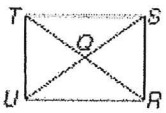
Key

Notes: Rectangles, Rhombi and Squares

Properties of Rectangles A rectangle is a quadrilateral with four right angles. Here are the properties of rectangles.



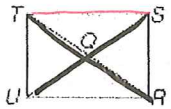
A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles. — $\angle UTS$, $\angle TSR$, $\angle SRU$, and $\angle RUT$ are right angles.
- The diagonals are congruent. — $\overline{TR} \cong \overline{US}$

Example 1 In rectangle $RSTU$ above, $US = 6x + 3$ and $RT = 7x - 2$. Find x.



$US = RT$ rectangle diagonals are $\cong$

$6x+3 = 7x-2$

$3 = x-2$

$\boxed{5 = x}$

Example 2 In rectangle $RSTU$ above, $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$. Find $m\angle STR$.

a rectangle has 4 right $\angle$s

$\angle STU = 90$

$\angle STU = \angle STR + \angle UTR$ angle addition

$90 = 8x+3+16x-9$

$90 = 24x - 6$

$96 = 24x$

$\boxed{4 = x}$

$\angle STR = 8(4) + 3$

$\boxed{\angle STR = 35°}$

3. If $AE = 3x + 3$ and $EC = 5x - 15$, find AC.

$AE = EC$ diagonals of a rectangle bisect each other

$3x+3 = 5x-15$

$18 = 2x$

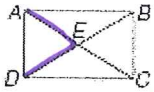
$9 = x$

$AC = AE + EC$ Segment addition

$AC = 3(9)+3+5(9)-15$

$\boxed{AC = 60}$

4. If $DE = 6x - 7$ and $AE = 4x + 9$, find DB.



$DB = AC$ diagonals of a rectangle are $\cong$

$DE + EB = DB$, $AE + EC = AC$ Seg add.

$DE + DE = AE + AE$

$6x - 7 + 6x - 7 = 4x + 9 + 4x + 9$

$12x - 14 = 8x + 18$

$4x = 32$

$\boxed{x = 8}$

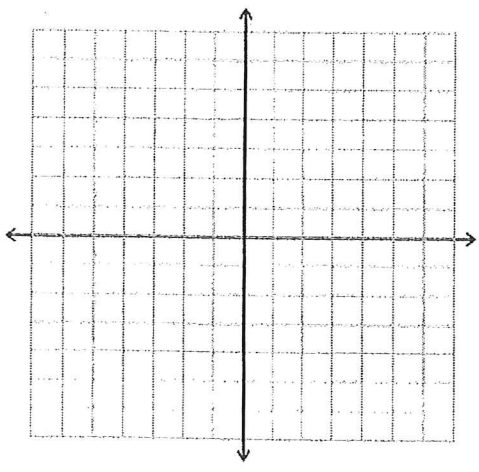
$DB = DE + DE$

$DB = 6(8) - 7 + 6(8) - 7$

$\boxed{DB = 82}$

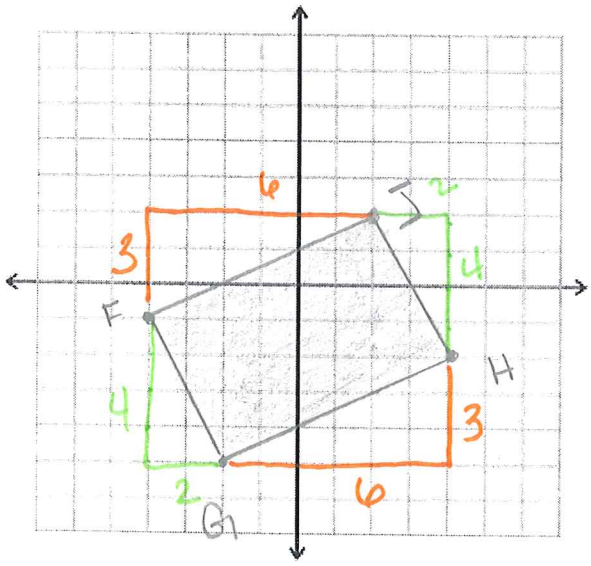
5. Determine whether the figure with vertices F(-4,-1), G(-2,-5), H(4,-2) and J(2,2) is a rectangle.

To be a rectangle, you must test for _____



2. Determine whether the figure with vertices F(-4,-1),G(-2,-5), H(4,-2) and J(2,2) is a rectangle.

To be a rectangle, you must test for All 4 Right Angles



Check to see if Consecutive Sides are ⊥

Slopes:

$JH = -\frac{4}{2} = -2$

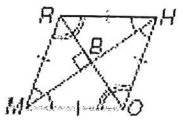
$HG = \frac{3}{6} = \frac{1}{2}$

$FG = -\frac{4}{2} = -2$

$FJ = \frac{3}{6} = \frac{1}{2}$

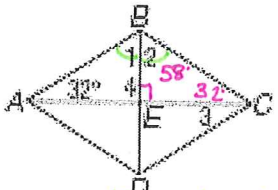
Yes, consecutive sides are ⊥ ∴ FGHJ has all 4 right angles.

Properties of Rhombi A rhombus is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.



The diagonals are perpendicular.	$\overline{MH} \perp \overline{RO}$
Each diagonal bisects a pair of opposite angles.	$\overline{MH}$ bisects $\angle RMO$ and $\angle RHO$. $\overline{RO}$ bisects $\angle MRH$ and $\angle MOH$.
If the diagonals of a parallelogram are perpendicular, then the figure is a rhombus.	If $RHOM$ is a parallelogram and $\overline{RO} \perp \overline{MH}$, then $RHOM$ is a rhombus.

Example In rhombus $ABCD$, $m\angle BAC = 32$. Find the measure of each numbered angle.



$\angle 4 = 90°$ diagonals of a rhombus are $\perp$

$\angle 1 + 90 + 32 = 180$ $\triangle$ sum theorem

$\angle 1 = 58°$

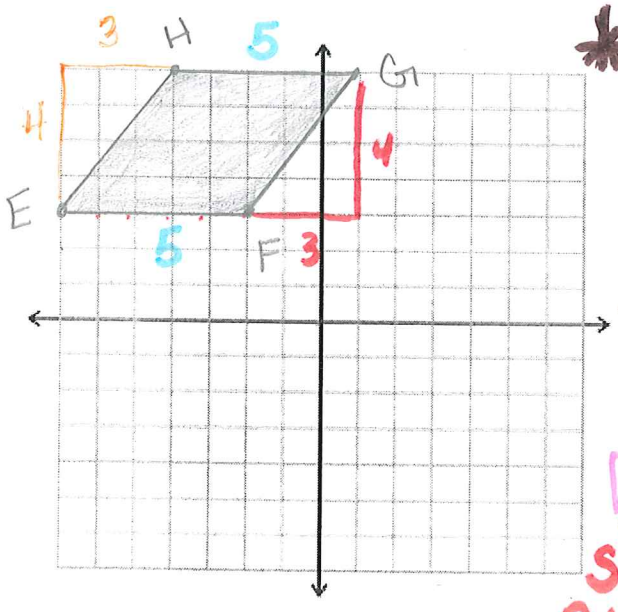
$\angle 1 \cong \angle 2$ each diagonal of a rhombus bisect the angle

$\angle 2 = 58°$

$\angle 3 = 32°$ ↑ same justification

2. Determine whether the figure with vertices E(-7,3), F(-2,3), G(1,7) H(-4,7) is a rhombus.

To be a rhombus, you must test for 4 ≅ sides



* Check distances *

$HG = 5$

$EF = 5$

$EH = 4^2 + 3^2$

$= 16 + 9$

$= \sqrt{25}$

$EH = 5$

$GF = 4^2 + 3^2$

$= 16 + 9$

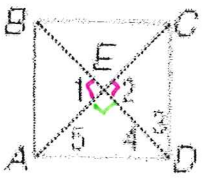
$= \sqrt{25}$

$GF = 5$

Since ALL four sides = 5 units, ALL four sides are ≅ ∴ Rhombus

Properties of Squares A square has all the properties of a rhombus and all the properties of a rectangle.

Example Find the measure of each numbered angle of square *ABCD*.



$\angle 1 = 90°$, $\angle 2 = 90°$ > diagonals of a square are ⊥

$\angle 4 \cong \angle 3$ diagonals of a square bisect the angles

$\angle 4 + \angle 3 = \angle ADC$ angle addition

$\angle ADC = 90$ a square has 4 Right ∠s

$\angle 4 + \angle 3 = 90$

$\angle 4 + \angle 4 = 90$

$2\angle 4 = 90$

$\angle 4 = 45°$

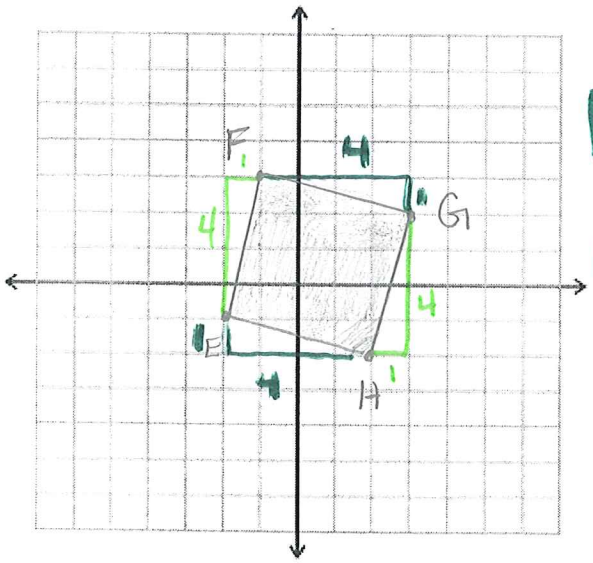
$\angle 3 = 45°$

$\angle 5 + 45 + 90 = 180$ Δ Sum Theorem

$\angle 5 = 45°$

4. Determine whether the figure with vertices E(-2,-1), F(-1,3), G(3,2), and H(2,-2) is a square.

To be a square, you must test for 4 ≅ sides + 4 right angles



Check Distance:

$FG = 4^2 + 1^2$

$FG = \sqrt{17}$

$GH = 1^2 + 4^2$

$GH = \sqrt{17}$

$EH = 4^2 + 1^2$

$EH = \sqrt{17}$

$FE = 1^2 + 4^2$

$FE = \sqrt{17}$

∴ ALL four sides are ≅

Check slopes to see if ⊥

$GH = \frac{4}{1} = 4$, $FE = \frac{4}{1} = 4$

$EH = -\frac{1}{4}$, $FG = -\frac{1}{4}$

∴ Four right angle

Since all 4 sides ≅ and all consecutive sides are ⊥, this means Quadrilateral EFGH is a SQUARE