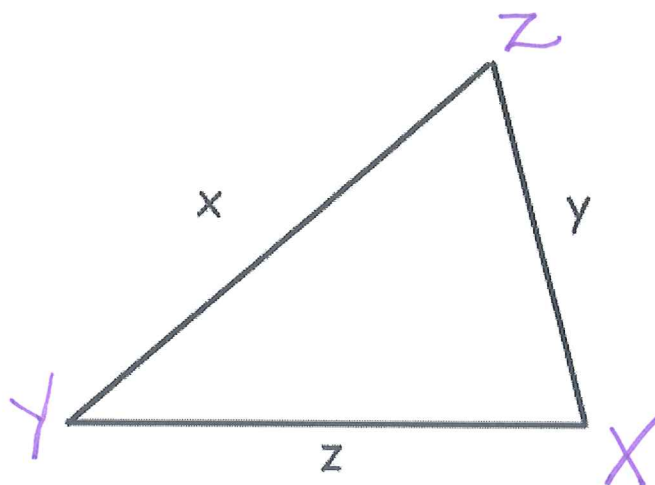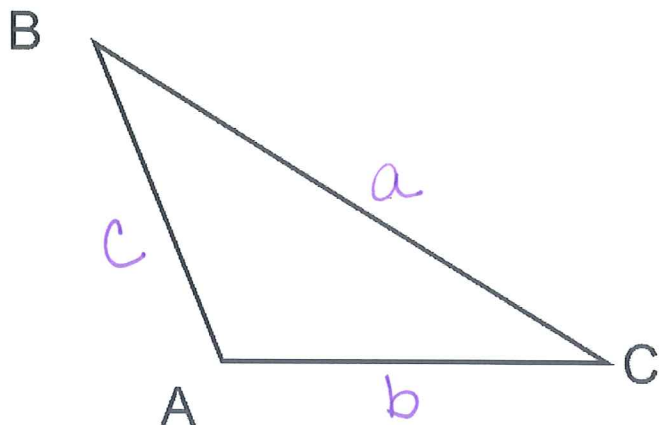# Law of Sines Notes

Review: Place the opposite sides.   Review: Place the opposite angles.

The Law of Sines In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.
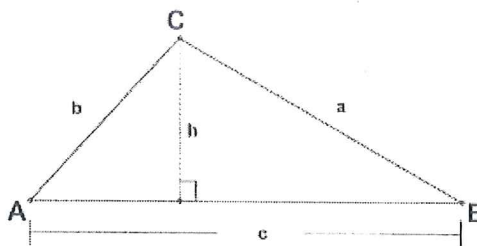
Law of Sines	$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Prove the law of sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

Step 1:

$\sin A = \dfrac{h}{b}$ $\sin B = \dfrac{h}{a}$

Step 2: get h alone $b \sin A = \dfrac{h}{b} \cdot b$ $a \sin B = \dfrac{h}{a} \cdot a$
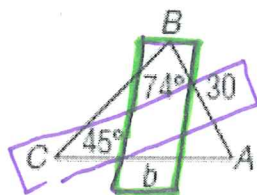
$b \sin A = h$ $a \sin B = h$

Step 3: If two things are equal to the same thing, then they are.... equal to each other.

$\dfrac{b \sin A}{ab} = \dfrac{a \sin B}{ab}$

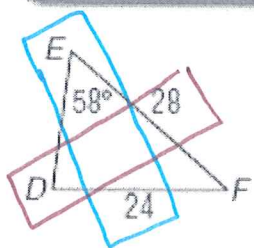$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

Example 1   In △ABC, find b.

$$\frac{Sin C}{c} = \frac{Sin B}{b}$$

$$\frac{Sin(45)}{30} = \frac{Sin(74)}{b}$$

$$\frac{b \, Sin(45)}{Sin(45)} = \frac{30 \cdot Sin(74)}{Sin(45)}$$

$$b = 30 \cdot Sin(74) \div Sin(45) \text{ in}$$
$$\text{calculator!}$$

$$b \approx 40.78$$

Example 2   In △DEF, find m∠D.

$$\frac{Sin E}{e} = \frac{Sin D}{d}$$

$$28 \cdot \frac{Sin(58)}{24} = \frac{Sin D}{28} \cdot 28$$

$$\frac{28 \cdot Sin(58)}{24} = Sin D$$

* Recall to solve for an angle you take the inverse!

$$\angle D = Sin^{-1}\left(\frac{28 \cdot sin(58)}{24}\right)$$

Plug in calculator:

$$sin^{-1}(28 \cdot Sin(58) \div 24) \boxed{enter}$$
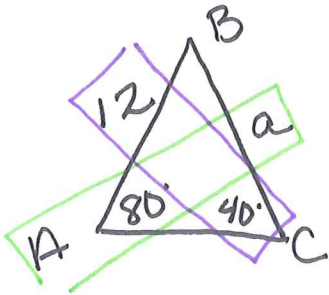
$$\boxed{\angle D = 81.65°}$$

Name: _____   Hour: _____

Now, set up the triangle yourself.

It is expected that you draw the triangle when one is not given to you.

Find each measure using the given measures of Triangle ABC. Find the measure.

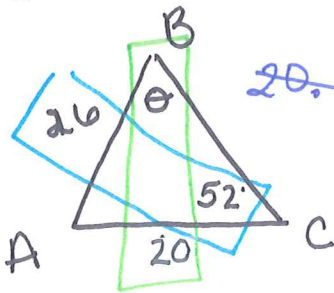3. If $c = 12$, $m\angle A = 80$, and $m\angle C = 40$, find a.

$$\frac{\sin(80)}{a} \bowtie \frac{\sin(40)}{12}$$

$$\frac{12 \cdot \sin(80)}{\sin(40)} = \frac{a \sin(40)}{\sin(40)}$$

$$\boxed{a = 18.39}$$

4. If $b = 20$, $c = 26$, and $m\angle C = 52$, find $m\angle B$.
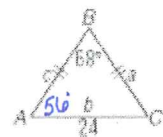
$$\frac{\sin B}{20} = \frac{\sin(52)}{26} \cdot 20$$

$$\sin B = \frac{20 \cdot \sin(52)}{26}$$

$$\angle B = \sin^{-1}\left(\frac{20 \cdot \sin(52)}{26}\right)$$

$$\angle B = 37.31°$$

5. **Example**   Isosceles $\triangle ABC$ has a base of 24 centimeters and a vertex angle of 68°. Find the perimeter of the triangle.

The vertex angle is 68°, so the sum of the measures of the base angles is 112 and $m\angle A = m\angle C = 56$.

Find $a = c$

$$\frac{\sin(56)}{a} = \frac{\sin(68)}{24}$$

Perimeter
$$P = a + 24 + c$$
$$P = 21.46 + 24 + 21.46$$
$$\boxed{P = 66.92 \text{ cm}}$$

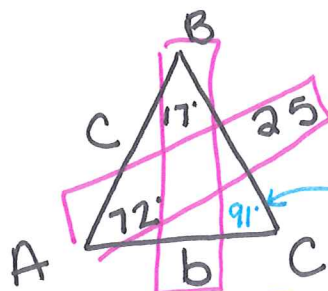$$\frac{24 \cdot \sin(56)}{\sin(68)} = \frac{a \sin(68)}{\sin(68)}$$

$$\boxed{a \approx 21.46 \text{ cm}}$$

Practice: solve the triangle. ← Find Everything that is missing!

6. If $a = 25$, $m\angle A = 72$, and $m\angle B = 17$

So Find
1.) b
2.) <c
3.) C

To Find ∠C
use Δ sum theorem
$17 + 72 + \angle C = 180$
$\boxed{\angle C = 91°}$

$b = \underline{7.69}$

$\angle C = \underline{91°}$

$C = \underline{26.28}$

Find b $\boxed{b = 7.69}$

$\dfrac{Sin(17)}{b} \nearrow \dfrac{Sin(72)}{25}$

$\dfrac{25 \cdot Sin(17)}{Sin(72)} = \dfrac{b \sin(72)}{\sin(72)}$

To Find c

$\dfrac{Sin(91)}{c} = \dfrac{Sin(72)}{25}$

$\dfrac{25 \cdot Sin(91)}{Sin(72)} = \dfrac{C \cdot Sin(72)}{Sin(72)}$

$\boxed{C \approx 26.28}$

7. If $b = 12$, $m\angle A = 89$, and $m\angle B = 80$

missing ∠C, a, c

To Find ∠C
use Δ sum theorem
$180 = 80 + 89 + \angle C$
$\boxed{\angle C = 11°}$

To Find side a

$\dfrac{Sin(89)}{a} = \dfrac{Sin(80)}{12}$

$\dfrac{12 \sin(89)}{Sin(80)} = \dfrac{a \sin(80)}{Sin(80)}$

$\boxed{a \approx 12.18}$

$\angle C = \underline{11°}$

$a = \underline{12.18}$

$C = \underline{2.33}$

Find side c:

$\dfrac{Sin(11)}{C} = \dfrac{Sin(80)}{12}$

$\dfrac{12 \sin(11)}{Sin(80)} = \dfrac{C \sin(80)}{Sin(80)}$ $\boxed{C \approx 2.33}$

8. If $a = 30$, $c = 20$, and $m\angle A = 60$,

Find ∠C, ∠B, b

$\dfrac{Sin C}{20} = \dfrac{Sin(60)}{30}$

$Sin C = \dfrac{20 Sin(60)}{30}$

$\angle C = Sin^{-1}\left(\dfrac{20 \sin(60)}{30}\right)$

$\boxed{\angle C = 35.26°}$

Find ∠B
Δ Sum
$\boxed{\angle B = 84.74°}$

$\angle C = \underline{35.26°}$

$\angle B = \underline{84.74°}$

$b = \underline{34.50}$

Find side b

$\dfrac{Sin(84.74)}{b} = \dfrac{Sin(60)}{30}$

$\dfrac{30 \sin(84.74)}{Sin(60)} = \dfrac{b \sin(60)}{Sin(60)}$

$\boxed{b \approx 34.50}$