

GEO Notes: Isosceles and Equilateral Triangles- Algebra Based

Properties of Isosceles Triangles An isosceles triangle has two congruent sides. The angle formed by these sides is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

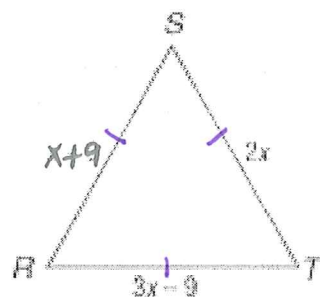
- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (**Isosceles Triangle Theorem**)
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.
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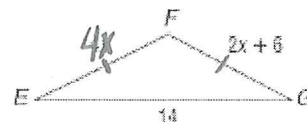
Example 1. Find x and the measure of each side of equilateral triangle RST .

$SR = ST$ def of equilateral Δ
 $x + 9 = 2x$
 $9 = x$
 $ST = 18$
 $SR = 18$
 $RT = 18$ } You check your work so these are all \cong



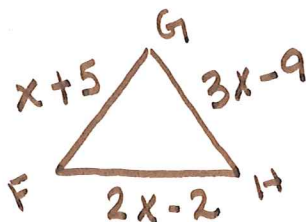
Example 2. Find x and the measure of each side of isosceles triangle EFG .

$EF = FG$ def of isosc. Δ
 $4x = 2x + 6$
 $2x = 6$
 $x = 3$
 $EF = 12$
 $FG = 12$
 $EG = 14$



Directions: Find x and the measure of each side of the triangle.

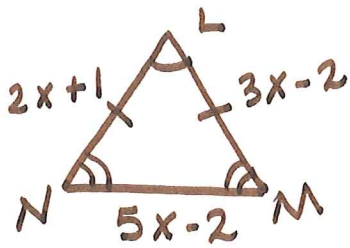
3. ΔFGH is equilateral with $FG = x + 5$, $GH = 3x - 9$, and $FH = 2x - 2$.



$FG = GH$ def of eq. Δ
 $x + 5 = 3x - 9$
 $14 = 2x$
 $7 = x$

$x = \underline{7}$
 $FG = \underline{12}$
 $GH = \underline{12}$
 $FH = \underline{12}$

4. $\triangle LMN$ is isosceles, $\angle L$ is the vertex angle, $LM = 3x - 2$, $LN = 2x + 1$, and $MN = 5x - 2$.



$$LM = LN$$

$$3x - 2 = 2x + 1$$

$$x = 3$$

$$x = \underline{3} \quad LM = \underline{7}$$

$$LN = \underline{7} \quad MN = \underline{13}$$

5. COORDINATE GEOMETRY Find the measures of the sides of $\triangle DEC$. Classify the triangle by sides.

Use the Distance Formula to find the lengths of each side.

$$DE^2 = 6^2 + 8^2$$

$$DE = \sqrt{100}$$

$$DE = \underline{10}$$

$$EC^2 = 1^2 + 7^2$$

$$EC = \sqrt{50}$$

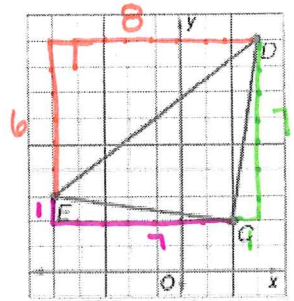
$$EC = \underline{5\sqrt{2}}$$

$$CD^2 = 1^2 + 7^2$$

$$CD = \sqrt{50}$$

$$CD = \underline{5\sqrt{2}}$$

$EC \cong CD \therefore$ by def $\triangle DEC$ is an isosceles \triangle .



6. Find the measures of the side of $\triangle KPL$ and classify the triangle by its sides.
 $K(-3, 2)$, $P(2, 1)$, $L(-2, -3)$

$$KL^2 = 5^2 + 1^2$$

$$KL = \sqrt{26}$$

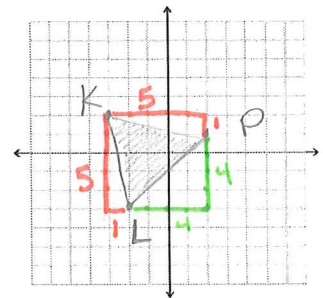
$$KP^2 = 5^2 + 1^2$$

$$KP = \sqrt{26}$$

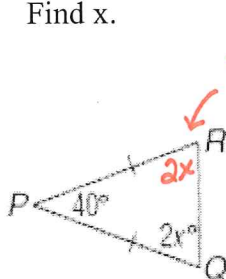
$$PL^2 = 4^2 + 4^2$$

$$PL = \underline{4\sqrt{2}}$$

$KL \cong KP \therefore$ by def $\triangle KPL$ is an isosceles \triangle .



7. Find x .



base \angle s of isos $\triangle \cong$

$$\angle P + \angle R + \angle Q = 180 \quad \triangle \text{ sum}$$

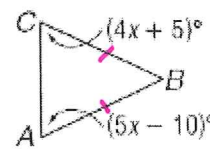
$$40 + 2x + 2x = 180$$

$$4x + 40 = 180$$

$$4x = 140$$

$$x = \underline{35}$$

8. Find x . If $BC \cong BA$.



$\angle C \cong \angle A$
 base \angle s of isosc \triangle are \cong

$$4x + 5 = 5x - 10$$

$$15 = x$$

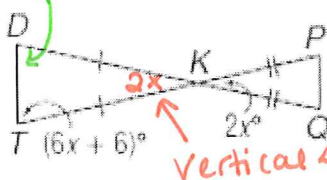
9. Find x .

$6x + 6$ base \angle s of isosc \triangle

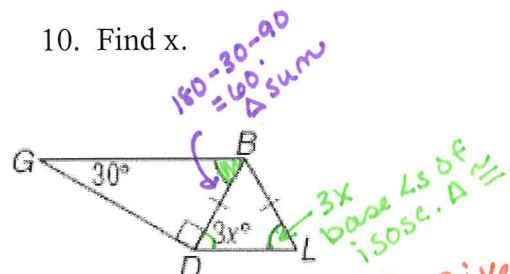
$$\angle D + \angle DKT + \angle T = 180 \quad \triangle \text{ sum}$$

$$6x + 6 + 2x + 6x + 6 = 180$$

$$x = \underline{12}$$



10. Find x .



$180 - 30 - 90 = 60$
 $\triangle \text{ sum}$

$\angle L \cong \angle GBD$ given

$$3x = 60$$

$$x = \underline{20}$$