

GEOMETRY: SIMILARITY BOOKLET

Name: _____

Ratios

A *ratio* is the comparison of two numbers written as a fraction.

For example, 1: Your school's basketball team has won 7 games and lost 3 games. What is the *ratio* of wins to losses?

Example 2: Using Ratios

The ratio of the side lengths of a triangle is 4:7:5, and its perimeter is 96 cm. What is the length of the shortest side?



Example 3: Using Ratios

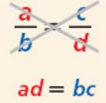
The ratio of the angles of a triangle is 4:7:5. What is the measure of the largest angle?

Proportion – an equation that equates two ratios

$$\frac{a}{b} = \frac{c}{d}$$

Cross Products Property

In a proportion, if $\frac{a}{b} = \frac{c}{d}$ and b and $d \neq 0$, then $ad = bc$.


$$ad = bc$$

EXAMPLE 1 Using Cross Products

$$\frac{10.4}{22.1} = \frac{41.6}{n}$$

EXAMPLE 2 Using Cross Products

$$\frac{t+3}{8} = \frac{t}{3}$$

EXAMPLE 3 Using Cross Products

$$\frac{2y}{9} = \frac{8}{4y}$$

EXAMPLE 4 Using Cross Products

$$\frac{x+3}{4} = \frac{9}{x+3}$$

Solve each question.

1) $\frac{4}{8} = \frac{x}{10}$

2) $\frac{4}{3} = \frac{2}{b-9}$

3) $\frac{8}{7} = \frac{v+8}{v}$

4) $\frac{3}{10} = \frac{x-10}{7x+1}$

5) $\frac{z-4}{5} = \frac{20}{z-4}$

6) In a triangle, the ratio of the measures of three sides is 5:12:13, with a perimeter of 90 cm. Find the length of the shortest side of the triangle.

7) In a triangle, the ratio of the measures of three angles is 5:7:8, find the measure of the smallest angle.

8) Out of a survey of 1000 households. 460 had at least one dog or cat as a pet. What is the ratio of pet owners to households?

9) In 2002, the Chicago Cubs baseball team never tied a game and won 67 games out of 162 played. What is the ratio of games won to games lost?

NAME _____ DATE _____ PERIOD _____

7-1 Practice

Proportions

- 1. NUTRITION** One ounce of cheddar cheese contains 9 grams of fat. Six of the grams of fat are saturated fats. Find the ratio of saturated fats to total fat in an ounce of cheese.
- 2. FARMING** The ratio of goats to sheep at a university research farm is 4:7. The number of sheep at the farm is 28. What is the number of goats?
- 3. ART** Edward Hopper's oil on canvas painting *Nighthawks* has a length of 60 inches and a width of 30 inches. A print of the original has a length of 2.5 inches. What is the width of the print?

Solve each proportion.

4. $\frac{5}{8} = \frac{x}{12}$

5. $\frac{x}{1.12} = \frac{1}{5}$

6. $\frac{6x}{27} = \frac{4}{3}$

7. $\frac{x+2}{3} = \frac{8}{9}$

8. $\frac{3x-5}{4} = \frac{-5}{7}$

9. $\frac{x-2}{4} = \frac{x+4}{2}$

Find the measures of the sides of each triangle.

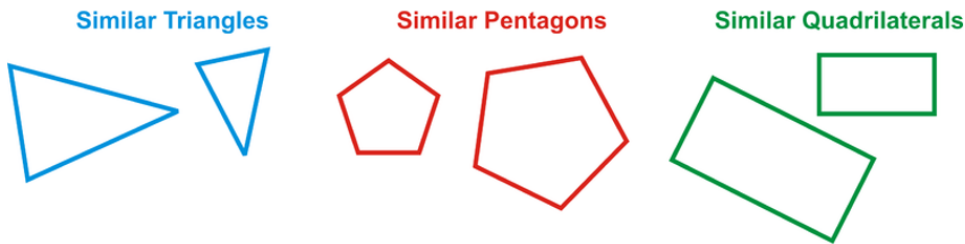
- 10.** The ratio of the measures of the sides of a triangle is 3:4:6, and its perimeter is 104 feet.
- 11.** The ratio of the measures of the sides of a triangle is 7:9:12, and its perimeter is 84 inches.
- 12.** The ratio of the measures of the sides of a triangle is 6:7:9, and its perimeter is 77 centimeters.

Find the measures of the angles in each triangle.

- 13.** The ratio of the measures of the angles is 4:5:6.
- 14.** The ratio of the measures of the angles is 5:7:8.
- 15. BRIDGES** The span of the Benjamin Franklin suspension bridge in Philadelphia, Pennsylvania, is 1750 feet. A model of the bridge has a span of 42 inches. What is the ratio of the span of the model to the span of the actual Benjamin Franklin Bridge?

Similar Polygons: Definition and Notation

Similar Polygons: Polygons that are exactly the same shape but not necessarily the same size.



There is more to mathematically showing similar figures than just looking at them, but for an intro demonstration, these are examples to show similar shape but not exact size.

Notation: The notation for similar is like a congruence symbol but without the =.

$\triangle ABC \sim \triangle XYZ$ reads "triangle ABC is similar to triangle XYZ".

Similar Polygons: Side Length Ratio and Scale Factor

Side Length Ratio and Scale Factor:

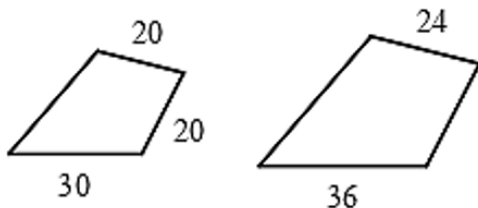
When you compare the lengths of the corresponding sides of similar figures, you get a numerical ratio.

This ratio is called the scale factor or Side Length Ratio (SLR) for the two figures.

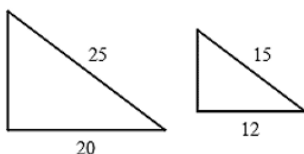
Vocab: SLR's being equal means scale factors are the same.

Similar Polygons: Side Length Ratio and Scale Factor

Ex1: Find the Side Length Ratio (SLR) of the figure on the left to the right.



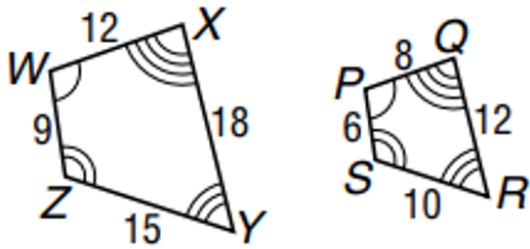
Ex2: Find the Scale Factor (SF) of the figure on the left to the right.



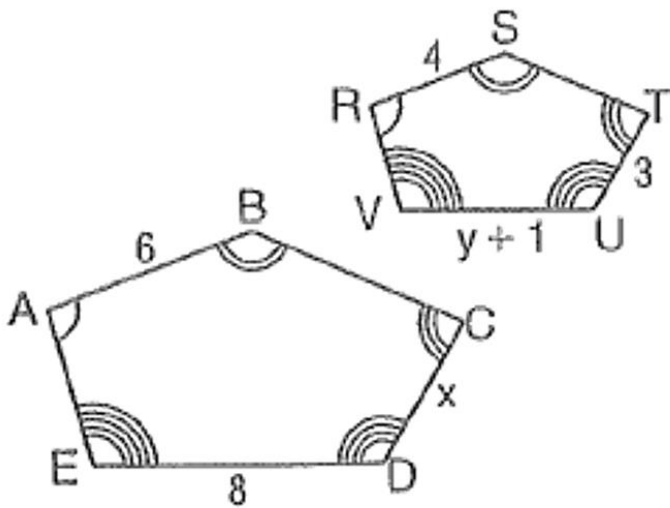
How to determine if polygons are similar

Example 3: Determining if figures are similar:

- 1) All corresponding angles are congruent
- 2) You may here "Sides are proportional" It means: All side length ratios are equal (Same SF)



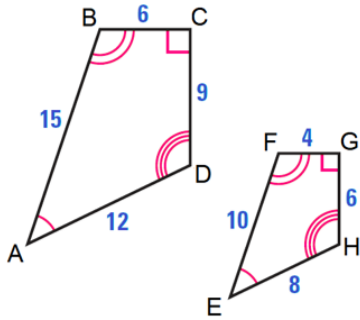
Example 4: Each pair of polygons is similar. Write a similarity statement, SLR (Scale Factor) and find x , the measure of the indicated side: UV.



Similar Figures: Independent Practice

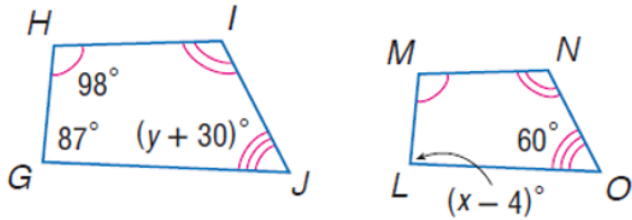
Recall: Determining if figures are similar:

- 1) All corresponding angles are congruent
- 2) You may here "Sides are proportional" It means: All side length ratios are equal (Same SF)



7.2 Ratios, Proportions, and Similar Figures Warm-up

1. Given the two polygons are similar, find x and y .



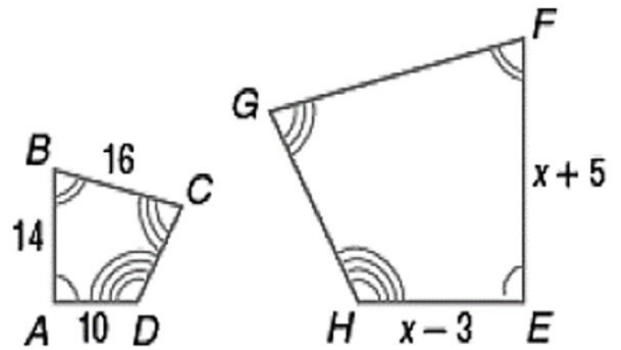
2. The two quadrilaterals are similar.

A. Write the similarity statement.

B. Find the scale factor.

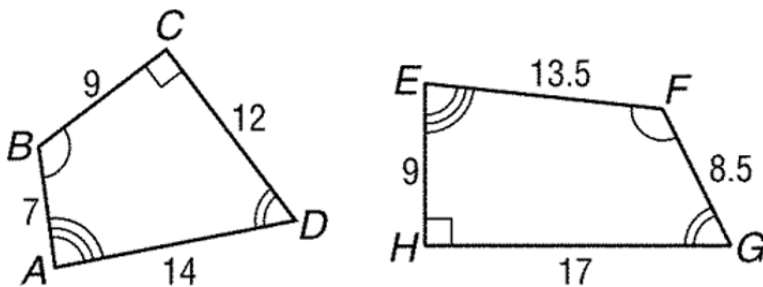
C. Find x .

D. Find the measure of the indicated side.



A. Quad $ABCD \sim$ _____ B. _____ C. _____ D. $GF =$ _____

3. Determine whether quadrilateral $ABCD \sim$ quadrilateral $EFGH$. Justify your answer.

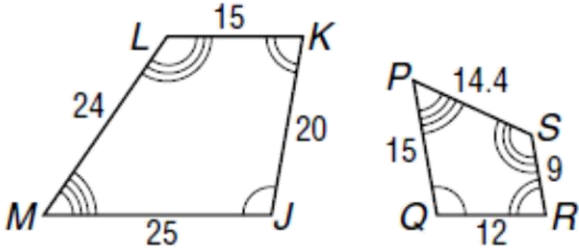


Similar Polygons Focus Intervention

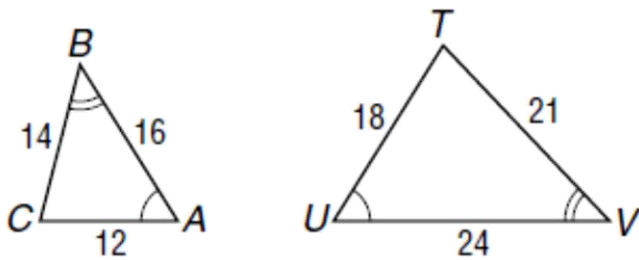
1. What two concepts must be true in order for polygons to be similar?

a.) _____ b.) _____

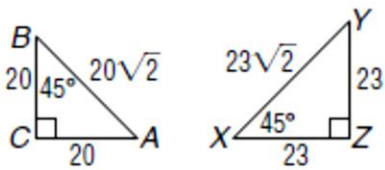
2. Determine if the figures are similar. Justify your reasoning as modeled in class.



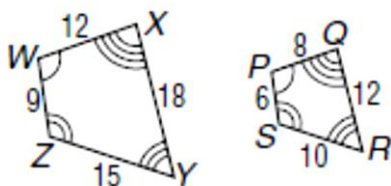
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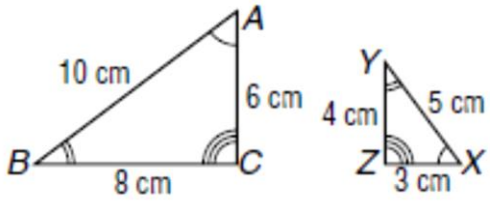
4. Determine if the figures are similar. Justify your reasoning as modeled in class.



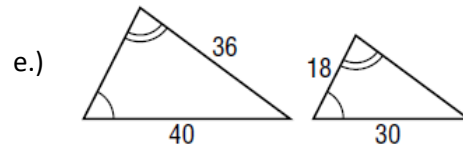
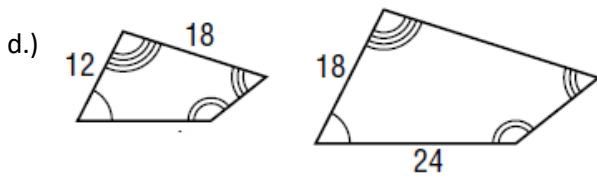
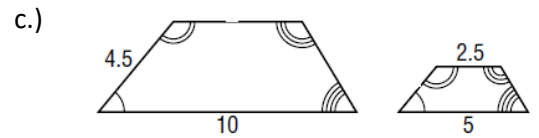
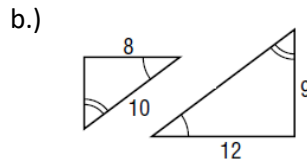
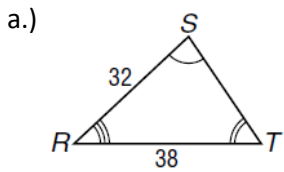
5. Determine if the figures are similar. Justify your reasoning as modeled in class.



6. Determine if the figures are similar. Justify your reasoning as modeled in class.



7. Find the scale factor of the following similar figures.



8. The two polygons are similar.

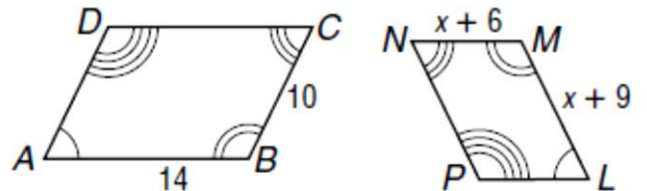
A. Write a similarity statement

B. Find the scale factor.

C. Find x .

D. Find MN and ML .

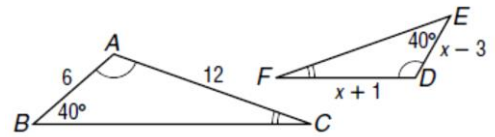
\overline{LM} and \overline{MN}



9. The two polygons are similar.

- A. Write a similarity statement
- B. Find the scale factor.
- C. Find x .
- D. Find DE and DF .

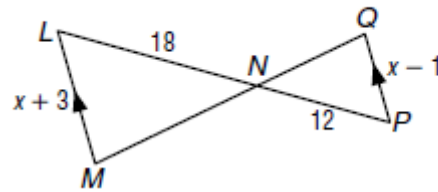
\overline{DE} and \overline{DF}



10. The two polygons are similar.

- A. Write a similarity statement
- B. Find the scale factor.
- C. Find x .
- D. Find LM and QP .

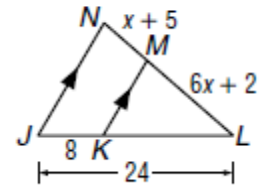
\overline{LM} and \overline{QP}



11. The two polygons are similar.

- A. Write a similarity statement
- B. Find the scale factor.
- C. Find x .
- D. Find NL and ML .

\overline{NL} and \overline{ML}



7.3 SIMILAR TRIANGLES

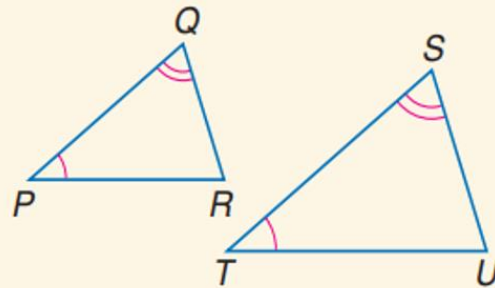
Notes

POSTULATE 7.1

Angle-Angle (AA) Similarity

If the two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Example: $\angle P \cong \angle T$ and $\angle Q \cong \angle S$,
so $\triangle PQR \sim \triangle TSU$.



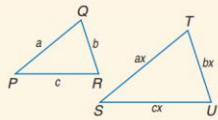
○ **AA Similarity:** If 2 PAIRS of corresponding angles are congruent, then the triangles are similar.

Other Shortcuts that we don't always use:

○ **SSS Similarity:** If all three pairs of corresponding SLR's are equal, then the triangles are similar.

7.1 Side-Side-Side (SSS) Similarity If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

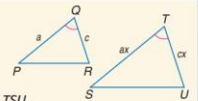
Example: $\frac{PQ}{ST} = \frac{QR}{SU} = \frac{RP}{UT}$, so $\triangle PQR \sim \triangle TSU$.



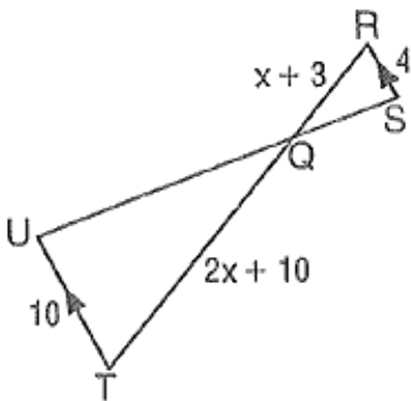
○ **SAS Similarity:** If 2 pairs of corresponding SLR's are equal and their included angles are congruent, then the triangles are similar.

7.2 Side-Angle-Side (SAS) Similarity If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

Example: $\frac{PQ}{ST} = \frac{QR}{SU}$ and $\angle Q \cong \angle S$, so $\triangle PQR \sim \triangle TSU$.



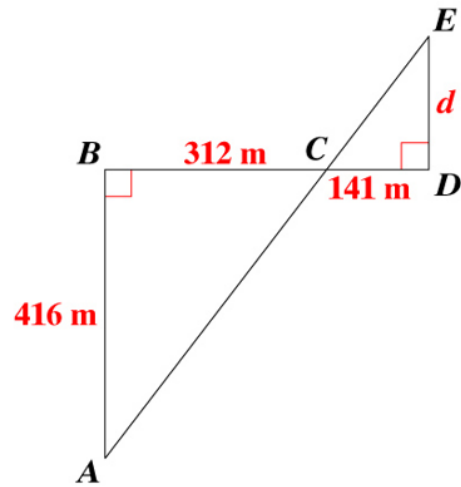
Example 1: Determine if the triangles are similar. If so, write a similarity statement and explain why the two triangles are similar, and find x , the measure(s) of the indicated side(s) RQ and TQ, and the scale factor (SLR).



Similarity and Indirect Measurement

2 EXAMPLE

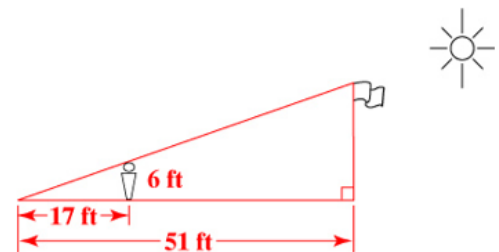
In the figure below, $\triangle ABC \sim \triangle EDC$. Find d .



Similarity and Indirect Measurement

Example 3. When a 6-ft student casts a 17-ft shadow, a flagpole casts a shadow that is 51 ft long. Find the height of the flagpole.

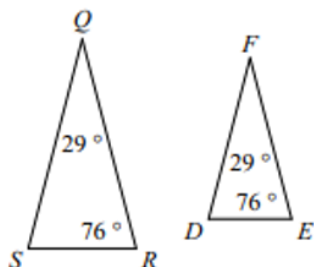
Set up a proportion for the similar triangles.



7.3 Warm Up

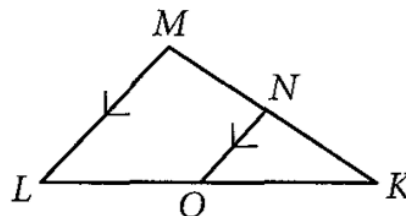
State if the triangles in each pair are similar. If so, state how you know they are similar and complete the similarity statement.

1.



$\triangle QRS \sim$ _____

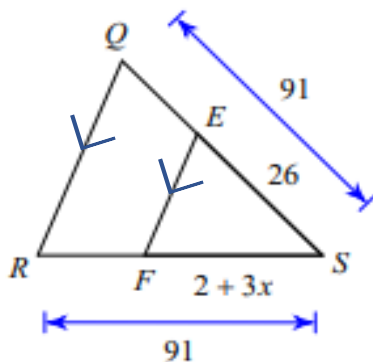
2.



$\triangle QSR \sim \triangle$ _____

Identify the Similar triangles, how you know they are similar, find x .

3.

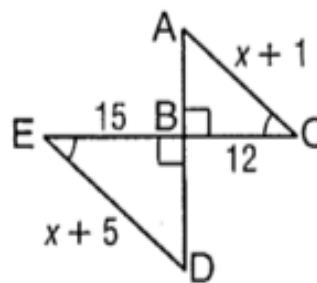


$\triangle QSR \sim \triangle$ _____

Bc _____

$x =$ _____

4.



$\triangle ABC \sim \triangle$ _____

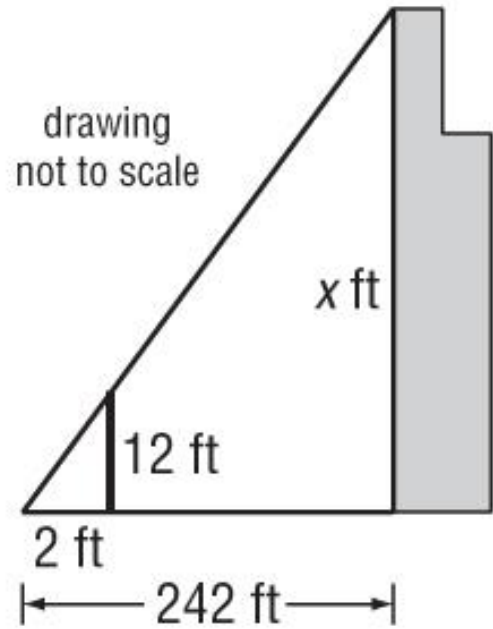
BC _____

$x =$ _____

Indirect Measurement

Example 5. A flagpole that is 11 feet tall casts a 5 and a half foot shadow. At the same time of day, a nearby building casts a 10 ft, 7 in shadow. How tall is the building?

Example 6. Josh wanted to measure the height of the Sears Tower in Chicago. He used a 12-foot light pole and measured its shadow at 1 p.m. The length of the shadow was 2 feet. Then he measured the length of the Sears Tower's shadow and it was 242 feet at the same time. What is the height of the Sears Tower?



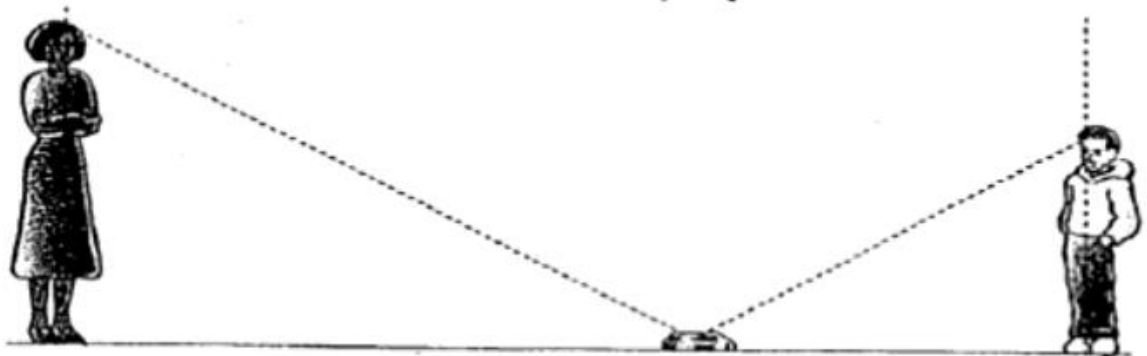
Example 7.

The principal asked Hank to demonstrate what he was learning in math class. Hank decided to use the mirror method to estimate the principal's height. Here are the measurements Hank recorded. Use them to find the principal's height.

Height from the ground to Hank's eyes = 1.5 m

Distance from the center of the mirror to Hank = 3 m

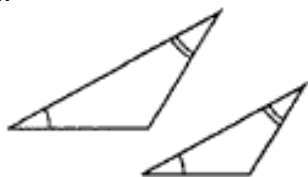
Distance from the center of the mirror to the principal = 3.7 m



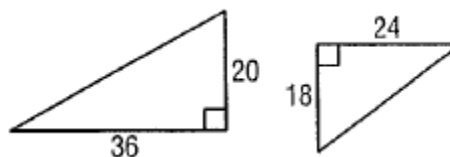
7.3 TRIANGLE SIMILARITY HOMEWORK

Directions: Determine whether the pairs of triangles are similar by AA Similarity.

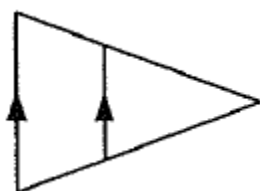
1.



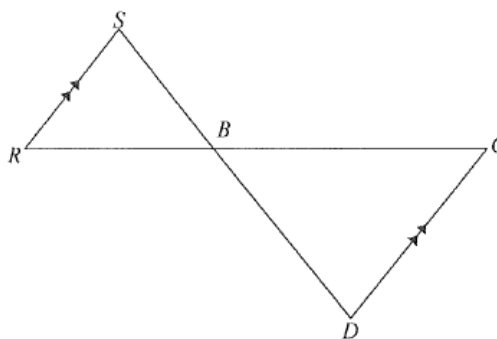
2.



3.



4.



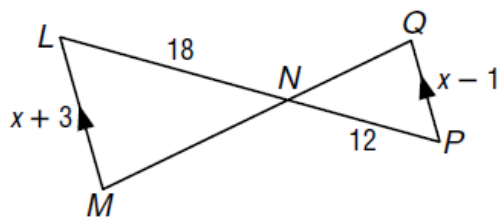
Directions: The two triangles given are similar. Complete the following for each problem:

A. Write the similarity statement.

B. Find x .

C. Find the measure of the indicated side

5. \overline{LM} and \overline{QP}

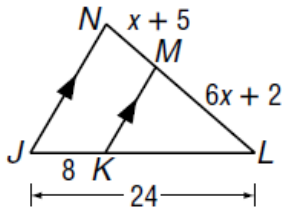


a) $\triangle LNM$ _____

b) $x =$ _____

c) $LM =$ _____, $QP =$ _____

6. \overline{NL} and \overline{ML}

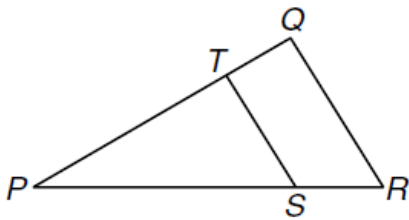


a) $\triangle LMK$ _____

b) $x =$ _____

c) $NL =$ _____, $ML =$ _____

7. If $\overline{TS} \parallel \overline{QR}$, $TS = 6$, $PS = x + 7$, $QR = 8$, and $SR = x - 1$, find PS and PR .

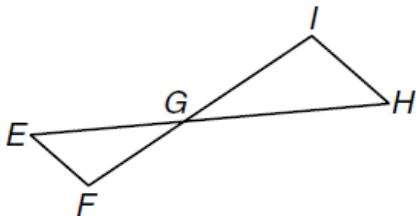


a) $\triangle PQR$ _____

b) $x =$ _____

c) $PS =$ _____, $PR =$ _____

8. If $\overline{EF} \parallel \overline{HI}$, $EF = 3$, $EG = x + 1$, $HI = 4$, and $HG = x + 3$, find EG and HG .



a) $\triangle EFG$ _____

b) $x =$ _____

c) $EG =$ _____, $HG =$ _____

Indirect Measurement Assignment

Directions: For each real world question, draw and label the figure, set up your proportions, and leave your answer in the indicated units. YOU MUST SHOW ALL WORK!

1. At a specific time of day, a flag pole has a shadow 40 ft long, at the same time of day Jose who is 6 ft tall has a shadow of 8ft long. Find the height of the flagpole to the nearest tenth of a foot if needed.

The height of the flagpole is: _____ ft

2. Sam places a mirror on the ground between herself and an apartment building and stands so that she can see the top of a window on the 10th floor. The mirror is 1.34 meters from her feet and 7.68 meters from the base of the building. Samantha's eye is 1.43 meters above the ground. How high is the window to the nearest meter?

The window is _____ meters high.

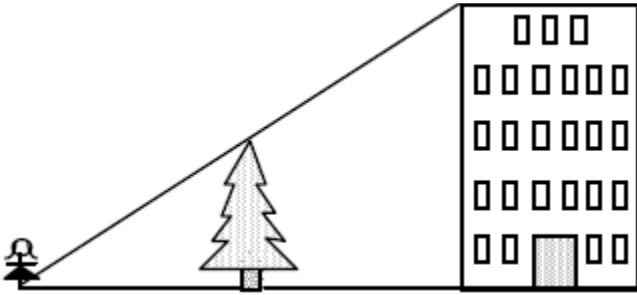
3. A flagpole that is 15 feet tall casts a 7 and a half foot shadow. At the same time of day, a nearby building casts a 18 feet, 4 in. shadow. How tall is the building? Show your answer in inches then convert to the feet.

The building is _____ total inches.

My conversion is:

_____ feet, _____ inches.

4. Anna wants to find the height of the tallest building in her city. She stands 384 feet away from the building. There is a tree 31 feet in front of her that is 20 feet tall. How tall is the building to the nearest foot?



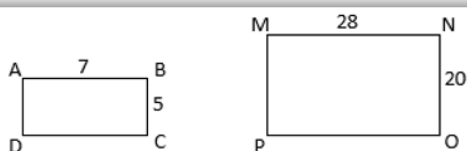
The building is _____ feet.

5. Nicole wants to find out the height of her favorite pine tree so that she can fit it in her house for Christmas. She stands within the tree's shadow and walks until her shadow meets the tree's shadow. Nicole is 152.4cm tall. Her feet are 91.44cm from the base of the tree. She also knows that the tree has a shadow of 274.32cm long at this time of day. Help Nicole find the height of her favorite tree. If she can fit a 250cm tree in her living room, will this tree fit for the holidays? Please leave final answer in cm.

Height of tree: _____

Can she use the tree for Christmas? _____

Perimeter Examples



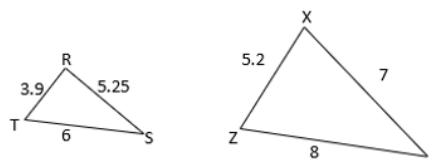
1. Find the scale factor of ABCD to MNOP.

2. Find the perimeter of each rectangle.

ABCD=_____ MNOP=_____

3. What is the ratio of the perimeter ABCD to MNOP?

4. What do you notice about the ratios?



5. Find the scale factor of ΔRST to ΔXYZ .

6. Find the perimeter of each triangle.

ΔRST =_____ ΔXYZ =_____

7. What is the ratio of the perimeter ΔRST to ΔXYZ ?

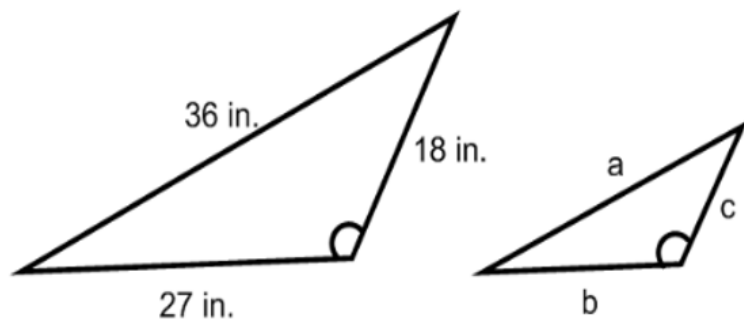
8. What do you notice about the ratios?

- Perimeter Ratio is equal to the Side Length Ratio

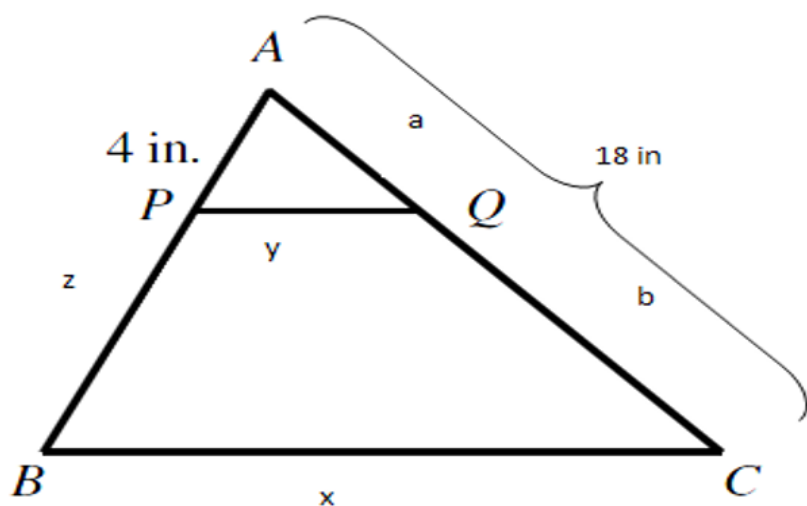
- Equivalent Ratios:

- SLR (Side Length Ratio)
- PR (Perimeter Ratio)
- SF (Scale Factor)

Example 1: The two triangles below are similar. Find a , b , and c if the perimeter of the small triangle is 21 in. Find the perimeter ratio of the larger triangle to the smaller triangle first. Round to the nearest tenth if needed.



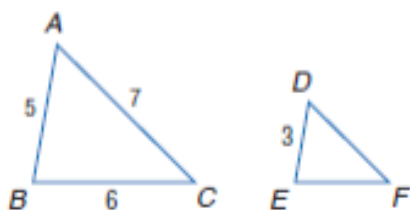
Example 2: The perimeter of $\triangle ABC = 54$ inches and the perimeter of $\triangle APQ = 18$ inches. $\triangle ABC \sim \triangle APQ$. Find a , b , x , y , and z .



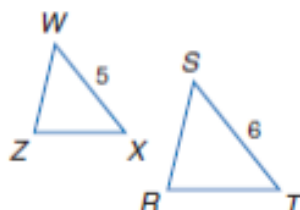
Geo 7.5 Book Solutions

Pg. 419 #1,2, 8-10, 12, 13

1. $\triangle DEF$, if $\triangle ABC \sim \triangle DEF$,
 $AB = 5$, $BC = 6$, $AC = 7$,
 and $DE = 3$

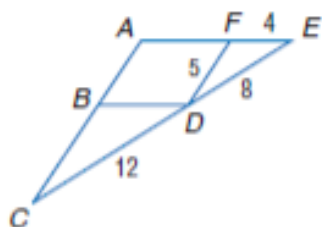


2. $\triangle WZX$, if $\triangle WZX \sim \triangle SRT$, $ST = 6$,
 $WX = 5$, and the perimeter of
 $\triangle SRT = 15$

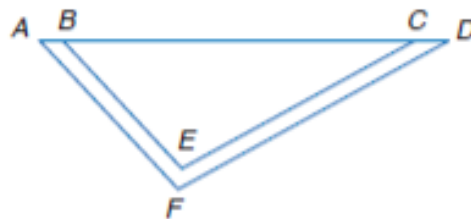


Find the perimeter of the given triangle.

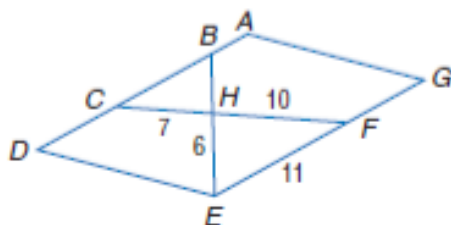
8. $\triangle BCD$, if $\triangle BCD \sim \triangle FDE$,
 $CD = 12$, $FD = 5$, $FE = 4$,
 and $DE = 8$



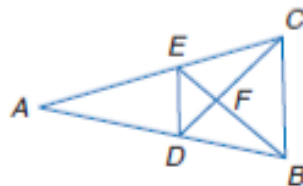
9. $\triangle ADF$, if $\triangle ADF \sim \triangle BCE$,
 $BC = 24$, $EB = 12$, $CE = 18$,
 and $DF = 21$



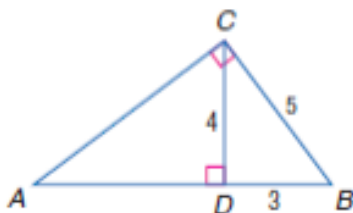
10. $\triangle CBH$, if $\triangle CBH \sim \triangle FEH$,
 $ADEG$ is a parallelogram,
 $CH = 7$, $FH = 10$, $FE = 11$,
 and $EH = 6$



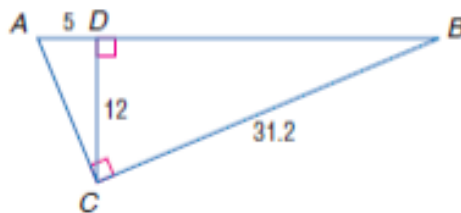
11. $\triangle DEF$, if $\triangle DEF \sim \triangle CBF$,
 perimeter of $\triangle CBF = 27$,
 $DF = 6$, $FC = 8$



12. $\triangle ABC$, if $\triangle ABC \sim \triangle CBD$,
 $CD = 4$, $DB = 3$, and $CB = 5$



13. $\triangle ABC$, if $\triangle ABC \sim \triangle CBD$,
 $AD = 5$, $CD = 12$, $BC = 31.2$



9.5 DILATIONS

WHAT IS IT??

A **dilation** is any **re-sizing (Enlargement or Reduction)**

The **scale factor** controls how large or **small** the figure will become. We dilate according to the **scale factor**.

More specifically, we **multiply** EVERY coordinate by the scale factor.

Dilations

- Will have a center point where it will enlarge or reduce from.
- Enlargements will be multiplied by a scale factor that is greater than one.

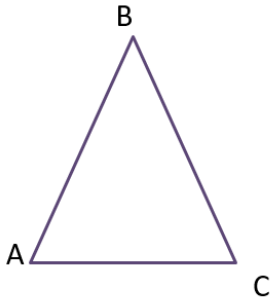


- Reductions will be figures multiplied by scale factors between 0 and 1.



- Dilations with a scale factor of one will be congruent to the pre-image.



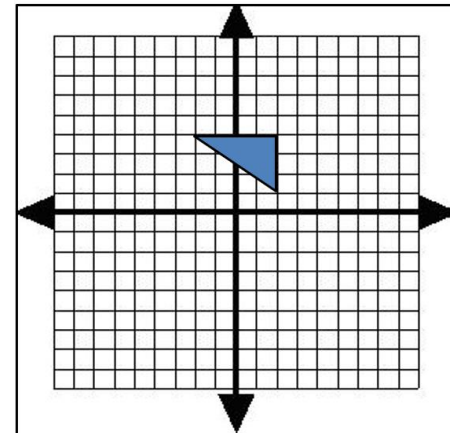


D

On a coordinate plane, we multiply EVERY coordinate of the pre-image by the scale factor to produce the image.

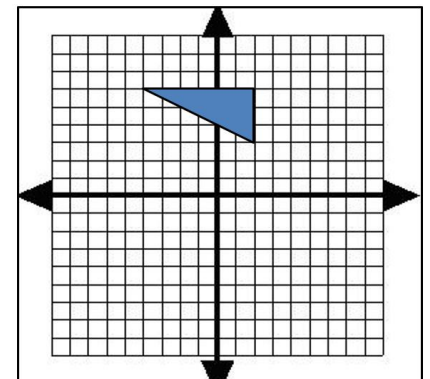
Directions: Plot the original points as indicated. Connect the points to make a right triangle. Then, perform the given dilation.

Original Coordinates:	A (-2, 4)	B (2, 4)	C (2, 1)
Dilate by Scale Factor of 2	A (,)	B (,)	C (,)



Directions: Plot the original points as indicated. Connect the points to make a right triangle. Then, perform the given dilation.

Original Coordinates:	A (-4, 6)	B (2, 6)	C (2, 3)
Dilate by Scale Factor of 1/2	A (,)	B (,)	C (,)



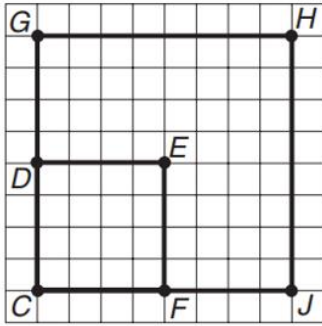
Soooo, when a figure is dilated by a scale factor **GREATER** than one, the image gets **BIGGER**.

However, when a figure is dilated by a scale factor **LESS** than one (fraction), the image gets **smaller**.

Let's Practice!

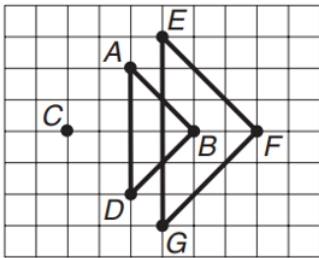
Determine the scale factor for each dilation with center C . Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.

$CGHJ$ is a dilation image of $CDEF$.



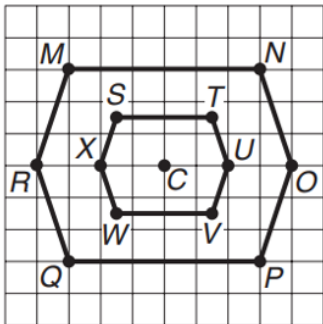
Show what you learned:

$\triangle EFG$ is a dilation image of $\triangle ABC$.



1. Is this an enlargement, reduction or congruence dilation?
2. What is the scale factor?

$STUVWX$ is a dilation image of $MNOPQR$.



3. Is this an enlargement, reduction or congruence dilation?
4. What is the scale factor?

5. What is the image of $P(-3,4)$ if it is dilated by a scale factor of 3 centered at the origin?

6. What is the image of $P(-6,-12)$ if it is dilated by a scale factor of $1/3$ centered at the origin?

9-5 Skills Practice

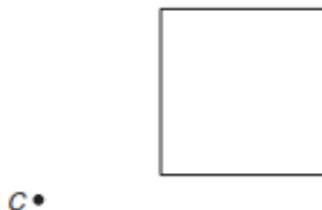
Dilations

Draw the dilation image of each figure with center C and the given scale factor.

1. $r = 2$



2. $r = \frac{1}{4}$



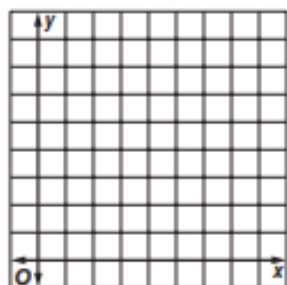
Find the measure of the dilation image $\overline{M'N'}$ or of the preimage \overline{MN} using the given scale factor.

3. $MN = 3, r = 3$

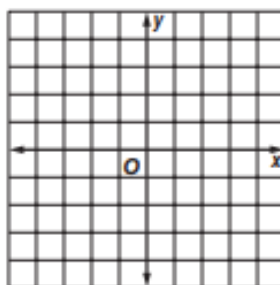
4. $M'N' = 7, r = 21$

COORDINATE GEOMETRY Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of 2. Then graph a dilation centered at the origin with a scale factor of $\frac{1}{2}$.

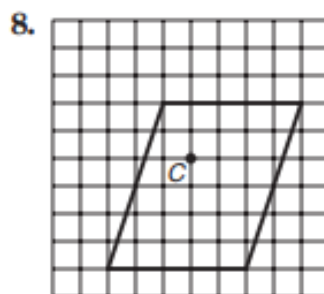
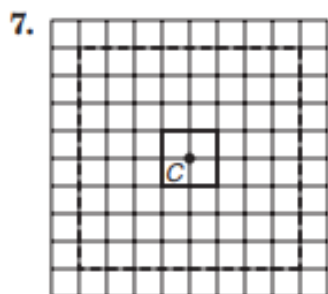
5. $J(2, 4), K(4, 4), P(3, 2)$



6. $D(-2, 0), G(0, 2), F(2, -2)$



Determine the scale factor for each dilation with center C . Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*. The dashed figure is the dilation image.



9-5 Practice

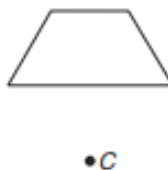
Dilations

Draw the dilation image of each figure with center C and the given scale factor.

1. $r = \frac{3}{2}$



2. $r = \frac{2}{3}$



Find the measure of the dilation image $\overline{A'T'}$ or of the preimage \overline{AT} using the given scale factor.

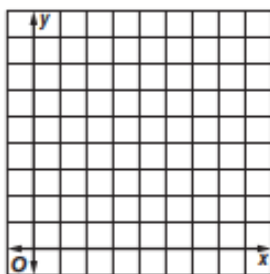
3. $AT = 15, r = \frac{3}{5}$

4. $AT = 30, r = -\frac{1}{6}$

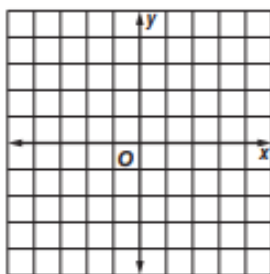
5. $A'T' = 12, r = \frac{4}{3}$

COORDINATE GEOMETRY Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of 2. Then graph a dilation centered at the origin with a scale factor of $\frac{1}{2}$.

6. $A(1, 1), C(2, 3), D(4, 2), E(3, 1)$

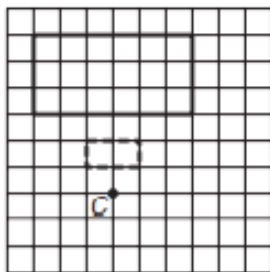


7. $Q(-1, -1), R(0, 2), S(2, 1)$

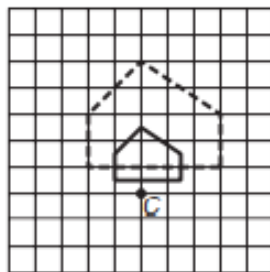


Determine the scale factor for each dilation with center C . Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*. The dotted figure is the dilation image.

8.



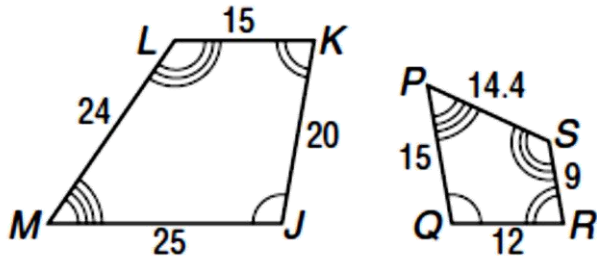
9.



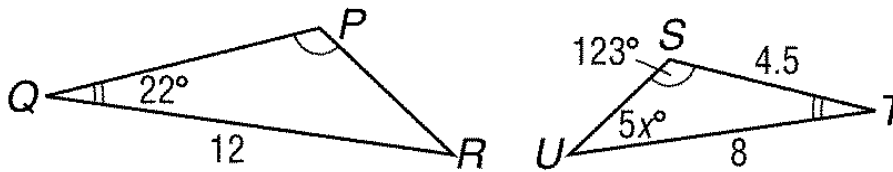
10. **PHOTOGRAPHY** Estebe enlarged a 4-inch by 6-inch photograph by a factor of $\frac{5}{2}$. What are the new dimensions of the photograph?

Similarity Review 2020/2021 Remote/Hybrid Learning

1. Determine if the figures below are similar. Explain why or why not.

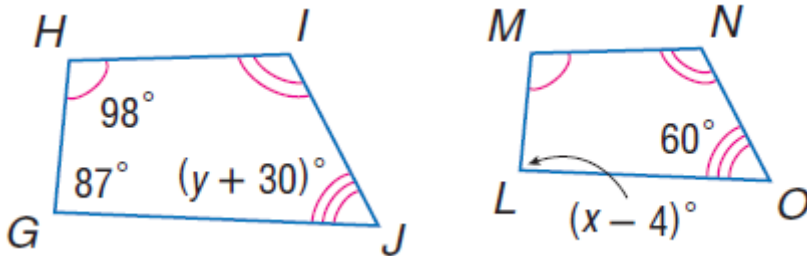


2. Given $\Delta STU \sim \Delta PQR$, find x .



$x =$ _____

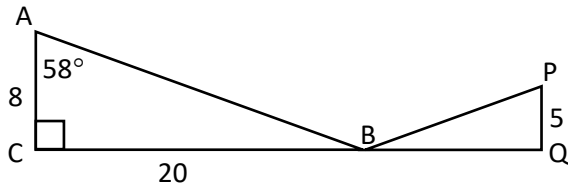
3. Given Quadrilateral HIJG \sim Quadrilateral MNOL, find x and y .



$x =$ _____

$y =$ _____

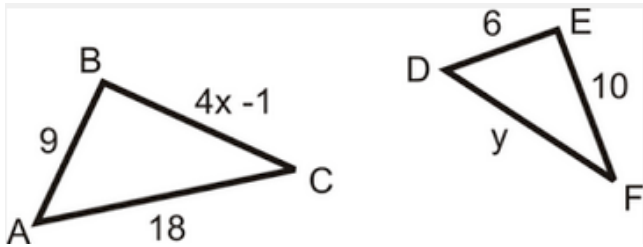
4. $\Delta ABC \sim \Delta PBQ$. Find $\angle PBQ$ and BQ . Round to the nearest tenth.



$\angle PBQ =$ _____

$BQ =$ _____

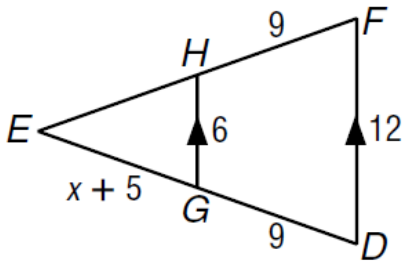
5. If $\triangle ABC \sim \triangle DEF$, find the perimeter of $\triangle ABC$. What is the ratio of ABC to DEF?



Perimeter of ABC = _____

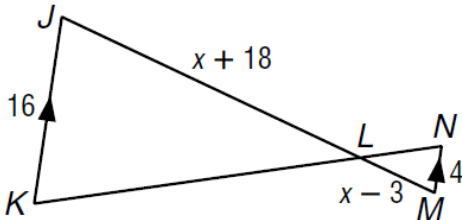
6. Identify the Similar triangles, how you know they are similar, find the variable(s) and the measures of the indicated sides.

\overline{EH} and \overline{EF}



7. Identify the Similar triangles, how you know they are similar, find the variable(s) and the measures of the indicated sides.

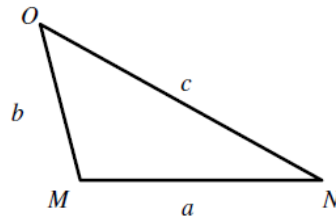
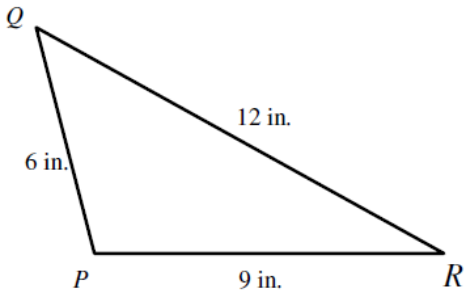
\overline{JL} and \overline{LM}



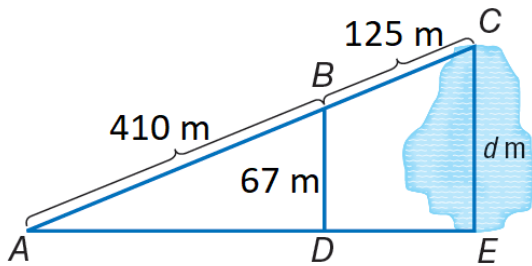
8. A flagpole 5 meters tall casts a 3-meter shadow. At the same time of day, a nearby building casts a 32-meter shadow. How tall is the building?

9. $\triangle QPR \sim \triangle OMN$

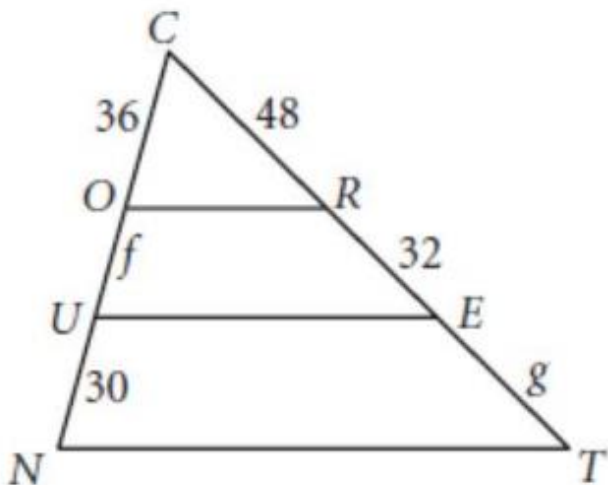
Find a , b , and c if the perimeter of $\triangle OMN$ is 18 inches. All measurements are in inches.



10. In the figure, triangle DBA is similar to triangle ECA . Ramon wants to know the distance across the lake. Find d and round to the nearest hundredth if needed.

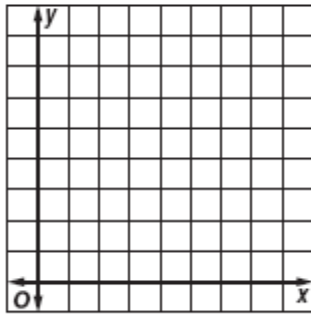


11. $OR \parallel UE \parallel NT$. Find f and g .

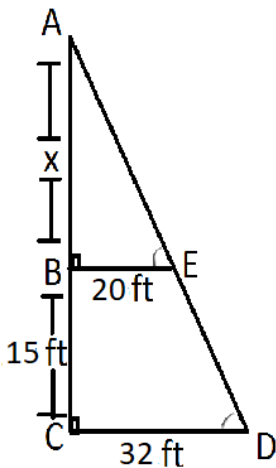


12. Find the image of the polygon, given the vertices, after a dilation centered at the origin with a scale factor of 2, 3, $\frac{1}{2}$, and $\frac{1}{3}$.

$J(2, 4), K(4, 4), P(3, 2)$



13. Find x .



14. Given $\triangle ABC \sim \triangle APQ$. If the perimeter of ABC is 51 in and the perimeter of triangle APQ is 34 in. Find all variables.

