

Name: \_\_\_\_\_

## Notes: Rectangles, Rhombi and Squares

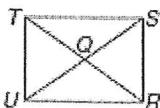
**Properties of Rectangles** A rectangle is a quadrilateral with four right angles. Here are the properties of rectangles.

A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles.
- The diagonals are congruent.



**Example 1** In rectangle  $RSTU$  above,  $US = 6x + 3$  and  $RT = 7x - 2$ . Find  $x$ .

$$\begin{aligned} US &\cong TR \text{ diags of} \\ &6x+3=7x-2 \text{ a rectangle} \\ &x=5 \end{aligned}$$

**Example 2** In rectangle  $RSTU$  above,  $m\angle STR = 8x + 3$  and  $m\angle UTR = 16x - 9$ . Find  $m\angle STR$ .

$$\begin{aligned} \angle STR + \angle UTR &= \angle UTS \\ 8x+3 + 16x-9 &= 90 \\ x=4 \\ \angle STR &= 8(4) + 3 \\ \angle STR &= 35^\circ \end{aligned}$$

$\angle UTS = 90$   
by def of a rectangle

3. If  $AE = 3x + 3$  and  $EC = 5x - 15$ , find  $AC$ .

$$\begin{aligned} AE &= EC \text{ diag. of a rect} \\ 3x+3 &= 5x-15 \text{ bisect each other} \\ x=9 \\ AC &= 3(9)+3+5(9)-15 \\ AC &= 60 \end{aligned}$$

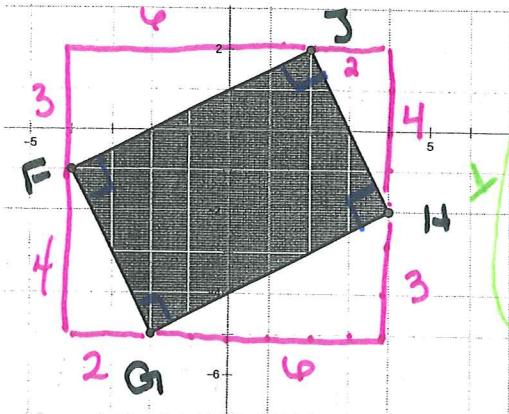
4. If  $DE = 6x - 7$  and  $AE = 4x + 9$ , find  $DB$ .

$$\begin{aligned} AE &= DE \text{ diag. of a rect are } \cong \text{ And bisect each other} \\ 4x+9 &= 6x-7 \\ x=8 \\ DB &= 2(6(8)-7) \\ DB &= 82 \end{aligned}$$

5. Determine whether the figure with vertices  $F(-4, -1)$ ,  $G(-2, -5)$ ,  $H(4, -2)$  and  $J(2, 2)$  is a rectangle.

To be a rectangle, you must test for Consecutive  $\perp$  sides

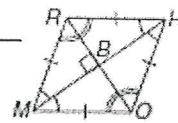
Check slopes



$$\begin{aligned} \text{Slope } JH &= -\frac{4}{2} = -2 > \perp \\ \text{Slope } GH &= \frac{3}{6} = \frac{1}{2} > \perp \\ \text{Slope } FG &= -\frac{4}{2} = -2 > \perp \\ \text{Slope } FJ &= \frac{3}{6} = \frac{1}{2} > \perp \end{aligned}$$

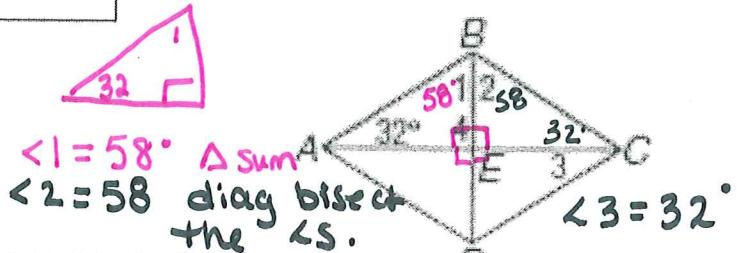
FGHI is a rectangle because con. sides are  $\perp$  which make 4 right  $\angle s$

**Properties of Rhombi** A rhombus is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.



The diagonals are perpendicular.	$MH \perp RO$
Each diagonal bisects a pair of opposite angles.	$MH$ bisects $\angle RMO$ and $\angle RHO$ . $RO$ bisects $\angle MRH$ and $\angle MOH$ .
If the diagonals of a parallelogram are perpendicular, then the figure is a rhombus.	If $RHOM$ is a parallelogram and $RO \perp MH$ , then $RHOM$ is a rhombus.

**EX1.** In rhombus ABCD,  $m\angle BAC = 32^\circ$ . Find the measure of each numbered angle.

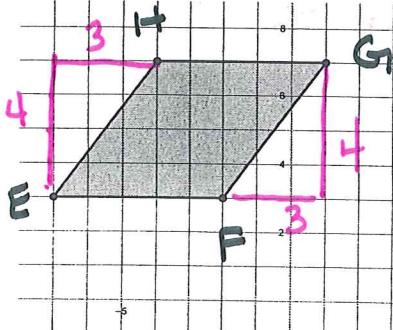


2. Determine whether the figure with vertices  $E(-7,3)$ ,  $F(-2,3)$ ,  $G(1,7)$ ,  $H(-4,7)$  is a rhombus.

To be a rhombus, you must test for

$4 \cong \text{Sides}$

\*Find all distances\*



$$HG = 5$$

$$EF = 5$$

$$EH^2 = 3^2 + 4^2$$

$$\sqrt{EH^2} = \sqrt{25}$$

$$EH = 5$$

$$GF^2 = 3^2 + 4^2$$

$$GF = 5$$

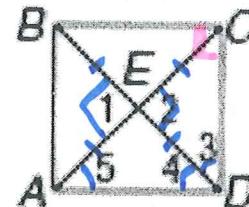
$$HG \cong EF \cong EH \cong GF$$

So all 4 sides are

$\cong \therefore EFGH$  is a Rhombus.

**Properties of Squares** A square has all the properties of a rhombus and all the properties of a rectangle.

1. **Example** Find the measure of each numbered angle of square ABCD.



$$\angle 1 = 90^\circ$$

$$\angle 2 = 90^\circ$$

2. Determine whether the figure with vertices  $E(-2,-1)$ ,  $F(-1,3)$ ,  $G(3,2)$ , and  $H(2,-2)$  is a square.

$$\angle 5 = 45^\circ$$

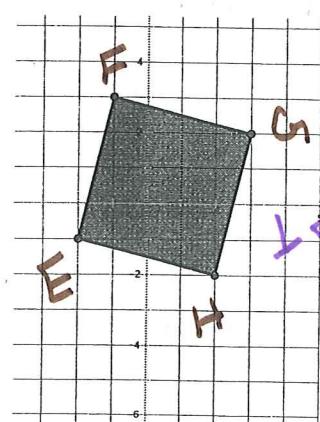
$$\angle 4 = 45^\circ$$

$$\angle 3 = 45^\circ$$

To be a square, you must test for

$4 \rightleftharpoons \text{right } \angle s$

$4 \cong \text{Sides}$



slopes:

$$\text{Slope } GH = \frac{4}{1} = 4 > 1$$

$$\text{Slope } EH = -\frac{1}{4} > 1$$

$$\text{Slope } FE = 4 > 1$$

$$\text{Slope } FG = -\frac{1}{4} > 1$$

distances:

$$FG = \sqrt{17}$$

$$EH = \sqrt{17}$$

$$GH = \sqrt{17}$$

$$FE = \sqrt{17}$$

$$\} \cong$$

ALL 4  $\angle s$  are Right  $\angle s$  because consecutive sides are  $\perp$  and  $FG \cong EH \cong GH \cong FE$  so ALL 4 sides are  $\cong$ . It is a square.