Notes: Conditional Statements and Logic 2.3 Examples

Directions: Identify the hypothesis and conclusion for each statement. State if the conditional is true or false for each false statement, provide a counter example.

1. If you live in California, then you live in San Diego.

Hypothesis: p

q

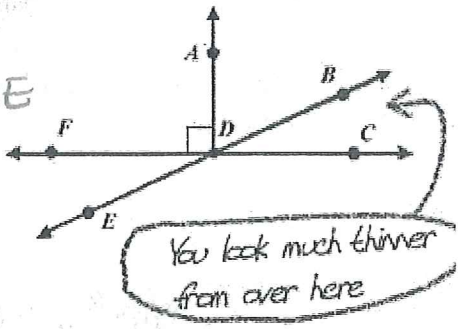
Conclusion:

False: I live in LA

2. If two angles sum of their measures is 90°, then they are complementary angles.

Hypothesis:

Def = TRUE



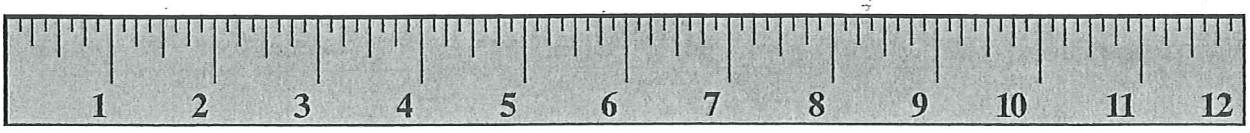
Conclusion:

I do not go to School (DHS class), if it is Sunday.

3. If two angles are supplementary angles, then they form a linear pair.

Hypothesis:

Conclusion:



$p \rightarrow q$

State the inverse, converse, and contrapositive of the following statements.

1. If the worker is injured, then the family sues.

$\sim p \rightarrow \sim q$ Inverse: If the worker is NOT injured, then the family does NOT sue.

$q \rightarrow p$ Converse: If the family sues, then the worker is injured

$\sim q \rightarrow \sim p$ Contrapositive: If the family does not sue, then the worker is not injured.

2. If the enemy retreats, then the general will pursue them.

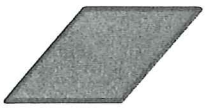
Inverse: _____

Converse: _____

Contrapositive: _____

Given the following statements, decide their truth values, and then decide the truth values of its inverse, converse, and contrapositive.

3. If a polygon has four sides, then it is a pentagon.



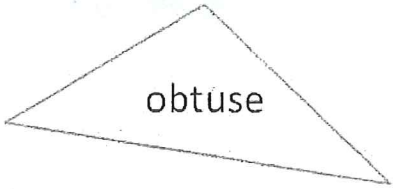
Statement: F

Inverse: If a polygon doesn't have 4 sides, then it is not a Pentagon.

Converse: If the polygon is a pentagon, then it has 4 sides.

Contrapositive: If the poly is not a pentagon, then it does not have 4 sides.

4. If a triangle is obtuse, then it has one obtuse angle.



Statement: _____

Inverse: _____

Converse: _____

Contrapositive: _____

Name Key

Biconditional Statements

Ashley began a new summer job, earning \$10 an hour. If she works over 40 hours a week, she earns time and a half, or \$15 an hour. If she earns \$15 an hour, she has worked over 40 hours a week.

p: Ashley earns \$15 an hour
q: Ashley works over 40 hours a week

$p \rightarrow q$: If Ashley earns \$15 an hour, she has worked over 40 hours a week.
$q \rightarrow p$: If Ashley works over 40 hours a week, she earns \$15 an hour.

In this case, both the conditional and its converse are true. The conjunction of the two statements is called a biconditional statement

KEY CONCEPT — **Biconditional Statement**

Words A biconditional statement is the conjunction of a conditional and its converse.

Symbols $(p \rightarrow q) \wedge (q \rightarrow p)$ notation $(p \leftrightarrow q)$

If and only if can be abbreviated *iff*.

So, the biconditional statement is as follows.

$p \leftrightarrow q$: Ashley earns \$15 an hour *if and only if* she works over 40 hours a week.

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

a. **Two angle measures are complements if and only if their sum is 90.**
Conditional: If two angle measures are complements, then their sum is 90.
Converse: If the sum of 2 angle measures is 90, then they are complements.
Both the conditional and the converse are true, so the biconditional is true.

b. $x > 9$ iff $x > 0$
Conditional: If $x > 9$ then $x > 0$ Converse: If $x > 0$ then $x > 9$
The conditional is true, but the converse is not. Let $x = 2$. Then $2 > 0$ but $2 \not> 9$. So, the biconditional is false.

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

1. A calculator will run if and only if it has batteries.

Conditional: If a calculator runs, then it has batteries

Converse: If a calculator has batteries, then it will run. False, solar powered or issue w/ calculator

2. Two lines intersect if and only if they are not vertical.

Conditional: If two lines intersect, then they are not vertical

Converse: If two lines are not vertical, then they intersect

False - 2 parallel horizontal lines will not intersect

3. Two angles are congruent if and only if they have the same measure.

Cond: If 2 angles are $\cong$, then they have the same measure

Conv: If 2 $\angle$s have same measure, then they are $\cong$

TRUE

4. $3x - 4 = 20$ iff $x = 7$.

Cond: If $3x-4=20$, then $x=7$

Conv: If $x=7$, then $3x-4=20$

False $3x-4=17$ when $x=7$

Always, Sometimes, never true.

a.) $\overleftrightarrow{GH}$ contains 3 noncollinear points. Never

b.) If plane T contains $\overleftrightarrow{EF}$ and $\overleftrightarrow{EF}$ contains point G, then plane T contains point G. Always

c.) The intersection of plane Q and plane Y is point T. Never

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EXTRA PRACTICE 2.3 LOGICAL REASONING

Use the statement "All squares are rectangles" for Exercises 1–5.

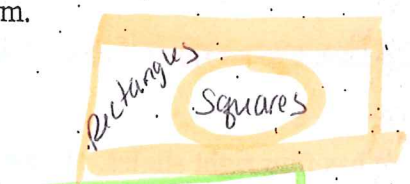
1. Rewrite the statement as a conditional.

If the figure is a square, then it is a rectangle

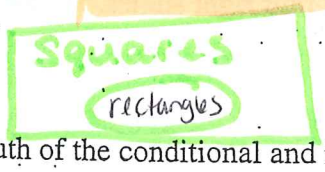
2. Write the converse of the conditional.

If a figure is a rectangle, then it is a square

3. Illustrate the conditional with a Venn diagram.



4. Illustrate the converse with a Venn diagram.



5. What conclusion can you make about the truth of the conditional and its converse from the Venn diagrams?

The conditional is true but the converse is not as there are rectangles which are not squares

For Exercises 6–7, write the converse of the conditional. Combine the conditional and its converse to write a biconditional.

6. If a figure is a line segment, then it contains at least two distinct points.

Converse: If a figure contains at least 2 distinct pts, then it is a line seg. Bicon: A figure is a line segment iff it contains at least 2 distinct pts

7. If it is snowing, then it is cold outside. Converse: If it is cold outside, then it is snowing.

It is snowing iff it is cold outside

Use the statement "If the temperature is at or below 32°F, then water will freeze" for Exercises 8–9.

8. State whether the conditional is true or false. If it is false, give a counterexample.

True

9. Write the converse and state whether it is true or false. If it is false, give a counterexample.

If water will freeze, then the temperature is at or below 32° F. True

Also do
Pg 108 #11-14, 17-19
Pg 94 #6-10 all

NAME Key CLASS DATE

EXTRA PRACTICE 2.2 DEDUCTIVE REASONING

Write the hypothesis and conclusion of each conditional statement.

1. If a figure is part of a line, then it is a line segment.

2. If B is on $\overline{AC}$ such that $AB = BC$, then B is the midpoint of $\overline{AC}$.

3. You will earn more money if you work more hours.

4. A square has four right angles. If a figure is a square, then it has 4 right $\angle$s

Write the following statements as conditionals.

5. Marvin turns his car heater on when it is cold.
   If it is cold then Marvin turns on his car heater

6. Kira always writes a check when she buys groceries.
   If Kira buys groceries, then she writes a check

7. Intersecting lines form two pairs of vertical angles.
   If two lines intersect, then they form vertical angles

Identify each of the following as an example of inductive or deductive reasoning.

8. If the product of two numbers is 1, the numbers are reciprocals. deductive

9. One out of four students were in the marching band last year. One out of four students will be in the marching band this year.
   inductive

Use deductive reasoning to draw conclusions from the following statements.

10. If two integers are even, then their sum is even. One integer is -6 and the other is 2.
    The sum of -6 and 2 is even

11. Every box of cereal made with oats contains a prize. Rick bought a box of cereal made with oats.
    Rick received a prize.

6.) True 7.) TRUE 8.) False

10.) Conditional: make this 1st
If you are riding in an airplane, then you are safer than riding in a car.
p → q

(F) inverse: ~p → ~q
If you are not riding in an airplane, then you are not safer than riding in a car.

(F) converse: If you are safer than riding in a car, then you are flying in an airplane

(T) contrap: If you are not safer than riding in a car, then you are not flying in an airplane.

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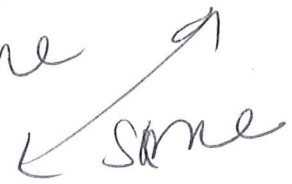
12.) Always

14.) Sometimes, collinear.

18.) If 2 pts lie in a plane then the whole line lies in the plane



20.) line & all points on the line in the plane.