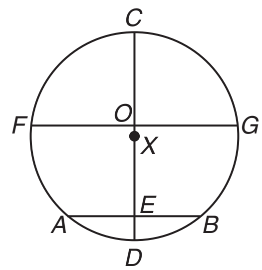


42. Given: $\overline{CD}$ is the perpendicular bisector of chord $\overline{AB}$ in $\odot X$.
Prove: $\overline{CD}$ contains point X.



Proof:
Suppose X is not on $\overline{CD}$. Draw $\overline{XE}$ and radii $\overline{XA}$ and $\overline{XB}$. Since $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$, E is the midpoint of $\overline{AB}$ and $\overline{AE} \cong \overline{EB}$. Also, $\overline{XA} \cong \overline{XB}$, since all radii of a $\odot$ are $\cong$. $\overline{XE} \cong \overline{XE}$ by the Reflexive Property. So, $\triangle AXE \cong \triangle BXE$. By CPCTC, $\angle XEA \cong \angle XEB$. Since $\angle XEA$ and $\angle XEB$ are congruent adjacent angles that make up AEB, $\overline{XE} \perp \overline{AB}$. Then $\overline{XE}$ is the perpendicular bisector of $\overline{AB}$. But $\overline{CD}$ is also the perpendicular bisector of $\overline{AB}$. This contradicts the uniqueness of a perpendicular bisector of a segment. Thus, our assumption is false, and center X must be on $\overline{CD}$.

43. Given: In $\odot X$, X is on $\overline{CD}$ and $\overline{FG}$ bisects $\overline{CD}$ at 0.
Prove: Point O is point X.



Proof:
Since point X is on $\overline{CD}$ and C and D are on $\odot X$, $\overline{CD}$ is a diameter of $\odot X$. Since $\overline{FG}$ bisects $\overline{CD}$ at 0, O is the midpoint of $\overline{CD}$. Since the midpoint of a diameter is the center of a circle, O is the center of the circle. Therefore point O is point X.

46. About 17.3; call the point where $\overline{PQ}$ and $\overline{AB}$ intersect S. Since point P is the center of $\odot P$, $\overline{PS}$ lies on a radius of $\odot P$. Since $\overline{PS}$ is perpendicular to chord $\overline{AB}$, by Theorem 10.3, $\overline{PS}$ bisects $\overline{AB}$. So $\overline{AS} \cong \overline{SB}$. By segment addition $AS + SB = AB$. By substitution, $AS + AS = 10$, so $AS = 5$. Since $\overline{PS}$ is perpendicular to chord $\overline{AB}$, $\angle PSA$ is a right angle. So $\triangle PSA$ is a right triangle. By the Pythagorean theorem, $PS = \sqrt{PA^2 - AS^2}$. By substitution, $PS = \sqrt{11^2 - 5^2}$ or $\sqrt{96}$. Similarly, $\triangle ASQ$ is a right triangle with $SQ = \sqrt{AQ^2 - AS^2} = \sqrt{9^2 - 5^2}$ or $\sqrt{56}$. Since $PQ = PS + SQ$, $PQ = \sqrt{96} + \sqrt{56}$ or about 17.3.

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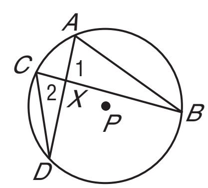
10. Given: $\odot P$
Prove: $\triangle AXB \sim \triangle CXD$
Proof:
<u>**Statements (Reasons)**</u>

1. $\odot P$ (Given)
2. $\angle A \cong \angle C$ (Inscribed $\angle$s intercepting same arc are $\cong$.)
3. $\angle 1 \cong \angle 2$ (Vertical $\angle$s are $\cong$.)
4. $\triangle AXB \sim \triangle CXD$ (AA Similarity)



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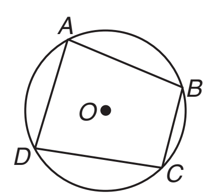
37. Given: quadrilateral $ABCD$ inscribed in $\odot O$
Prove: $\angle A$ and $\angle C$ are supplementary.
$\angle B$ and $\angle D$ are supplementary.



Proof: By arc addition and the definitions of arc measure and the sum of central angles, $m\widehat{DCB} + m\widehat{DAB} = 360$. Since $m\angle C = \frac{1}{2} m\widehat{DAB}$ and $m\angle A = \frac{1}{2} m\widehat{DCB}$, $m\angle C + m\angle A = \frac{1}{2}(m\widehat{DCB} + m\widehat{DAB})$, but $m\widehat{DCB} + m\widehat{DAB} = 360$, so $m\angle C + m\angle A = \frac{1}{2}(360)$ or 180. This makes $\angle C$ and $\angle A$ supplementary.

Because the sum of the measures of the interior angles of a quadrilateral is 360, $m\angle A + m\angle C + m\angle B + m\angle D = 360$. But $m\angle A + m\angle C = 180$, so $m\angle B + m\angle D = 180$, making them supplementary also.

39. Isosceles right triangle; sides are congruent radii making it isosceles and $\angle AOC$ is a central angle for an arc of 90°, making it a right angle.

40. Square; each angle intercepts a semicircle, making them 90° angles. Each side is a chord of congruent arcs, so the chords are congruent.

41. Square; each angle intercepts a semicircle, making them 90° angles. Each side is a chord of congruent arcs, so the chords are congruent.

42. The measures of an inscribed angle and a central angle for the same intercepted arc can be calculated using the measure of the arc. However, the measure of the central angle equals the measure of the arc, while the measure of the inscribed angle is half the measure of the arc.

44. Use the properties of trapezoids and inscribed quadrilaterals to verify that $ABCD$ is isosceles.
$m\angle A + m\angle D = 180$ (same side interior angles = 180)
$m\angle A + m\angle C = 180$ (opposite angles of inscribed quadrilaterals = 180)
$m\angle A + m\angle D = m\angle A + m\angle C =$ (Substitution)
$m\angle D = m\angle C$ (Subtraction Property)
$\angle D \cong \angle C$ (Def. of $\cong$ ∠s)

45. Sample answer: The socket is similar to an inscribed polygon because the vertices of the hexagon can be placed on a circle that is concentric with the outer circle of the socket. An inscribed polygon is one in which all of its vertices are points on a circle. The side of the regular hexagon inscribed in a circle $\frac{3}{4}$ inch wide is $\frac{3}{8}$ inch.

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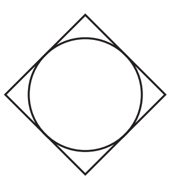
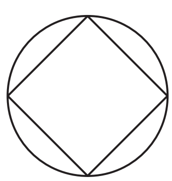
33. Sample answer:

polygon circumscribed about a circle:

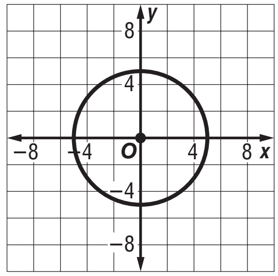


polygon inscribed in a circle:

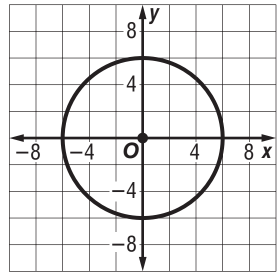


Pages 617–618, Lesson 10-8

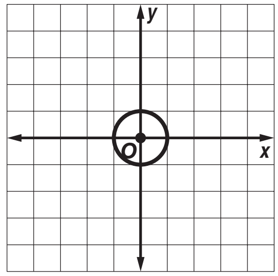
22.



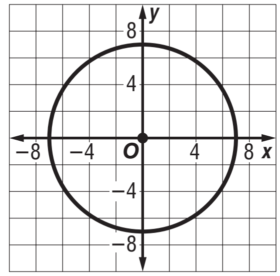
23.



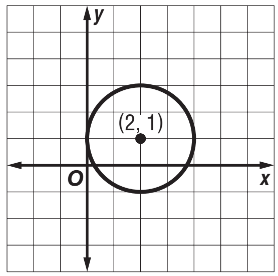
24.



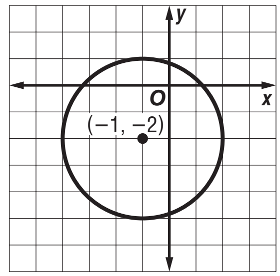
25.



26.

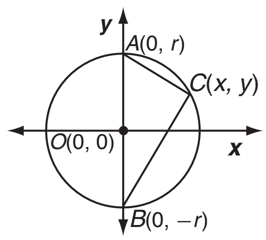


27.



32. The slope of $\overline{AC}$ is $-\frac{1}{4}$, so the slope of its bisector is 4. The midpoint of $\overline{AC}$ is (0, 5). Use the slope and the midpoint to write an equation for the bisector of $\overline{AC}$: $y = 4x + 5$. The slope of $\overline{BC}$ is $-\frac{9}{2}$, so the slope of its bisector is $\frac{2}{9}$. The midpoint of $\overline{BC}$ is (−2, −3). Use the slope and the midpoint to write an equation for the bisector of $\overline{BC}$: $y = \frac{2}{9}x - \frac{23}{9}$. Solving the system of equations, $y = 4x + 5$ and $y = \frac{2}{9}x - \frac{23}{9}$, yields (−2, −3), which is the circumcenter. Let (−2, −3) be D, then $DA = DB = DC = \sqrt{85}$.

43. Given: $\overline{AB}$ is a diameter of ⊙O and C is a point on ⊙O.
Prove: $\angle ACB$ is a right angle.



Proof:
$\overline{AC}$ has slope $\frac{y - r}{x}$ and $\overline{CB}$ has slope $\frac{y - (-r)}{x}$ or $\frac{y + r}{x}$.

$\frac{y - r}{x} \cdot \frac{y + r}{x}$.

$= \frac{y^2 - r^2}{x^2}$ Multiply.

Pages 617–618, Lesson 10-8 *(continued)*

$= \frac{y^2 - (x^2 + y^2)}{x^2}$ $\quad r^2 = x^2 + y^2$

$= \frac{y^2 - x^2 - y^2}{x^2}$ $\quad -(x^2 + y^2) = -x^2 - y^2$

$= \frac{-x^2}{x^2}$ or -1 $\quad$ Simplify.

Since the product of the slope of $\overline{AC}$ and $\overline{CB}$ is -1, $\overline{AC} \perp \overline{CB}$ and $\angle ACB$ is a right angle.

Page 625, Practice Test

22. Given: $\odot X$ with diameters $\overline{RS}$ and $\overline{TV}$

Prove: $\widehat{RT} \cong \widehat{VS}$

Proof:

Statements (Reasons)

1. $\odot X$ with diameters $\overline{RS}$ and $\overline{TV}$ (Given)
2. $\angle RXT \cong \angle VXS$ (Vertical $\angle$s are $\cong$.)
3. $m\angle RXT = m\angle VXS$ (Def. of $\cong$ $\angle$s)
4. $m\widehat{RT} = m\angle RXT$, $m\widehat{VS} = m\angle VXS$
(Measure of arc equals measure of its central angle.)
5. $m\widehat{RT} = m\widehat{VS}$ (Substitution)
6. $\widehat{RT} \cong \widehat{VS}$ (Def. of $\cong$ arcs)

Notes