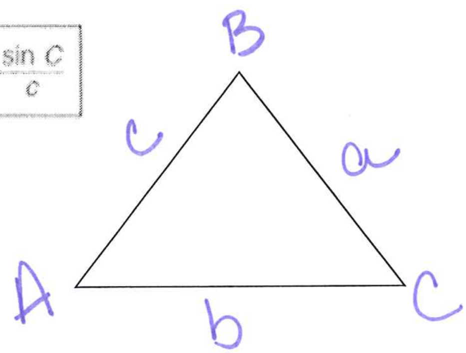
Law of Sines and Cosines Notes

Use for NON-RIGHT Triangles Only!

The Law of Sines In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



The Law of Cosines Another relationship between the sides and angles of any triangle is called the **Law of Cosines**. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

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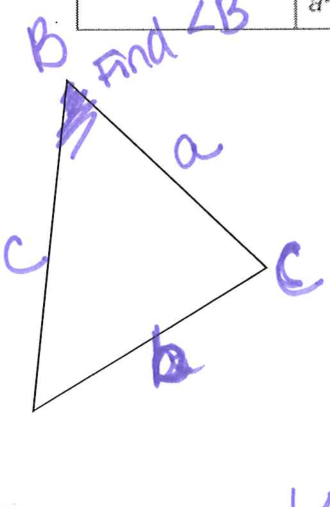
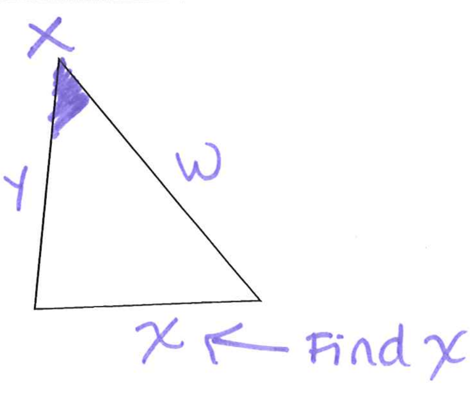
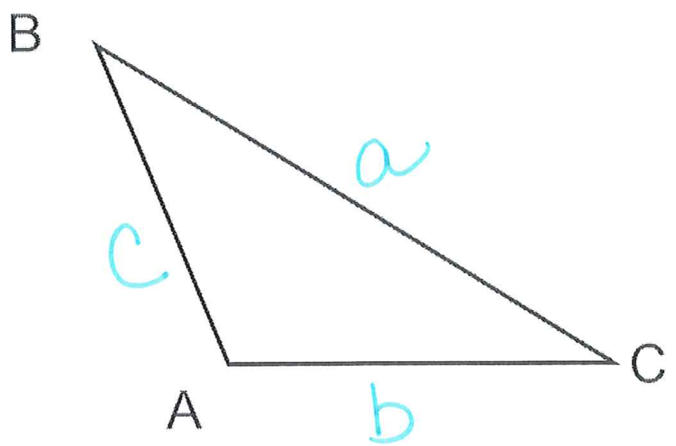




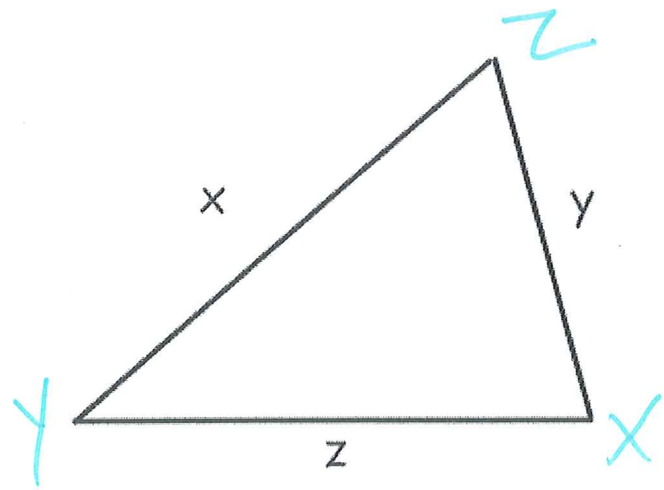
Name : Key

Law of Sines Notes

Review: Place the opposite sides.



Review: Place the opposite angles.



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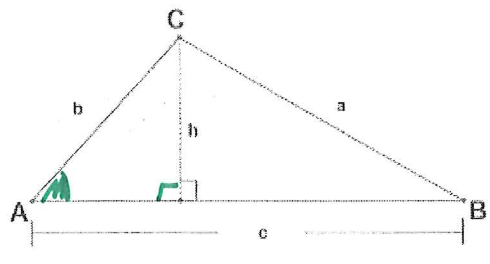


Prove the law of sines: $\frac{\sin A}{a} = \frac{\sin B}{b}$

Step 1:

Sin A= $\frac{h}{b}$ Sin B= $\frac{h}{a}$

Step 2: get h alone $h = b \cdot \sin A$

$h = a \cdot \sin B$

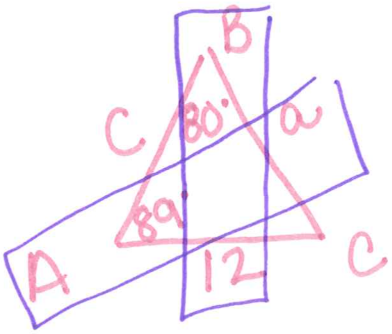
Step 3: If two things are equal to the same thing, then they are....

$b \sin A = a \cdot \sin B$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Solve ΔABC for all variables. ← Find all ∠s and all sides.

Ex1. If $b = 12$, $m\angle A = 89$, and $m\angle B = 80$



$$\frac{\sin(80)}{12} = \frac{\sin(89)}{a}$$

$$a\sin(80) = 12\sin(89)$$

$$a = \frac{12 \cdot \sin(89)}{\sin(80)}$$

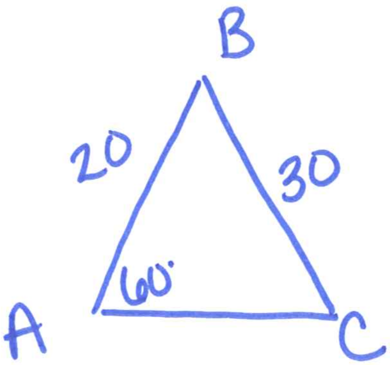
$$\boxed{a \approx 12.2}$$

$\boxed{\angle C = 11°}$ by Δ sum

$$\frac{\sin(11)}{c} = \frac{\sin(80)}{12}$$

$$\boxed{c \approx 2.3}$$

Ex2. If $a = 30$, $c = 20$, and $m\angle A = 60$,



$$\frac{\sin C}{20} = \frac{\sin(60)}{30}$$

$$\sin C = \frac{20\sin(60)}{30}$$

$$\angle C = \sin^{-1}\left(\frac{20 \cdot \sin(60)}{30}\right)$$

$$\boxed{\angle C \approx 35.7°}$$

$\boxed{\angle B \approx 84.3°}$ by Δ sum

$$\frac{\sin(84.3)}{b} = \frac{\sin(60)}{30}$$

$$\boxed{b \approx 34.5}$$

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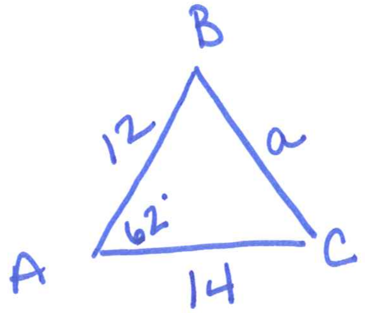
Ex 3. If m<A = 62°, b = 14, and c = 12. Solve the triangle.



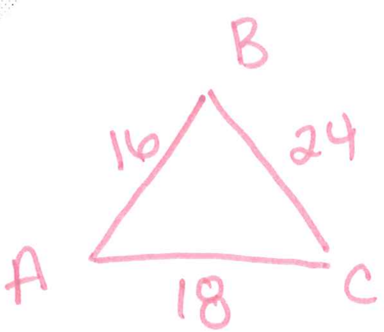
$a^2 = 14^2 + 12^2 - 2\cdot 14\cdot 12\cos(62)$

$a = \sqrt{14^2 + 12^2 - 2\cdot 14\cdot 12\cos(62)}$

$\boxed{a \approx 13.5}$

$\frac{\sin(62)}{13.5} = \frac{\sin C}{12}$ $\boxed{\angle C \approx 70.6^\circ}$ $\angle B \approx 47.4^\circ$

Ex 4. If a = 24, b = 18, and c = 16. Solve the triangle.



ops.

$24^2 = \underbrace{16^2 + 18^2}_{\text{CLT}} - 2\cdot 16\cdot 18\cos A$

$576 = 580 - 576\cos A$

$-4 = -576\cos A$

$\frac{-4}{-576} = \cos A$

$\angle A = \cos^{-1}\left(\frac{-4}{-576}\right)$

$\boxed{\angle A \approx 89.6^\circ}$

$\frac{\sin B}{18} = \frac{\sin(89.6)}{24}$

$\sin B = \frac{18\cdot\sin(89.6)}{24}$

$\angle B = \sin^{-1}\left(\frac{18\cdot\sin(89.6)}{24}\right)$

$\boxed{\angle B \approx 48.6^\circ}$

$\angle C \approx 41.8^\circ$ by Δ sum.