Name: Key - Schmidt DUE: _____ HOUR: _____

2018/2019 Accelerated Geometry Final Exam Review

Note to students:
The semester exam will assess several content themes: right triangle trigonometry (24%), vectors (12%), circles (22%), area, surface area and volume (26%), and trigonometric functions/unit circle (16%).

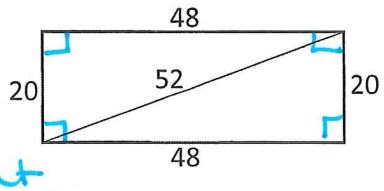
Right Triangle Trigonometry "Need to Know"

- What is the Pythagorean Theorem? What is important to remember about the hypotenuse? $a^2+b^2=c^2$ c = longest across from 90°
- What is a Pythagorean Triple? works for pyth. w/ whole #s
- What are the side lengths for a 30-60-90 triangle with a hypotenuse of length 2?
- What are the side lengths for a 45-45-90 triangle with a leg of length 1?
- What are the Law of Sines and Law of Cosines? When do you use these laws?

$\frac{\sin A}{a} = \frac{\sin B}{b}$ 2 op.

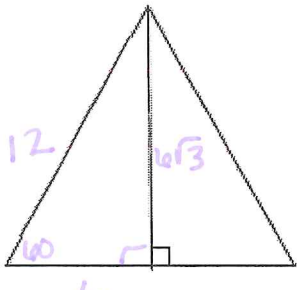
$c^2 = a^2+b^2-2ab\cos C$ one pair of ops.

1. Is the quadrilateral in the diagram a rectangle? Explain how you know.



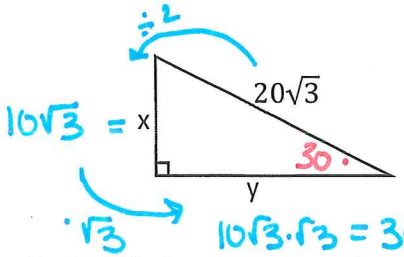
$20^2+48^2=2704$ we know
$52^2=2704$ we have Right Angles! (by the converse of pyth thm)

2. If the altitude is $6\sqrt{3}$, what is the perimeter of the equilateral triangle?



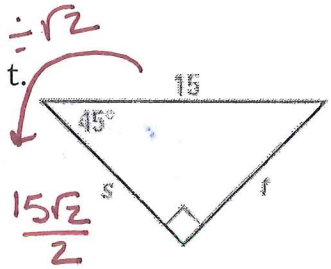
12 x 3 = 36 units

3. Find x and y. (This needs a ~~30°~~ 30°-60°-90° Label)



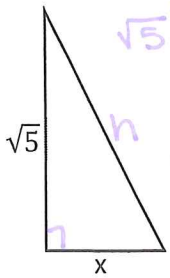
x = 10√3
y = 30

4. Find s and t.

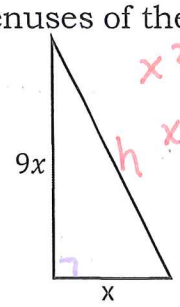


$s = \frac{15\sqrt{2}}{2}$
$t = \frac{15\sqrt{2}}{2}$

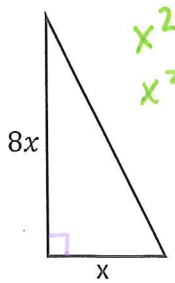
5. Find the lengths of the hypotenuses of these right triangles. not 30°60°90°!



$\sqrt{5}^2+x^2=h^2$
$5+x^2=h^2$
$h=\sqrt{5+x^2}$

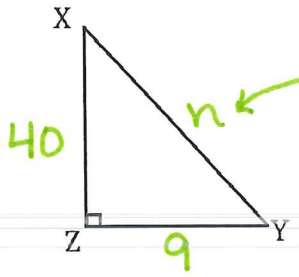


$x^2+(9x)^2=h^2$
$x^2+81x^2=h^2$
$\sqrt{82x^2}=\sqrt{h^2}$
$x\sqrt{82}=h$



$x^2+(8x)^2=h^2$
$x^2+64x^2=h^2$
$65x^2=h^2$
$\sqrt{65x^2}=h$
$x\sqrt{65}=h$

6. In the figure, $\tan x = \frac{9}{40}$. Find cos Y, cos X, sin Y, sin X.



$9^2+40^2=n^2$

$41=n$

$\cos Y = \frac{9}{41}$

$\cos x = \frac{40}{41}$

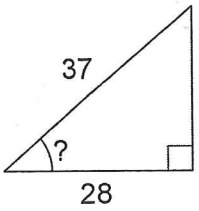
$\sin Y = \frac{40}{41}$

$\sin X = \frac{9}{41}$

notice the relationship :)

7. Find the measure of the missing angle. Round to the nearest degree.

a.

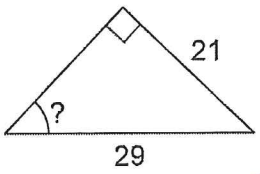


$\cos\theta = \frac{28}{37}$

$\theta = \cos^{-1}\left(\frac{28}{37}\right)$

$\theta = 40.821°$

b.



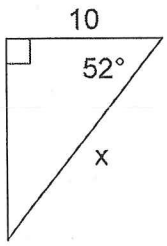
$\sin\theta = \frac{21}{29}$

$\sin^{-1}\left(\frac{21}{29}\right) = \theta$

$\theta \approx 46.397°$

8. Solve to find each missing side. Round to the nearest tenth.

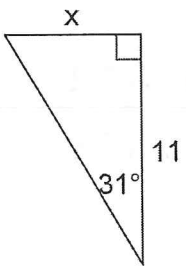
a.



$\cos(52°) = \frac{10}{x}$

$x \approx 16.2$

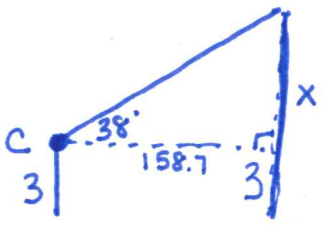
b.



$\tan(31°) = \frac{x}{11}$

$x \approx 6.6$

9. A digital camera with a panoramic lens is described as having a view with an angle of elevation of 38 degrees. If the camera is on a 3 foot tripod aimed directly at a monument 158.7 feet away, how tall is the monument?



$\tan(38) = \frac{x}{158.7}$

$x \approx 123.990$

$+ 3$

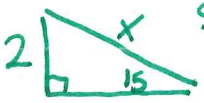
126.990 ft tall

10. Sierra doesn't know why she is not calculating the correct answer for her work. Find, describe, and correct her error.

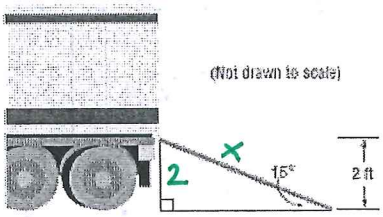
Find the length of the ramp to the nearest foot.

$\tan 15° = \frac{2\ ft}{?}$

The ramp is 7.5 ft long.

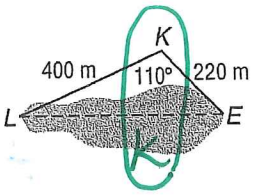


$\sin 15 = \frac{2}{x}$



Should have used sine $x = 7.7$ feet.

11. To approximate the length of a pond, a surveyor walks 400 meters from point L to point K, then turns and walks 220 meters from point K to point E. If m∠LKE = 110°, find the length LE of the pond to the nearest tenth of a meter.

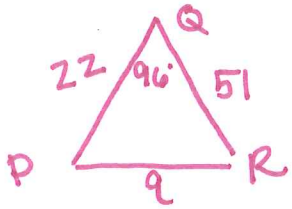


one pair of opps. Law of Cosines

$k^2 = 400^2 + 220^2 - 2 \cdot 400 \cdot 220 \cos(110°)$

$LE \approx 518.262\text{m}$

12. Solve △PQR for r = 22, p = 51, and m∠Q = 96°. Round angle measures to the nearest degree and side measures to the nearest tenth.



$q^2 = 22^2 + 51^2 - 2 \cdot 22 \cdot 51 \cdot \cos 96$

$q \approx 57.6$

$\frac{\sin R}{22} = \frac{\sin 96}{57.6}$

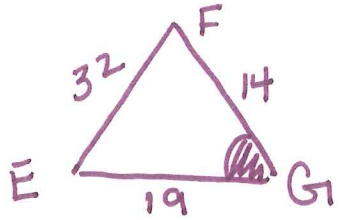
$\angle R \approx 22.3°$

△sum:

$\angle P + 22.3 + 96 = 180$

$\angle P \approx 61.7°$

13. In ΔEFG, $e = 14$, $f = 19$, $g = 32$, find the measure of the largest angle.



$32^2 = 14^2 + 19^2 - 2 \cdot 14 \cdot 19 \cos G$

$1024 = 557 - 532 \cos G$

$-557 \quad -557$

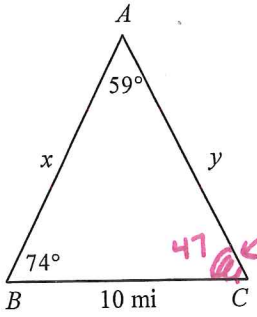
$467 = -532 \cos G$

$\cos G = \frac{467}{-532}$

$\angle G = \cos^{-1}\left(\frac{467}{-532}\right)$

$\angle G \approx 151.380°$

14. Three aircraft are flying in a triangular shape in the sky. In the figure below, A, B, and C represent the position of the aircraft. Find the values of x and y to the nearest mile.



△sum

$\frac{\sin(47°)}{x} = \frac{\sin(59°)}{10}$

$x \approx 8.532\text{mi}$

$\frac{\sin(74°)}{y} = \frac{\sin(59°)}{10}$

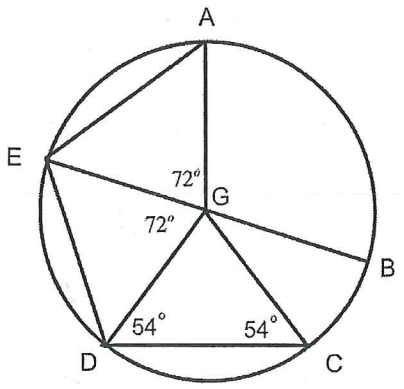
$y \approx 11.214\text{mi}$

Circles "Need to Know"

- Know the vocabulary chord, tangent, secant, inscribed angle, and central angle.
- Know how to find circumference and area. $d\pi = C$ $A = \pi r^2$
- What is the relationship between inscribed or central angle and arc measure?
- How do you calculate arc length, given arc measure? $\ell = \frac{a}{360} \cdot C$
- If a quadrilateral is inscribed in a circle, what do you know about the opposite angles of the quadrilateral? Suppl.
- What can you tell about two tangent segments that meet at a point outside of a circle? $\cong$ seg.
- What angle is formed by a line tangent to a circle and the radius of the circle? $\perp$
- Translate between the geometric description and the equation of a circle.

$(x-h)^2 + (y-k)^2 = r^2$

15. Given: EB is a diameter of circle G. True or False?
If false, correct the statement to make it true.



False a. $\angle BGC = 72^\circ$ $\angle BGC = 36^\circ$

False b. AE // CD $\overline{AE} \cong \overline{CD}$

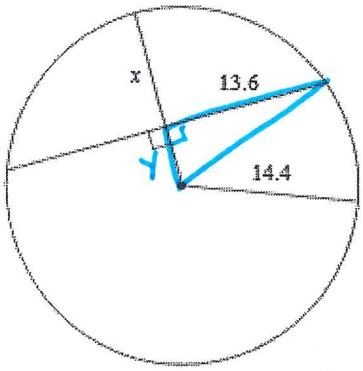
c. $\angle GED = \angle GDC$ TRUE!!!

False d. $\angle DGC = 54^\circ$ $\angle DGC = 72^\circ$

False e. $\overline{ED} \cong \overline{AG}$ $\overline{EG} \cong \overline{AG}$

f. $\overline{AE} \cong \overline{CD}$ True!!!

16. Find the length of the segment indicated. Round your answer to the nearest tenth if necessary.



Find y 1st:

$y^2 + 13.6^2 = 14.4^2$

$y^2 = 22.4$

$y = \sqrt{22.4} \approx 4.733$

$14.4 - y = x$

$\boxed{x \approx 9.667}$

17. Find the measure of each numbered angle. The measure of arc QR = 40° and the measure of arc TS = 110°.

$\angle 1 = \frac{1}{2}110^\circ = 55^\circ$

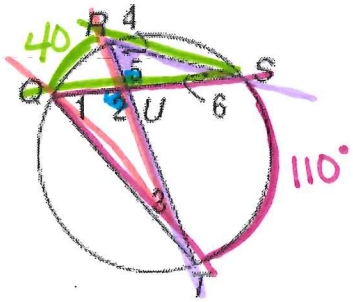
$\angle 2 = 105^\circ$ by Δ sum

$\angle 3 = \frac{1}{2}40 = 20^\circ$

$\angle 4 = \frac{1}{2}110 = 55^\circ$

$\angle 5 = 105$ by Δ sum

$\angle 6 = \frac{1}{2}40 = 20^\circ$



18. The diameter $\overline{GE}$ of $\odot C$ is 32 units long. Find the length of arc DHE if $m\angle DCE = 90°$.

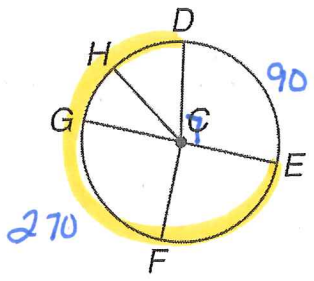
$\ell = \frac{a}{360} \cdot C$ $\quad C = d\pi$

$a = 270°$
$d = 32$

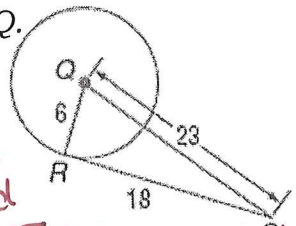
$\ell = \frac{270}{360} \cdot 32\pi$

$\ell = \frac{8640\pi}{360}$

$\ell = 24\pi$ units



19. Determine if segment *RS* is tangent to circle *Q*.

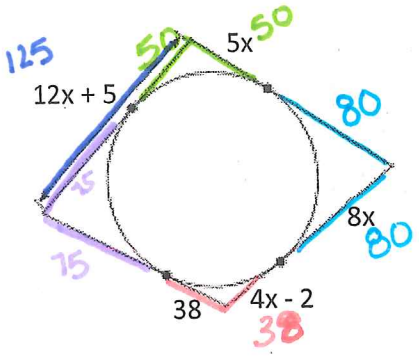


Is $\angle R$ a Right $\angle$?

$6^2 + 18^2 = 360 \neq 23^2 = 529$ $\quad a^2 + b^2 \neq c^2$

$\therefore \angle R$ is not a Right $\angle$ and Radius QR is Not $\perp$ to ST $\therefore$ ST is not tangent.

20. Find the perimeter of the polygon. Assume lines which appear to be are tangent.



Find x

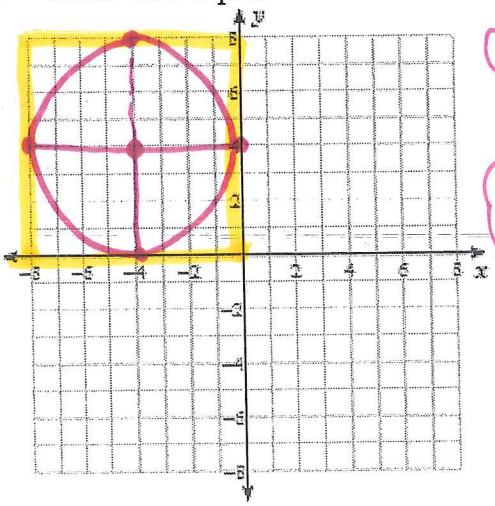
$38 = 4x - 2$

$40 = 4x$

$10 = x$

$P = 486$

21. The points of a square are (0, 0), (–8, 0), (–8, 8) and (0, 8). Graph the square and write the equation of a circle that would be inscribed in the square.



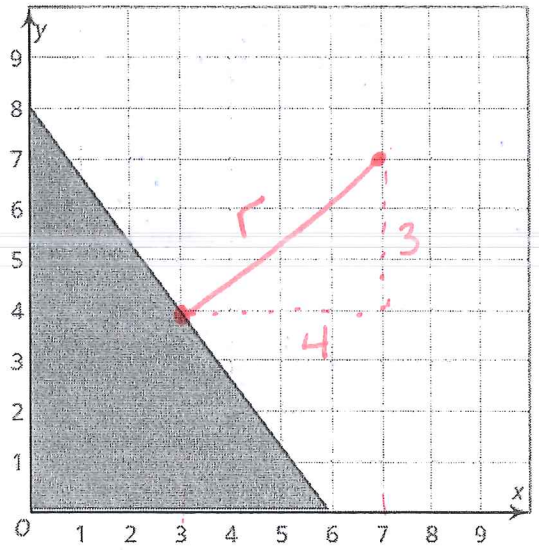
Center (–4, 4)

$r = 4$

$(x+4)^2 + (y-4)^2 = 16$

xy hk

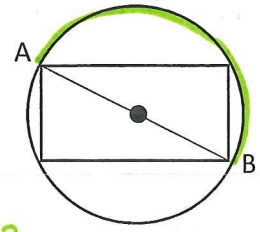
22. A circle is tangent to the triangle at (3, 4). The center of the circle is at (7, 7). Write the equation of the circle.



Find r $5=r$ $3^2+4^2=r^2$

$(x-7)^2+(y-7)^2=25$

23. Find the exact area of the circle if arc length AB is 10π.



$A=\pi r^2$ Find r:

$\ell = 10\pi$ $\frac{180}{360}\cdot d\pi = 10\pi$

$d\pi = 20\pi$

$d=20$

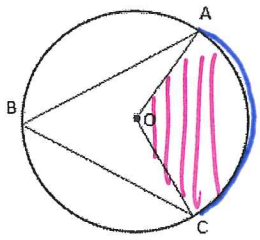
$r=10$

Area:

$A=\pi 10^2$

$A=100\pi$ units^2

24. The length of arc AC is equal to one–third of the circumference of circle O and the arc length is 4π meters. Find the radius, m∠AOC, m∠ABC, and the area of the sector defined by ∠AOC and arc AC to the nearest tenth.



$\ell = 4\pi$

$\frac{120}{360}d\pi = 4\pi$

$d=12$

$r=6m$

$\angle AOC = \frac{1}{3}360$

$\angle AOC = 120°$

$\angle ABC = \frac{1}{2}\angle AOC$

$\angle ABC = \frac{1}{2}120°$

$\angle ABC = 60°$

radius = 6m

m∠AOC = 120°

m∠ABC = 60°

sector area = $12\pi_{m^2} \approx 37.699$

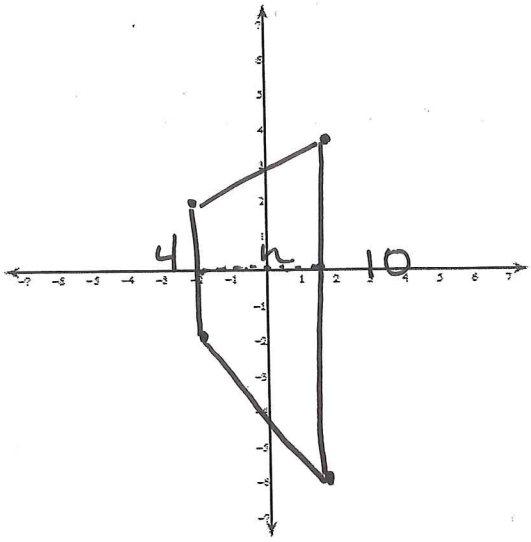
Sector Area:

$A_S = \frac{120}{360}\pi 6^2$

Area, SA, and Volume "Need to Know"

You will be provided with any formulas necessary to solve problems. You will have to understand how to apply the formulas to the assessment items.

25. Given the coordinates of the vertices, find the area of the figure.
(–2, 2) (2, 4) (2, –6) (–2, –2)



$A = \frac{1}{2}h(b_1 + b_2)$

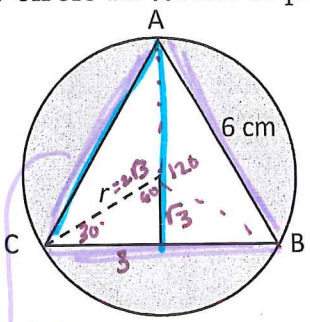
$A = \frac{1}{2}4(10+4)$

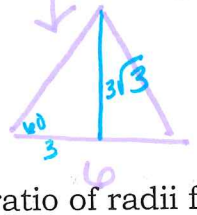
$A = \frac{1}{2}4 \cdot 14$

$A = 28 \text{ units}^2$

26. Find the area of the circle in terms of pi and the area of the shaded region.

$r = 2\sqrt{3}$





area of the circle: $12\pi \text{ cm}^2$

$A_c = \pi(2\sqrt{3})^2 = \pi 2\sqrt{3} \cdot 2\sqrt{3}$

$= \pi 4\sqrt{3}^2 = \pi 4 \cdot 3$

area of the shaded region: $12\pi - 9\sqrt{3} \text{ cm}^2 \approx 22.111 \text{ cm}^2$

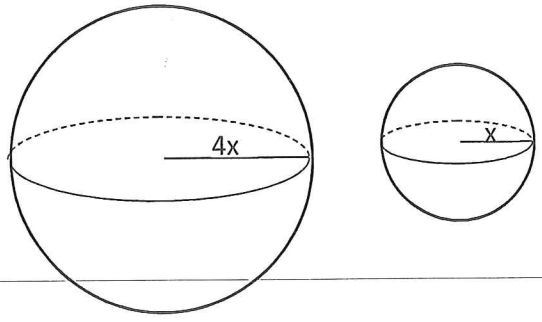
$\pi(2\sqrt{3})^2 - \frac{1}{2}6 \cdot 3\sqrt{3}$

$12\pi - 9\sqrt{3}$

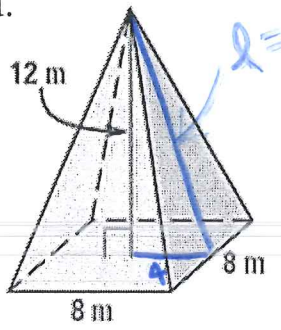
27. a. What is the ratio of radii for the two spheres? $4x : 1x \Rightarrow 4:1$ or $1:4$

b. Compare the ratio of surface area for the two spheres to the ratio of radii. 16:1

c. Compare the ratio of volume for the two spheres to the ratio of radii. 64:1



28. Find the SA of the square pyramid.

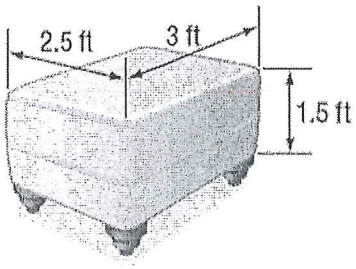


Base: $8 \cdot 8$

$+ 4 \cdot \frac{1}{2} 8 \cdot \sqrt{160}$

$SA \approx 266.386 m^2$

29. Jill wants to have her ottoman, shown below, reupholstered. Find the surface area that will be reupholstered. Do not count the area of the bottom.



Top: 2.5×3

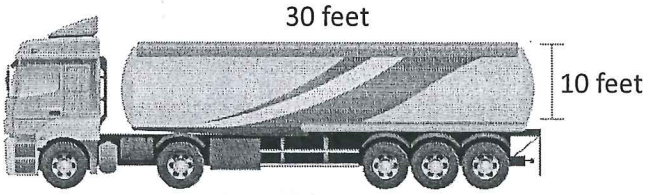
Front/Back + $2(3 \times 1.5)$

$+ 2(2.5 \times 1.5)$

$SA = 24 ft^2$ of fabric

30. A fuel tanker is in the shape of a right cylinder. The full load of the fuel inside will be delivered to two locations; station A will receive one–third of the fuel and station B will receive two–thirds of the fuel. How many gallons of fuel will be delivered to station B? $1 ft^3 = 7.5$ gallons. Round to the nearest gallon.

$V_T = \pi 5^2 \times 30 = 750\pi = V_T$



Station A:

$\frac{1}{3}(750\pi)$

$= 250\pi ft^3$

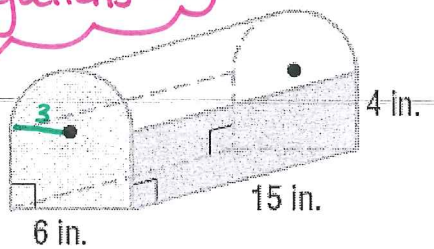
Station B:

$\frac{2}{3} 750\pi = 500\pi ft^3$

$500\pi \times 7.5 = 11{,}780.972$

Approx: 11,781 gallons to Station B

31. Find the volume of the composite solid. Round your answer to the nearest tenth.



$V = \frac{1}{2}$ Cylinder + Prism

$= \frac{1}{2}\pi 3^2 \cdot 15 + (6 \cdot 15) 4$

$V \approx 572.058 in^3$

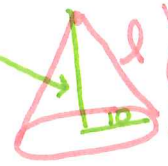
32. The volume of a cone is 460π cm². The cone has a diameter of 20 cm. Find the slant height of the cone. ← for ℓ not h... but we must find h 1st bc we are given volume.

$V = \frac{1}{3}\pi r^2 \cdot h$

$460\pi = \frac{1}{3}\pi 10^2 \cdot h$

$1380 = 100\pi h$

$h = 13.8\text{cm}$



$13.8^2 + 10^2 = \ell^2$

$\sqrt{290.44} = \ell$

$17.042\text{cm} = \ell$

SLANT HEIGHT

33. If a right circular cylinder has a radius of 4 inches and a surface area of 112π square inches, what is the height of the cylinder in inches?

$112\pi = 2\pi 4^2 + 2\pi 4 \cdot h$

$112\pi = 32\pi + 8\pi h$ ← Don't combine not like terms

$-32\pi \quad -32\pi$

$80\pi = 8\pi h$

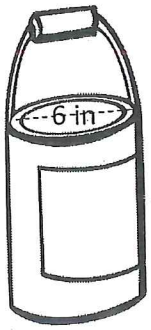
$h = 10\text{in}$

34. Compare the volume and tell which is greater: A rectangular prism that is 2 inches wide, 6 inches long and 1 inch deep or a square pyramid with a base that is 2 inches per side and is 6 inches high.

$V_{\text{of Rect. prism}} = 2 \cdot 6 \cdot 1 = 12\text{in}^3$

$V_{\text{of pyramid}} = \frac{1}{3} 2 \cdot 2 \cdot 6 = 8.\text{in}^3$] The prism has greater Volume

35. How many 1 x 1.5 x 2 inch ice "cubes" would be needed to melt into a cylindrical shaped ice bucket and fill it to the top without overflowing? The ice bucket has a diameter of 6 inches and a height of 10 inches. Cubes are whole ice cubes.



r = 3

$V_{cyl.} = \pi 3^2 \cdot 10 = 90\pi \approx 282.743\text{in}^3$

$V_{cube} = 1 \times 1.5 \times 2 = 3\text{in}^3$

94 cubes = 282in^3

95 cubes = 285in^3 which is too big, it will overflow

36. A cube has side length 10cm. What is the radius of a sphere with same volume? Round to the nearest tenth.

$V_C = 10 \cdot 10 \cdot 10$

$V = 1000\text{cm}^3$

$1000 = \frac{4}{3}\pi r^3$

$\frac{750}{\pi} = \frac{\pi r^3}{\pi}$

$238.732 = r^3$

$\sqrt[3]{238.732} = r$

$r = 6.204\text{cm}$

Trigonometric Functions/Unit Circle "Need to Know"

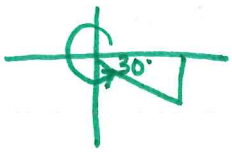
o/h a/h o/a h/o h/a a/o

- Ratios for sin, cos, tan, csc, sec, and cot
- The length of the radius of the unit circle → Always 1.
- Coordinates in the (x,y) plane correspond to which trigonometric ratios? cos = x, sin = y } on unit circle r = 1.
- Definition of a radian angle unit of measure. Arc length of a unit circle.
- How to convert between degrees and radians $\frac{\pi}{180} \cdot D = R$ $D = R\frac{180}{\pi}$
- Which quadrants have positive and negative coordinates? + | ++ / − | + r
- What does each component of a trigonometric function (such as $y = a \sin b(\theta - h) + k$) mean in terms of period, amplitude, phase shift, and the vertical shift?

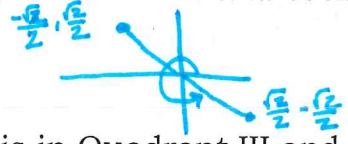
Amp ↑ Period change ↑ phase shift ↑ vert. shift

37. Identify an angle in Quadrant IV with a reference angle of 30°. Tell the measure of the angle in both degrees and radians.

330° and $\frac{11\pi}{6}$

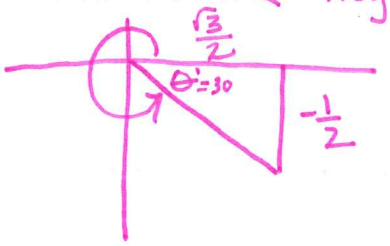


38. $\frac{3\pi}{4}$ is an angle whose sine and cosine are opposites. Tell the measure of the other angle whose sine and cosine are opposites in both degrees and radians.



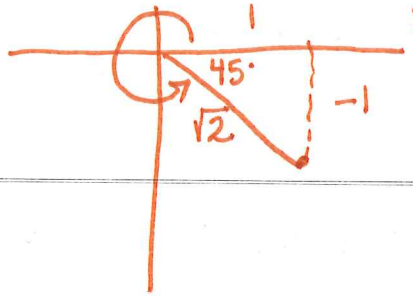
$\frac{7\pi}{4}$ (315°)

39. If θ is in Quadrant III and $\sin\theta = -\frac{1}{2}$, what other angle in a different quadrant will have the same sine? ← neg y ⇒ QIII and QIV



330° and $\frac{11\pi}{6}$

40. Graph the point (1, −1) in standard position so its terminal side is θ. Then find the reference angle, θ', and all exact trig ratio values. θ = 315°



$\sin\theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

θ' = 45°

sin θ' = $-\frac{\sqrt{2}}{2}$

cos θ' = $\frac{\sqrt{2}}{2}$

tan θ' = −1

41. Identify the point(s) on the unit circle where tangent is undefined.

where we ÷ by zero! $\frac{y}{x}$ or where x is zero.

90° and 270° $\frac{\pi}{2}$ or $\frac{3\pi}{2}$

Give the exact measurements for the following.

42. $\cos -45°$ $= \frac{\sqrt{2}}{2}$

43. $\tan 210°$ $= \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

44. $\cos 510°$ $= -\frac{\sqrt{3}}{2}$

45. $\sin\left(-\frac{5\pi}{4}\right)$

46. $\cos\frac{11\pi}{3} = \frac{1}{2}$

47. $\tan\frac{3\pi}{6}$

State the vertical shift, amplitude, period, and phase shift for each function. Then graph the function.

48. $y = \cos\theta$

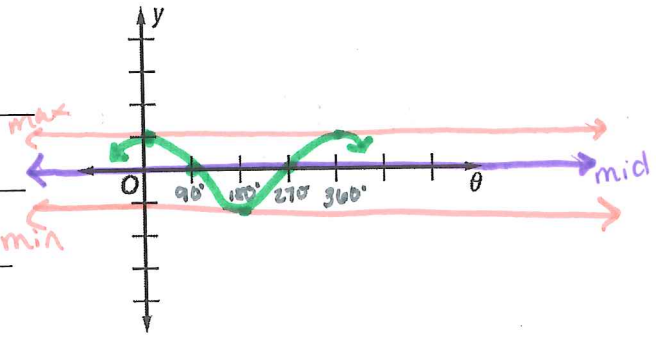
0 90° 180° 270° 360°

Vertical Shift: none

Amplitude: 1

Period: 360°

Phase Shift: none



49. $y = 2\sin\theta + 1$

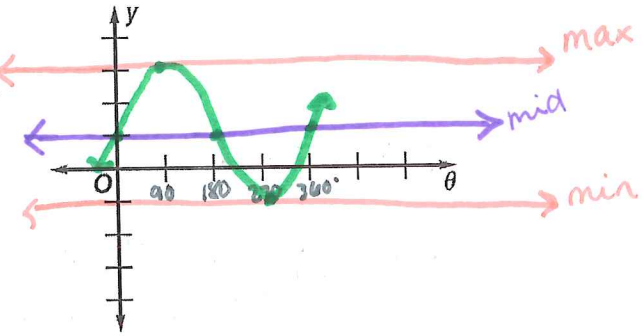
0 90 180 270 360

Vertical Shift: up 1

Amplitude: 2

Period: 360

Phase Shift: none



50. $y = \sin 4\theta$

0 90 180 270 360

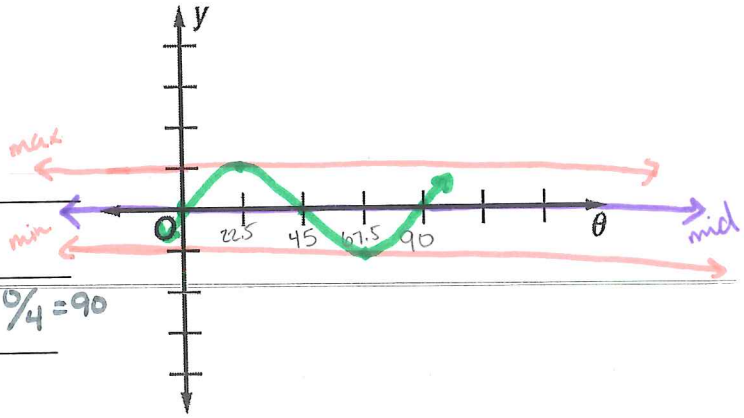
0 22.5 45 67.5 90

Vertical Shift: none

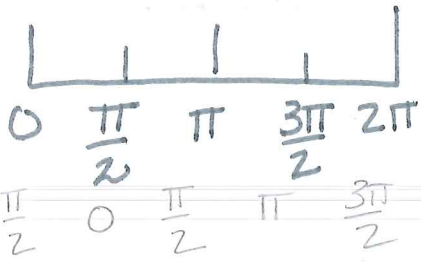
Amplitude: 1

Period: 90° $360/4 = 90$

Phase Shift: none



51. $y = \cos\left(\theta + \frac{\pi}{2}\right)$

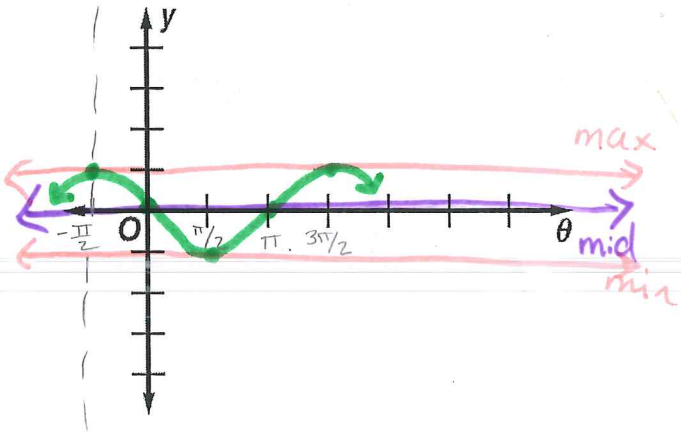


Vertical Shift: none

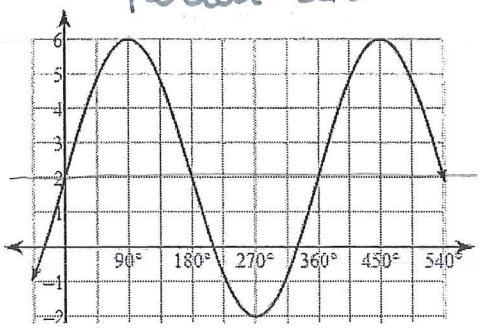
Amplitude: 1

Period: 2π

Phase Shift: none



52. Write a sine function for the graph below. Amp: 4 VS = up 2 Period = 360°



$y = 4\sin\theta + 2$

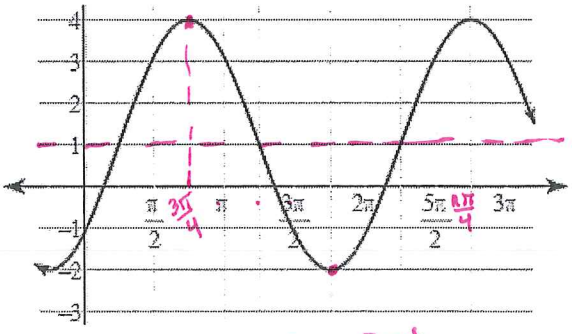
or

$y = -4\sin(\theta - 180^\circ) + 2$

or

$y = 4\sin(\theta - 360) + 2$

53. Write a cosine function for the graph below Amp = 3 midline @ 1



$y = 3\cos\left(\theta - \frac{3\pi}{4}\right) + 1$

or

$y = 3\cos\left(\theta - \frac{11\pi}{4}\right) + 1$

or

$y = -3\cos\left(\theta - \frac{7\pi}{4}\right) + 1$

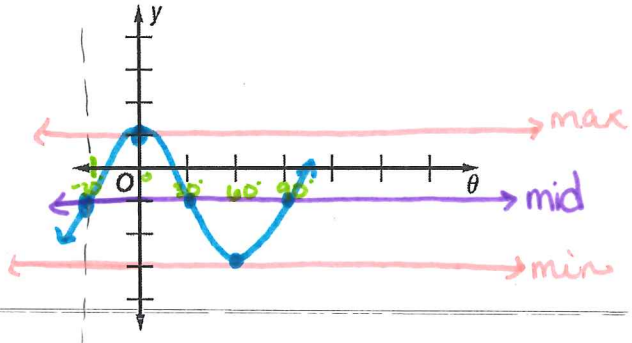
54. Graph the function $y = 2\sin3(\theta + 30°) - 1$.

Vertical Shift: down 1

Amplitude: 2

$\frac{360}{3} =$ Period: 120°

Phase Shift: left 30



0 90° 180° 270° 360°

0° 30° 60° 90° 120° Period 120°

-30 0 30° 60° 90°

Vectors "Need to Know"

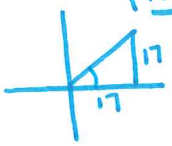
- How do you find the magnitude or direction of a vector? Length. Pythag. distance / Pos. rotation from pos x axis.
- If a vector is in standard position, the tail of the vector is at (0,0)?
- How do you find the component form of a vector given two points? Given magnitude and direction? $\langle x_2 - x_1, y_2 - y_1 \rangle$ and work backwards.
- When are two vectors equal? Same components.
- What are the different methods for adding vectors? para, tip to tail
- What is scalar multiplication of a vector? changes the size.
- What happens when a vector is multiplied by a negative constant? changes direction

Let $\vec{v} = \langle 3, 7 \rangle$ and $\vec{w} = \langle 4, -2 \rangle$. Compute the following, then find magnitude and direction.

55. $3\vec{v} + 2\vec{w}$

$\langle 9,21 \rangle + \langle 8,-4 \rangle$

$\langle 17, 17 \rangle$

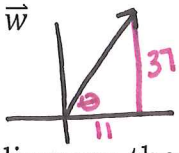


mag: $17\sqrt{2}$

direction: 45°

56. $5\vec{v} - \vec{w}$

$\langle 15, 35 \rangle - \langle 4, -2 \rangle = \langle 11, 37 \rangle$



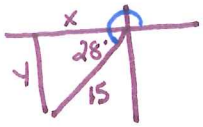
mag: $\sqrt{1490}$

direction: $\tan^{-1}\left(\frac{37}{11}\right) = 73.443°$

57. Consider vector $\vec{v}\langle -2,5 \rangle$ and vector $\vec{w}\langle 3,7 \rangle$. Draw a diagram that models $\vec{v} + \vec{w}$

Find the component form of vector v, with magnitude and direction given:

58. $\vec{v} = 15,\ \theta = 208°$



$\cos 28 = \frac{x}{15}$, $\sin 28 = \frac{y}{15}$

$-x = 13.244$

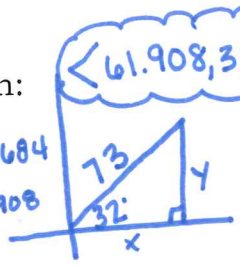
$y = -7.042$ Q III on coord. Plane so negative

$\langle -13.244, -7.042 \rangle$

59. $\vec{v} = 73,\ \theta = 32°$

$\sin 32 = \frac{y}{73}$ $y = 38.684$

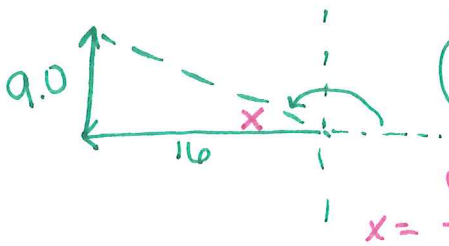
$\cos 32 = \frac{x}{73}$ $x = 61.908$



$\langle 61.908, 38.684 \rangle$

Draw each vector component to find the resultant. Find the direction and velocity.

60. A motorboat heads across a river due west at 16 m/s. The river current flows north at 9.0 m/s.



$9^2 + 16^2 = \vec{r}^2$

$\sqrt{337} = |\vec{r}|$

≈ 18.358 m/s

direction: $180 - x$

$x = \tan^{-1}\left(\frac{9}{16}\right)$ $x = 29.358°$

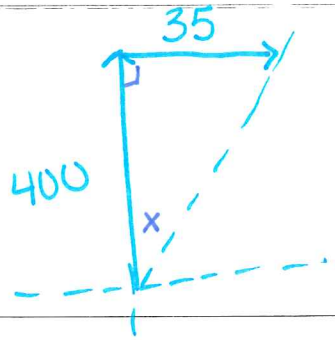
Direction

150.642°

OR

29.358° north of due west

61. An airplane is flying due north at 400 miles per hour. The wind is blowing due east of ~~north~~ at 35 miles per hour.



$400^2 + 35^2 = |\vec{r}|^2$

$\sqrt{161225} = |\vec{r}|$

≈ 401.528 mph

$\tan^{-1}\left(\frac{35}{400}\right)$

$x = 5.001$

direction: 84.999°

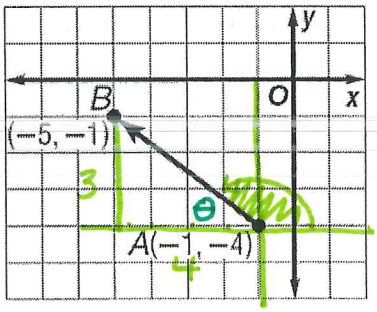
or 5.001 east of north

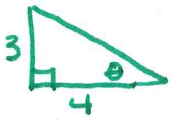
62. Write the component form of the vector and find the magnitude and direction of the vector.

$\vec{v} = \langle -4, 3 \rangle$ comp. form :)

mag: $4^2 + 3^2 = \vec{v}^2$

$5 = |\vec{v}|$





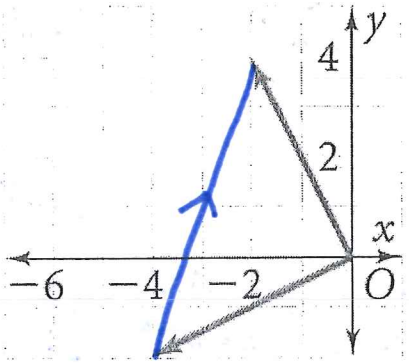
$\tan^{-1}\left(\frac{3}{4}\right) \approx 36.870°$

direction: $180 - \theta$

$143.130°$

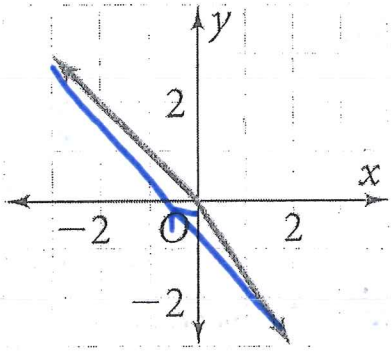
Draw the resultant. Write the resultant in component form.

63.



$\langle 2, 6 \rangle$

64.



$\langle -5, 6 \rangle$