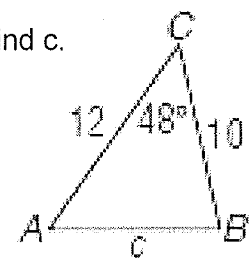
Law of Cosines Notes

The Law of Cosines Another relationship between the sides and angles of any triangle is called the **Law of Cosines**. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

Law of Cosines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $\quad b^2 = a^2 + c^2 - 2ac \cos B$ $\quad c^2 = a^2 + b^2 - 2ab \cos C$

1. Find c.



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 10^2 + 12^2 - 2(10)(12)\cos 48$

$c^2 = 100 + 144 - 240\cos 48$

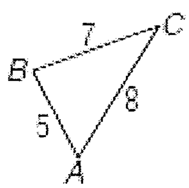
$c^2 = 244 - 240\cos 48$

$\sqrt{c^2} = \sqrt{83.41}$

$\boxed{c = 9.13}$

do we round to nearest whole * to simplify the radical? no! get decimal in calculator

2. Find the m<A.



$a^2 = b^2 + c^2 - 2bc\cos A$

$7^2 = 8^2 + 5^2 - 2(8)(5)\cos A$

$49 = 64 + 25 - 80\cos A$

$49 = \cancel{89} - 80\cos A$

$-89 \quad -\cancel{89}$

$\dfrac{-40}{-80} = \dfrac{-\cancel{80}\cos A}{\cancel{-80}}$

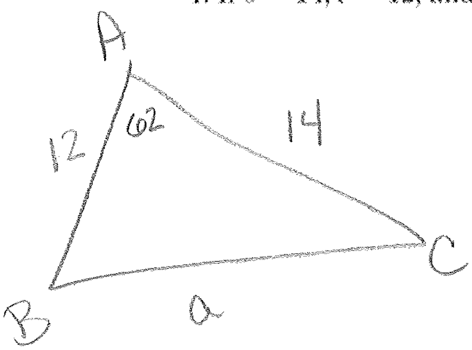
$\frac{1}{2} = \cos A$

$A = \cos^{-1}\left(\frac{1}{2}\right)$ $\quad \boxed{m\angle A = 60°}$

Law of Cosines Notes

Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $b = 14$, $c = 12$, and $m\angle A = 62$, find a.



$a^2 = b^2 + c^2 - 2(b)(c)\cos A$

$a^2 = 14^2 + 12^2 - 2(14)(12)\cos 62$

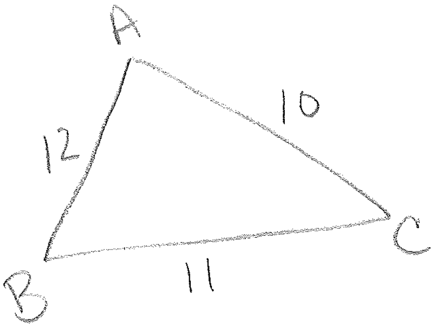
$a^2 = 196 + 144 - 336\cos 62$

$a^2 = 340 - 336\cos 62$

$\sqrt{a^{\cancel{2}}} = \sqrt{182.3}$

$\boxed{a = 13.5}$

2. If $a = 11$, $b = 10$, and $c = 12$, find $m\angle B$.



$b^2 = a^2 + c^2 - 2(a)(c)\cos B$

$10^2 = 11^2 + 12^2 - 2(11)(12)\cos B$

$100 = 121 + 144 - 264\cos B$

$100 = 265 - 264\cos B$

$-265 \quad -265$

$\frac{-165}{-264} = \frac{-\cancel{264}\cos B}{-\cancel{264}}$

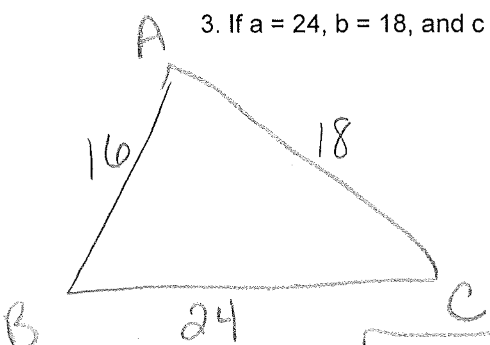
$\frac{5}{8} = \cos B$

$B = \cos^{-1}\left(\frac{5}{8}\right)$

$= 51.318$

$\boxed{m\angle B = 51^\circ}$

3. If a = 24, b = 18, and c = 16. Solve the triangle.



$a^2 = b^2 + c^2 - 2(b)(c)\cos A$

$24^2 = 18^2 + 16^2 - 2(18)(16)\cos A$

$576 = 580 - 576\cos A$

$-4 = -576\cos A$

$\frac{1}{144} = \cos A$

$\boxed{m\angle A = 89^\circ}$

$b^2 = a^2 + c^2 - 2(a)(c)\cos B$

$18^2 = 24^2 + 16^2 - 2(24)(16)\cos B$

$324 = 832 - 768\cos B$

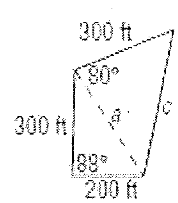
$-508 = -768\cos B$

$\frac{127}{192} = \cos B$

$\boxed{m\angle B = 49^\circ}$

$180 - 90 - 49 = 41$ $\boxed{m\angle C = 42^\circ}$

4. **Example** Ms. Jones wants to purchase a piece of land with the shape shown. Find the perimeter of the property.



$a^2 = 300^2 + 200^2 - 2(300)(200)\cos 88$

$\sqrt{a^2} = \sqrt{90000 + 40000 - 120000\cos 88}$

$\boxed{a = 355}$

$c^2 = 355^2 + 300^2 - 2(355)(300)\cos 80$

$\sqrt{c^2} = \sqrt{126025 + 90000 - 213000\cos 80}$

$\boxed{c = 423}$

Perimeter: $200 + 200 + 300 + 423 = \boxed{1223 ft}$