**Introduction to Logic Statements: Reading Homework**

**Declarative Sentences**

When we define and explain things in geometry, we use declarative sentences. For example, "Perpendicular lines intersect at a 90 degree angle" is a declarative sentence. A declarative sentence is a sentence that asserts the truth or falsehood of something. For example, "That car is red" is a declarative sentence. Other sentences can be interrogative, exclamatory, or imperative. Examples are, respectively, "Is that car red?", "Wow, a red car!", and "Drive that red car." Geometry most often concerns itself with declarative sentences.

**Statements**

A declarative sentence is also a sentence that can be classified in one, and only one, of two ways: true or false. Most geometric sentences have this special quality, and are known as statements. For example, "It is purple" is a declarative sentence, but we don't know what "it" is, so we cannot argue its truth or falsehood. "Fred is purple" is a declarative sentence that is definitely either true or false; it is the kind of declarative sentence we can study under the rules of logic. "An obtuse triangle is a triangle with one obtuse angle" is also a declarative sentence that is either true or false (we know it is true, of course) and so can be studied using the rules of logic. From this point forth, we will define a statement as a declarative sentence that is either true or false.

Are the following sentences statements, as defined above?

1. The car drove up the hill. \_\_\_\_\_\_\_\_\_\_

2. What time is it? \_\_\_\_\_\_\_\_\_\_

3. If music is played too loud, it will damage a person's ears. \_\_\_\_\_\_\_\_\_\_

4. The water is colored red. \_\_\_\_\_\_\_\_\_\_

5. He is lazy. \_\_\_\_\_\_\_\_\_\_

**Negations**

Every statement has a negation. Usually the negation of a statement is simply the same statement with the word "not" before the verb. The negation of the statement "The ball rolls" is "The ball does not roll." By definition, the negation of a statement has the opposite truth value of the original statement. The negation of the statement a is ~a (read "not a").

**Conditional Statements**

The most important way to combine two statements is by implication. The implication of two statements c and d takes the form, "if f, then g." The result of implication is called a conditional statement. It is symbolized by placing an arrow between the two letters symbolizing the two statements, as so:

Conditional statements don't necessarily imply cause and effect. They simply state that if one event happens, then another will happen. Much of geometry can be explained using conditional statements, and it is important to understand them. For example, "if a polygon has three sides, then it is a triangle" is a conditional statement.

A conditional statement has two parts, the hypothesis and the conclusion. The hypothesis is the "if" clause of the statement. It is the condition necessary for the conclusion to occur. The conclusion is the "then" clause of the statement. The conclusion is true every time the hypothesis is true. In the statement "If Julie runs fast, then she will win the race", the hypothesis is "Julie runs fast" and the conclusion is "she will win the race."

State the negation of the following statements:

1. Adrian loves rice. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2. The horse is not brown. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Given the following two statements, p and q, the implication "if p, then q."

1. p: The painting is colorful q: The painting is pretty

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2. p: Two lines form a right angle. q: Two lines are perpendicular.

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3. p: Frank is a doctor. q: The birds sing.

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Many different statements can be made by switching the hypothesis with the conclusion and using the negation of a statement instead of the original statement. In the next section, we'll look at some conditional statements with their parts changed in certain ways, and we'll explore the truth values of such statements.

The three most common ways to change a conditional statement are by taking its inverse, its converse, or it contrapositive. In each case, either the hypothesis and the conclusion switch places, or a statement is replaced by its negation.

**The Inverse**

The inverse of a conditional statement is arrived at by replacing the hypothesis and the conclusion with their negations. If a statement reads, "The vertex of an inscribed angle is on a circle", then the inverse of this statement is "The vertex of an angle that is not an inscribed angle is not on a circle." Both the hypothesis and the conclusion were negated. If the original statement reads "if j, then k", the inverse reads, "if not j, then not k."

The truth value of the inverse of a statement is undetermined. That is, some statements may have the same truth value as their inverse, and some may not. For example, "A four-sided polygon is a quadrilateral" and its inverse, "A polygon with greater or less than four sides is not a quadrilateral," are both true (the truth value of each is T). In the example in the paragraph above about inscribed angles, however, the original statement and its inverse do not have the same truth value. The original statement is true, but the inverse is false: it is possible for an angle to have its vertex on a circle and still not be an inscribed angle.

**The Converse**

The converse of a statement is formed by switching the hypothesis and the conclusion. The converse of "If two lines don't intersect, then they are parallel" is "If two lines are parallel, then they don't intersect." The converse of "if p, then q" is "if q, then p."

The truth value of the converse of a statement is not always the same as the original statement. For example, the converse of "All tigers are mammals" is "All mammals are tigers." This is certainly not true.

The converse of a definition, however, must always be true. If this is not the case, then the definition is not valid. For example, we know the definition of an equilateral triangle well: "if all three sides of a triangle are equal, then the triangle is equilateral." The converse of this definition is true also: "If a triangle is equilateral, then all three of its sides are equal." What if we performed this test on a faulty definition? If we incorrectly stated the definition of a tangent line as: "A tangent line is a line that intersects a circle", the statement would be true. But it's converse, "A line that intersects a circle is a tangent line" is false; the converse could describe a secant line as well as a tangent line. The converse is therefore a very helpful tool in determining the validity of a definition.

**The Contrapositive**

The contrapositive of a statement is formed when the hypothesis and the conclusion are interchanged, and both are replaced by their negation. In other words, the contrapositive of a statement is the same as the inverse of that statement's converse, or the converse of its inverse.

Take the statement, "Long books are fun to read." Its contrapositive is "Books that aren't fun to read aren't long." The statement "if p, then q" becomes "if not q, then not p."

The contrapositive of a statement always has the same truth value as the original statement. Therefore, the contrapositive of a definition is always true. For example, the statement "A triangle is a three-sided polygon" is true. Its contrapositive, "A polygon with greater or less than three sides is not a triangle" is also true.

**Summary**

Some basics about logic: When you're talking about converse, inverse,

and contrapositive, then you're dealing with sentences of the form if p then q.

The converse of any such sentence is "if q, then p"

The inverse is

"if not p, then not q"

And the contrapositive is

"if not q, then not p"