## ACC Geometry Chapter 8 Booklet

## Pythagorean Theorem

$a^{2}+b^{2}=c^{2}$ when you want to solve for a side of a right triangle and do not have any special angles!
Q: What if you are given 3 side lengths and you are asked if the triangle is a right triangle? What do you do? How do you know what a pythagorean Triple is?


Elevation and Depression angles


Law of Sines and Cosines:
Use for non-right triangles only!!!!

| $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ | $a^{2}=b^{2}+c^{2}-2 b c \cos A$ <br> $b^{2}=a^{2}+c^{2}-2 a c \cos B$ <br> $c^{2}=a^{2}+b^{2}-2 a b \cos C$ <br> Law of Sines |
| :---: | :---: |
| Law of Cosines |  |

Chapter 8 Practice from Textbook Extra Practice
Pg 436 \#27
Pg 444 \#13-23 odd, 31, 35
Pg 452 \#9-21odd, 33, 35
Pg 460 \#24 ( 4.1 mi ), 50(19.32ft), 51
Pg 463 \#4(d), 5
Pg 468 \#15, 16(162.3m), 18(158.7ft), 22(5.1mi)
Pg 475 \#5
Pg 476 \#27, 28(536ft), 29, 30(56.9)
Pg 482 \#4(98), 23, 30(43.5, 79.7,56.8), 37
You may also want to use the book's
Study Guide and Review pg 486 or the Practice Test on pg 491.

Q: What if you are given 3 side lengths and you are asked if the triangle is a right triangle? What do you do? How do you know what a pythagorean Triple is?


Directions: Find x and the area of each figure.
1.

2.

3.


Converse of the Pythagorean Theorem: If the sum of the squares of the two smaller sides is equal to the square of the longest side, then the triangle is a right triangle. Pythagorean Triples are whole numbers which follows the converse of the Pythagorean Theorem.
Classifying triangles by angles based on sides:
If sum of smaller squares $>3$ rd side $^{2}$ then: Acute Triangle
If sum of smaller squares $=3$ rd side ${ }^{2}$ then: Right Triangle
If sum of smaller squares < 3 rd $^{\text {side }}{ }^{2}$ then: Obtuse Triangle
Given the following lengths, do they form right, obtuse or acute triangles? Are they Pythagorean Triples?
Ex.1) 9,40,41
Ex2.) 6,7,8

Ex3.) Determine if $\triangle A B C$ is a right, obtuse, or acute triangles when $A(2,-3), B(3,1)$, and $C(5,-1)$. Explain.


## Worksheet 10.1

Find the length of the hypotenuse, leave answer in simplest radical form.
1)

2)

32

Find the unknown leg length, leave answer in simplest radi¿al form.
3)

4)


Find the area of the isosceles triangle in simplest radical form.
5)

6)


The given lengths are two sides of a right triangle. All three side lengths of the triangle are integers and together forr Pythagorean triple. Find the length of the third side and tell whether it is a leg or the hypotenuse.
7) 24 and 32
8) 24 and 45
9) 40 and 85
10) 49 and 168
11) 72 and 78

Find the area of the right triangle. Write your answer in simplest radical form.
12)

13)

14)

15) A shipping dock has a mobile ramp that is used to help load and unload cargo from trucks. The ram is 125 inches long and has a base that is 120 inches long. What is the height $h$ or the ramp?


Challenge, Find the value of x for each
16)

17)

18)


### 10.1 Part 2 Converse of Pythagorean Theorem

$\qquad$ Do Work on own sheet of paper.
Tell whether the triangle is a right triangle. If not a right triangle, then what kind?
1)

2)

3)

Decide whether the numbers can represent the side lengths of a triangle. If they can, classify the triangle as acute, right, or obtuse.
4) $6,8,10$
5) $5,7,9$
6) 8,910
7) $10,12,30$
8) $16,30,34$
9) $18,34,45$
10) $\sqrt{8}, 4,6$
11) $20,21,28$
12) $\sqrt{13}, 10,12$
13) $14,48,50$

Graph points $A, B$, and $C$. Connect the points to form $\triangle A B C$. Decide whether $\triangle A B C$ is right, acute, or obtuse.
14) $A(-3,5), B(0,-2), C(4,1)$

16) $A(0,5), B(3,6), C(5,1)$

15) $A(-8,-4), B(-5,-2), C(-1,-7)$

17) $A(-2,4), B(2,0), C(5,2)$


The sides and classification of a triangle are given below. The length of the longest side is the integer given. What value(s) of $x$ make the triangle?
20) $x, x, 8$; right
21) $x, x, 12$; obtuse
22) $x, x, 6$; acute
23) $x, x, 16$; right
24) $x, x, 10$; obtuse
25) $x, x, 15$; acute

Maps The distances between three towns are given in the diagram.
26) Is the triangle ( $\triangle A B C$ ) formed by the three towns a right triangle?
27) Town $B$ is directly west of town $C$. Is town $A$ directly north of town C ?


## Special Right Triangles

properties of $45^{\circ}-45^{\circ}-90^{\circ}$, and $30^{\circ}-60^{\circ}-90^{\circ}$ Triangles
Recall the Pythagorean Theorem
Ex1: Find the length of SV.

$45^{\circ}-45^{\circ}-90^{\circ}$

## Fill in the table:




Ratio of a $45^{\circ}-45^{\circ}-90^{\circ}$ triangle is:


Find the value of the missing sides (Legs of Hypotenuse).


Find the value of $\boldsymbol{x}$.

$\frac{\text { 2. }}{\frac{45^{\circ}}{} \underbrace{x}_{5}}$
3.

4.


## $30^{\circ}-60^{\circ}-90^{\circ}$

## Discovering the shortcuts using Pythagorean Theorem

Find the missing sides and angles of the equilateral triangles below.
1.

$\mathrm{c}=$ $\qquad$ $\mathrm{a}=$ $\qquad$
2.

3.

$\mathrm{a}=$ $\qquad$ $\mathrm{b}=$ $\qquad$

Half of an equilateral triangle is also known as a 30-60-90 triangle. Notice the relationship between the two legs and the hypotenuse is the same EVERY time!

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A 30-60-90 right triangle has two legs and a hypotenuse. The leg opposite the 30
angle is called the short leg. The leg opposite the 60' angle is called the long leg.
Hypotenuse = short leg }
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$\qquad$

```
Long leg \(=\) short leg \(\times\)
``` \(\qquad\)
```

Short leg $=$ hypotenuse $\div$

``` \(\qquad\)

x
Short leg \(=\) long leg \(\div\) \(\qquad\)

\section*{\(30^{\circ}-60^{\circ}-90^{\circ}\) Triangles}

\section*{Consider an equilateral \(\boldsymbol{\Delta}\)}


\section*{\(30^{\circ}-60^{\circ}-90^{\circ}\)}


Short Leg

Fill in the missing sides of the triangle.
2)

3)

6)

1) \(10 \sqrt{175}\)
2) \(3 \sqrt{80}\)
3) \(7 \sqrt{112}\)
4) \(4 \sqrt{40}\)
5) \(6 \sqrt{160}\)
6) \(2 \sqrt{490}\)
7) \(2 \sqrt{12}\)
8) \(3 \sqrt{200}\)
9) \(2 \sqrt{320}\)
10) \(6 \sqrt{63}\)

Rationalize the Denominator
\(w=\frac{2}{\sqrt{5}}\)

1. \(\frac{6}{\sqrt{3}}\)
4. \(\frac{6}{\sqrt{15}}\)
2. \(\frac{6}{5 \sqrt{3}}\)
5. \(\frac{4}{\sqrt{21}}\)
3. \(\frac{10}{\sqrt{7}}\)
6. \(\frac{12}{\sqrt{2}}\)

\section*{Day 2 Special Right Triangle Warm-Up}

Find the lengths of the indicated sides. SHOW ALL WORK.
1.

2.

3.

\(M N=\)
\(N P=\)
4.

5.

6.

7.

8.

30-60-90
Find the lengths of the indicated sides. SHOW ALL WORK.
1.

2.

3.

4.

1. The sides of a square are 12 inches long. What is the length of the diagonal?
2. An isosceles right triangle has a hypotenuse of \(8 \sqrt{2} \mathrm{~cm}\). What is the length of the legs of the triangle?
3. An equilateral triangle sides are 10 inches. What is the length of the altitude?
4. In a \(30^{\circ}-60^{\circ}-90^{\circ}\) triangle, the hypotenuse is \(6 \sqrt{3}\), what is the length of the legs of the triangle?
5. The length of the diagonal of a square is 12 inches. Find the length of one side of the square.
6. The length of one side of an equilateral triangle is \(6 \sqrt{3}\) meters. Find the length of the altitude of the triangle.
7. The length of the altitude of an equilateral triangle is 12 feet. Find the length of one side of the equilateral triangle. What is the perimeter of the equilateral triangle?
8. The perimeter of an equilateral triangle is 39 cm . Find the length of the altitude of the triangle.
9. The length of the diagonal of a square is 18 mm . Find the perimeter of the square.

Find the value of each variable. Leave your answers in simplest radical form.

\(x=\) \(\qquad\) \(y=\) \(\qquad\)
2.

3.

\(a=\) \(\qquad\) \(b=\) \(\qquad\)
\(c=\) \(\qquad\) \(d=\)
\(\qquad\)
4.

5.

6.

\[
x=
\]
\(\qquad\)
7.

\[
z=
\]
8.

9.

\(x=\) \(\qquad\) \(y=\) \(\qquad\)
\(s=\) \(\qquad\)
10. Find the length of the diagonal of a square with 30 cm on a side.
11. The hypotenuse of an isosceles right triangle is 8.4 in . find the length of a side to the nearest tenth.
12. In a \(30^{\circ}-60^{\circ}-90^{\circ}\) triangle, the shorter leg is 6 ft long. Find the length of the other two sides.

Algebra Find the value of each variable. Leave your answers in simplest radical form.

\[
\begin{aligned}
& w= \\
& x= \\
& y= \\
& z=
\end{aligned}
\]
14.

\[
\begin{aligned}
& a= \\
& b=
\end{aligned}
\]
15.

\(p=\) \(\qquad\)
\(q=\) \(\qquad\)
\(r=\) \(\qquad\)
\(s=\) \(\qquad\)
\(\qquad\)
Period \(\qquad\)

Choose the best method, and then solve for the indicated values. Leave answers in simplified radical form.
1. \(\mathrm{m}=\) \(\qquad\)

2. \(n=\) \(\qquad\)
3. \(t=\) \(\qquad\)

4. \(\mathrm{c}=\) \(\qquad\)

5. \(\mathrm{AX}=\) \(\qquad\)

6. \(x=\) \(\qquad\)

7. \(y=\) \(\qquad\)

8. \(x=\) \(\qquad\)
\(y=\) \(\qquad\)

9. \(\mathrm{x}=\) \(\qquad\)
\(y=\) \(\qquad\)

10. \(x=\) \(\qquad\)
\(y=\) \(\qquad\)

11. \(w=\) \(\qquad\)
\(x=\) \(\qquad\)
12. \(\mathrm{x}=\) \(\qquad\)


13. A square has a diagonal of length 8 cm . Find the length of each side.
14. An equilateral triangle has sides of length 14 cm . Find the length of the altitude
15. Find the value of \(x\).

17. Find the value of \(x\).

\(\pi\)
19. Find the value of \(x, y\) and \(z\).


Find the missing lengths for each triangle below.
1)

4)

2)

5)

3)

6)


12

Name \(\qquad\) Date \(\qquad\)

Find the value of each variable in the polygon.
1. Equilateral \(\triangle A B C\)

2. Square \(A B C D\)

3. Regular hexagon \(A B C D E F\)


Find the value of each variable. Write answers in simplest radical form.
4.

5.

6.

7.

8.

9.


Sketch the figure that is described. Find the requested length.
Round decimals to the nearest tenth.
10. The side length of an equilateral triangle is 20 centimeters. Find the length of an altitude of the triangle.
11. The perimeter of a square is 20 centimeters. Find the length of a diagonal.
12. The diagonal of a square is 10 inches. Find the length of a side.

Baseball In Exercises 13-15, use the diagram and the following information.
The infield of a baseball field is a square. The distance from home plate to first base is 90 feet.
13. What is the distance from home plate to second base?
14. What is the distance from third base to first base?
15. If the pitcher's mound is 60 feet 6 inches from home plate, is it the midpoint of the diagonal from home plate to second base?


\section*{Extra Practice \\ 9.4}

Name
In 1-9, find the value of each variable in radical form.
1.

2.

3.

4.

5.

6.

7.

8.

9.

10. The perimeter of a square is 36 . What is the length of the diagonal?
12. What is the length of a side of an isosceles right triangle if its hypotenuse is 16 ?
14. The perimeter of a rectangle is 66 . The length is twice the width. What is the length of the diagonal?
11. What is the length of a side of an equilateral triangle whose altitude has a length of 18 ?
13. The length of the diagonal of a square is \(\frac{5 \sqrt{2}}{2}\). What is the length of a side?
15. The perimeter of an equilateral triangle is 36. What is the length of an altitude?

In 16-18, use the diagram and the following information.
A point on the edge of a symmetrical canyon is 4500 ft above a river that cuts through the canyon floor. The angle of depression from each side of the canyon to the canyon floor is \(60^{\circ}\).
16. Find the distance across the canyon,
18. Is it more or less than a mile across the canyon?
17. Find the length of the canyon wall (from the edge to the river).

\section*{Trigonometry Notes Ratios}

TRIGONOMETRY IS THE STUDY OF THE RELATIONSHIPS BETWEEN THE SIDES AND angles of triangles. A TRIGONOMETRIC RATIO IS A RATIO BETWEEN 2 sides of a right Triangle.

\section*{\(\underline{\theta}\) is a symbol for an angle}

\({ }^{* * *}\) Your opposite and adjacent sides will depend on what non- \(90^{\circ}\) ANGLE you are using! NEVER determine opposite/adjacent using the \(9 \mathbf{0}^{\circ}\) (always hyp)***

WAYS TO REMEMBER THE TRIGONOMETRIC RATIOS:


EXAMPLE 1: Find and simplify the trigonometric ratios.

\(\qquad\)
\(\sin \angle A=\)
\(\cos \angle A=\) \(\qquad\)
\(\tan \angle A=\) \(\qquad\)
\(\tan \angle C=\) \(\qquad\)
\(\qquad\) and \(\sin \angle C\) \(\qquad\) ***

Example 2 If \(\cos \theta=\frac{10}{13}\), what is \(\sin \theta\) ? What is \(\tan \theta\) ?
Example 3: If \(\tan \theta=\frac{3}{8}\), what is \(\sin \theta\) ? What is \(\cos \theta\) ?

Acc Geometry
Name
ID: 1
02020 Kuta S oftware LL C
Trig Ratio Examples
Date \(\qquad\) Period \(\qquad\)
Find the value of each trigonometric ratio. Simplify all fractions.
1) \(\sin C\)
2) \(\tan X\)


3) \(\cos A\)

4) \(\cos X\)

\[
\text { 5) } \cos A
\]

6) \(\sin A\)


\section*{Trig Ratio HW}
\(\qquad\) Period \(\qquad\)
Find the value of each trigonometric ratio. Simplify all fractions.
1) \(\cos C\)

2) \(\cos X\)

3) \(\tan A\)

5) \(\tan A\)

7) \(\sin C\)


10) \(\tan Z\)


\section*{Trigonometry Notes- Finding Missing Sides and Angles}

Using Your Calculator with Trigonometry!
\(\qquad\)
1. \(\operatorname{Sin}\left(57^{\circ}\right)=\)
3. \(\operatorname{Sin}\left(47^{\circ}\right)=\frac{x}{9}\)
4. \(\tan \left(61^{\circ}\right)=\frac{9}{x}\)
5. \(\theta=\tan ^{-1}\left(\frac{5}{6}\right)\)
6. \(\theta=\sin ^{-1}\left(\frac{9}{17}\right)\)
7. \(\tan \theta=\frac{20}{35}\)
8. \(\cos \theta=\frac{5}{7}\)

\section*{Trigonometry - Finding Missing Angles}

\section*{What do we do if the angle of our triangle is unknown?}

USE "INVERSE TRIG" - now we will give the calculator the ratio and it will tell us the angle in degrees
INVERSE SINE
INVERSE COSINE
INVERSE TANGENT
\(\sin ^{-1}\left(\frac{O}{H}\right)=\theta\)
\(\cos ^{-1}\left(\frac{A}{H}\right)=\theta\)
\(\tan ^{-1}\left(\frac{O}{A}\right)=\theta\)

Example 1-3: Find the measure of Angle B in each triangle below. Round to the nearest tenth of a degree.
1.

2.

3.


\section*{Finding Missing Sides Notes}

\section*{(FROM WORKSHEET B)}

Find the missing side(s) of each right triangle. Round to the nearest tenth.
7.

8.

10.

11.

12. Special Right Triangle (find EXACTLY)


\section*{Trigonometry Practice}

For 1 - 6, use the figures given to find each trigonometric ratio. Express the answers as a fraction and simplify!
1. \(\cos A\)
2. \(\tan B\)
3. \(\sin A\)

4. \(\tan X\)
5. \(\sin Z\)
6. \(\cos X\)

8.
9.

10.

11.

12.

13. Use Triangle \(L M N\) to find \(\sin L, \cos L, \tan L, \sin M, \cos M\), and \(\tan M\). Express each ratio as a fraction. Simplify all answers!
A. \(\ell=15, m=36, n=39\)
\(\sin L=\) \(\qquad\) \(\sin M=\) \(\qquad\) \(\sin L=\) \(\qquad\) \(\sin M=\) \(\qquad\) \(\cos L=\) \(\qquad\) \(\cos M=\) \(\qquad\) \(\cos L=\) \(\qquad\) \(\cos M=\) \(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
\(\qquad\)
14. In the right triangle below, if \(\sin x=\frac{15}{17}\), what is \(\tan x\) ? What is \(\cos x\) ?

\(\cos x=\) \(\qquad\)
\(\tan x=\) \(\qquad\)
15. Diego used a theodolite to map a region of land for his class in geomorphology. To determine the elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of sight to the top of the formation. The distance was 43 meters and the angle was 36 degrees. What is the height of the formation to the nearest meter?

\(\qquad\) DATE \(\qquad\) PERIOD \(\qquad\)

\section*{8-4 Practice}

\section*{Trigonometry}

Use \(\triangle L M N\) to find \(\sin L, \cos L, \tan L, \sin M, \cos M\), and \(\tan M\). Express each ratio as a fraction and as a decimal to the nearest hundredth.
1. \(\ell=15, m=36, n=39\)
2. \(\ell=12, m=12 \sqrt{3}, n=24\)


Use a calculator to find each value. Round to the nearest ten-thousandth.
3. \(\sin 72.5\)
4. \(\tan 27.5\)
5. \(\cos 64.8\)

Use the figure to find each trigonometric ratio. Express answers as a fraction and-as a-deeimal rounded to the nearest ten-thousandth. Simplify!

6. \(\cos A\)
7. \(\tan B\)
8. \(\sin A\)

Find the measure of each acute angle to the nearest tenth of a degree.
9. \(\sin B=0.7823\)
10. \(\tan A=0.2356\)
11. \(\cos R=0.6401\)

Find \(x\). Round to the nearest tenth.
12.

13.

14.

15. GEOGRAPHY Diego used a theodolite to map a region of land for his class in geomorphology. To determine the elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of sight to
 the top of the formation. The distance was 43 meters and the angle was 36 degrees. What is the height of the formation to the nearest meter?

\section*{8-4 Skills Practice \\ Trigonometry}

Use \(\triangle R S T\) to find \(\sin R, \cos R, \tan R, \sin S, \cos S\), and \(\tan S\). Express each ratio as a fraction and as a decimal to the nearest hundredth.
1. \(r=16, s=30, t=34\)
2. \(r=10, s=24, t=26\)


Use a calculator to find each value. Round to the nearest ten-thousandth.
3. \(\sin 5\)
4. \(\tan 23\)
5. \(\cos 61\)
6. \(\sin 75.8\)
7. \(\tan 17.3\)
8. \(\cos 52.9\)

Use the figure to find each trigonometric ratio. Express answers as a fraction and as a-deeimal rounded-te-the neavest-ten-thousandth.
9. \(\tan C\)
10. \(\sin A\)
11. \(\cos C\)


Find the measure of each acute angle to the nearest tenth of a degree.
12. \(\sin B=0.2985\)
13. \(\tan A=0.4168\)
14. \(\cos R=0.8443\)
15. \(\tan C=0.3894\)
16. \(\cos B=0.7329\)
17. \(\sin A=0.1176\)

Find \(x\). Round to the nearest tenth.
18.

19.

20.

\(\qquad\)
\(\qquad\)
Find the value of each trigonometric ratio.
1) \(\tan X\)

2) \(\sin Z\)

3) \(\cos X\)


State if each triangle is acute, obtuse, or right.
4)


Find the missing side lengths. Leave your answers as radicals in simplest form.
5)

6)

7)

8)


Find the missing side of each triangle. Leave your answers in simplest radical form.
9)


13 in
Find the missing side. Round to the nearest tenth.
10)


Find the measure of the indicated angle to the nearest degree.
11)


Find the missing side lengths. Leave your answers as radicals in simplest form.
12)

13)

14)


Find the area of each triangle. Round intermediate values to the nearest tenth. Use the rounded values to calculate the next value. Round your final answer to the nearest tenth.
15)

16)


\section*{Accelerated Geometry Notes: Section 8-5 Angles of Elevation and Depression.}


\author{
Label the angle of Elevation X \\ Label the angle of Depression Y
}
1. A dog chasing some birds in the woods got away from its owner. If the owner is 30 feet lower than the dog and the angle of elevation from the owner to the \(\operatorname{dog}\) is \(10^{\circ}\), find the distance from the owner to the dog.
2. Pat was flying a kite when the string broke and the kite fell to the ground. Pat is 36 yards lower than the kite. The distance from Pat to the kite is 200 yards. What is the angle of elevation?
3. After flying at an altitude of 600 meters, a hot air balloon starts to descend when its ground distance from the landing pad is 10,000 meters. What is the angle of depression for this part of the flight?
4. A bird watcher spied a woodpecker. The bird watcher is 40 yards lower than the woodpecker. The distance from the bird watcher to the woodpecker is 175 yards. What is the angle of elevation?
5. A water slide is 400 yards long with a vertical drop of 36.3 yards. Find the angle of depression of the slide.
6. A hiker stops to rest and sees a deer in the distance. If the hiker is 48 yards lower than the deer and the angle of elevation from the hiker to the deer is \(15^{\circ}\), find the distance from the hiker to the deer.

\section*{Angle of Elevation and Depression HW}
1.

HIKING Ayana is hiking in a national park. A forest ranger is standing in a fire tower that overlooks a meadow. She sees Ayana at an angle of depression measuring \(38^{\circ}\). If Ayana is 50 feet away from the base of the tower, which is closest to the height
 of the fire tower?
2. SHADOWS Find the angle of elevation of the Sun when a 7.6 -meter flagpole casts a 18.2-meter shadow. Round to the nearest tenth of a degree.
3. The tailgate of a moving van is 3.5 feet above the ground. A loading ramp is attached to the rear of the van at an incline of \(10^{\circ}\). What is the length of the ramp?

4. AVIATION After flying at an altitude of 500 meters, a helicopter starts to descend when its ground distance from the landing pad is 11 kilometers. What is the angle of depression for this part of the flight?
5. ocean archaeology A salvage ship uses sonar to determine the angle of depression to a wreck on the ocean floor that is 40 meters below the surface. How far must a diver, lowered from the salvage ship, walk along the ocean floor
 to reach the wreck?
6. standardized test example From the top of a 150 -foot high tower, an air traffic controller observes an airplane on the runway. Which equation would be
 used to find the distance from the base of the tower to the airplane?
7. GOLF A golfer is standing at the tee, looking up to the green on a hill. If the tee is 36 yards lower than the green and the angle of elevation
 from the tee to the hole is \(12^{\circ}\), find the distance from the tee to the hole.
8.

TOURISM Crystal is on a bus in France with her family. She sees the Eiffel Tower at an angle of \(27^{\circ}\). If the tower is 986 feet tall, how far away is the bus? Round to the nearest tenth.

9.

SLEDDING A sledding run is 300 yards long with a vertical drop of 27.6 yards. Find the angle of depression of the run.
10.The top of a signal tower is 120 meters above sea level. The angle of depression for the top of the tower to a passing ship is \(25^{\circ}\). What is the distance from the foot of the tower to the ship?

11. The angle of elevation from point \(A\) to the top of a hill is \(49^{\circ}\). If point \(A\) is 400 feet from the base of the hill, how high is the hill?

12. Find the angle of elevation of the sun when a 12.5-meter-tall telephone pole casts an 18-meter-long shadow.

13. The angle of depression from the top of a sheer cliff to point \(A\) on the ground is \(35^{\circ}\). If point \(A\) is 280 feet from the base of the cliff, how tall is the cliff?

14. The angle of depression from a balloon on a 75 -foot string to a person on the ground is \(36^{\circ}\). How high is the balloon?

15. INDIRECT MEASUREMENT Kyle is at the end of a pier 30 feet above the ocean. His eye level is 3 feet above the pier. He is using binoculars to watch a whale surface. If the angle of depression of the whale is \(20^{\circ}\), how far is the whale from
 Kyle's binoculars? Round to the nearest tenth foot.
16.

INDIRECT MEASUREMENT Mr. Dominguez is standing on a 40 -foot ocean bluff near his home. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are \(34^{\circ}\) and \(48^{\circ}\), how far apart are the dogs to the nearest foot?

17. Jermaine and John are watching a helicopter hover above the ground.

SKIP \#17


Jermaine and John are standing 10 meters apart.
18. AMUSEMENT PARKS From the top of a roller coaster, 60 yards above the ground, a rider looks down and sees the merry-go-round and the Ferris wheel. If the angles of depression
 are \(11^{\circ}\) and \(8^{\circ}\), respectively, how far anart are the merrv-on-ronind and 208 yards. Find the angle of depression from the top of the ski run to the bottom.


2C From the top of a 120 -foot-high tower, an air traffic controller observes an airplane on the runway at an angle of depression of \(19^{\circ}\). How far from the base of the tower is the airplane?

21. A ladder leaning against a building makes an angle of \(78^{\circ}\) with the ground. The foot of the ladder is 5 feet from the building. How long is the ladder?


22 SHADOWS Suppose the sun casts a shadow off a 35 -foot building. If the angle of elevation to the sun is \(60^{\circ}\), how long is the shadow to the nearest tenth of a foot?

23. BALLOONING From her position in a hot-air balloon, Angie can see her car parked in a field. If the angle of depression is \(8^{\circ}\) and Angie is 38 meters above the ground, what is the straight-line distance from Angie to her car? Round to the nearest whole meter.
24.

LIGHTHOUSES Sailors on a ship at sea spot the light from a lighthouse. The angle of elevation to the light is \(25^{\circ}\).


The light of the lighthouse is 30 meters above sea level. How far from the shore is the ship? Round your answer to the nearest meter.

\section*{8.1-8.5 Reviewing: Pythagorean Theorem, Special Right Triangles \& Trigonometry Directions: Complete the following questions.}
1. A ladder leaning against a house creates an angle of elevation of \(60^{\circ}\). The foot of the ladder is 7 feet from the foundation of the house. How long is the ladder?
2. A ladder leaning against a house creates an angle of elevation of \(22^{\circ}\). The foot of the ladder is 7 feet from the foundation of the house. How long is the ladder?
3. A ladder leaning against a house. The foot of the ladder is 7 feet from the foundation of the house and 24 feet up the wall. How long is the ladder?
4. a. Determine the height of the flagpole shown in the figure.

5. At a point 500 miles north of a ship, the shoreline runs east and west. West of that point, the navigator sights a light house at an angle of \(60^{\circ}\). How far is the ship from the lighthouse?
7. A person hang gliding at an altitude of 300 feet is over a spot 2,250 feet from an area of soft grass where he would like to land. At what angle of depression should he see the grass?
b. Determine the height of the flagpole shown in the figure.

6. A boy flying a kite lets out 100 feet of string making an angle of elevation of \(40^{\circ}\). How high above the ground is the kite?
9. Find all trig ratios for \(<\mathrm{A}\) and \(<\mathrm{C}\).

8. The roof of a house is the shape of an isosceles right triangle. The slope of the roof is 24 feet, what is the height of
 the roof?
10. An electrician sets up a ladder to reach the top of an electric pole 9 feet above the ground. The base of the ladder is 5 feet from the pole. While the electrician is gathering tools, the foot of the ladder slides 1 foot farther from the pole. How far up the pole does the ladder reach? Use exact values

11. David must install fencing around a lot that is shaped like a right triangle. The side of the lot that runs east-west is 200 ft long. The side of the lot that runs north-south is 125 ft long. Calculate how many feet of fencing he will need to surround the entire lot.
13. An ornamental pin is in the shape of an equilateral triangle. The length of each side is 6 centimeters. Josh will attach the fastener to the back along AB. Will the fastener fit if it is 4 centimeters long?

15. A person is standing on the third floor of the Galleria and has an angle of depression of \(22^{\circ}\) to the edge of the ice skating rink. The rink starts under the person and is 110 feet long. How high is the person to the nearest foot.
17. Find all trig ratios for \(<\mathrm{A}\) and \(<\mathrm{C}\).

19. Find x and y .

21. Find the length of \(A B\) in this trapezoid.

12. Jana is cutting a square of material for a tablecloth. The table's diagonal is 36 inches. She wants the diagonal of the tablecloth to be an extra 10 inches so it will hang over the edges of the table. What size square should Jana cut to make the tablecloth?
14. The angle of elevation of a 110 foot crane is \(45^{\circ}\). How high can the crane raise building material?

16. Does lengths \(8,15,17\) form a right, acute or obtuse triangle? If it forms a right triangle is are the lengths Pythagorean Triples?
18. A ball is rolling down a hill that is 424 feet long and has a height of 38 feet. What is the angle of depression for the hill?
20. Find \(x\) and \(y\).

22. Find x .


\title{
Law of Sines and Cosines Notes
}

Use for NON-RIGHT Triangles Only!

The Law of Sines In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.


The Law of Cosines Another relationship between the sides and angles of any triangle is called the Law of Cosines. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.
\begin{tabular}{|l|l|}
\hline \multirow{3}{*}{ Law of Cosines } & \begin{tabular}{l} 
Let \(\triangle A B C\) be any triangle with \(a, b\), and \(c\) representing the measures of the sides opposite \\
the angles with measures \(A, B\), and \(C\), respectively. Then the following equations are true. \\
\(a^{2}=b^{2}+c^{2}-2 b c \cos A\)
\end{tabular}\(\quad b^{2}=a^{2}+c^{2}-2 a c \cos B \quad c^{2}=a^{2}+b^{2}-2 a b \cos C\) \\
\hline
\end{tabular}


Review: Place the opposite sides.
B


A

Review: Place the opposite angles.


The Law of Sines In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.
\begin{tabular}{|l|l|}
\hline Law of Sines & \(\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}\) \\
\hline
\end{tabular}

Prove the law of sines: \(\frac{\sin A}{a}=\frac{\sin B}{b}\) Step 1:
\(\operatorname{Sin} A=\)
\(\operatorname{Sin} B=\)


Step 2: get h alone

Step 3: If two things are equal to the same thing, then they are....

Solve \(\triangle A B C\) for all variables.

Ex1. If \(b=12, m \angle A=89\), and \(m \angle B=80\)

Ex2. If \(a=30, c=20\), and \(m \angle A=60\),

The Law of Cosines Another relationship between the sides and angles of any triangle is called the Law of Cosines. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.
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\(a^{2}=b^{2}+c^{2}-2 b c \cos A \quad b^{2}=a^{2}+c^{2}-2 a c \cos B \quad c^{2}=a^{2}+b^{2}-2 a b \cos C\)
\end{tabular} \\
\hline
\end{tabular}

Ex 3. If \(m<A=62^{\circ}, b=14\), and \(c=12\). Solve the triangle.

Ex 4. If \(a=24, b=18\), and \(c=16\). Solve the triangle.

\section*{Law of Sines and Cosines HW\#1}

It is expected that you draw the triangle when one is not given to you.
Find each measure using the given measures of Triangle ABC. Find the measure.
1. If \(c=12, m \angle A=80\), and \(m \angle C=40\), find \(a\).
2. If \(b=20, c=26\), and \(m \angle C=52\), find \(m \angle B\).
3. Isosceles \(\triangle A B C\) has a base of 20 cm and a vertex angle of \(68^{\circ}\). Find the perimeter of the triangle.
4. If \(\mathrm{a}=25, \mathrm{~m}<\mathrm{A}=72^{\circ}\), and \(\mathrm{m}<\mathrm{B}=17^{\circ}\), solve the triangle.
5. SOLVE the triangle. (Triangle PQR)
\[
m \angle P=89, p=16, r=12
\]
6. In a scalene triangle, one side is 12 feet and another side is 20 feet. The angle opposite 20 feet is 83 degrees. Find the measure of the angle opposite the side that is 12 feet.
7. The famous Leaning Tower of Pisa was originally 184.5 feet high. At a distance of 123 feet from the base of the tower, the angle of elevation to the top of the tower is found to be \(60^{\circ}\). Solve the triangle and round to the nearest tenth.

\(\qquad\)
\(\mathrm{m}<\mathrm{A}=\) \(\qquad\)
\(C B=\) \(\qquad\)
8. Find \(c\).

9. Find the \(m<A\).

10. If \(a=24, b=18\), and \(c=16\). Solve the triangle.
11. Ms. Jones wants to purchase a piece of land with the shape shown here. Find the perimeter of the property.


Directions: Complete \#1-6 on a separate sheet of paper using the Law of Cosines! In \(\triangle R S T\), given the following measures, find the measure of the missing side.
1. \(r=5, s=8, m \angle T=39\)
2. \(r=6, t=11, m \angle S=87\)
3. \(r=9, t=15, m \angle S=103\)
4. \(s=12, t=10, m \angle R=58\)

In \(\triangle H I J\), given the lengths of the sides, find the measure of the stated angle to the nearest tenth.
5. \(h=12, i=18, j=7 ; m \angle H\)
6. \(h=15, i=16, j=22 ; m \angle I\)
7. Mrs. Burge is planting a raspberry garden in the shape of a triangle to pay homage to her favorite subject in school. The side lengths are as follows: 50 feet, 60 feet, 100 feet. Mr. Giannini wants to find the measure of the largest angle. Help him!!! ©
8. Ms. Schmidt, who lives behind Mrs. Burge, is constructing a tomato garden. The garden will connect to the smallest side of the raspberry garden. An adjacent side is congruent to the smallest side of the raspberry garden. The third side is 72 feet. Ms. Labadie is laughing at their farming skills. Find the measure of the angle between the two congruent sides.
9. AIRCRAFT From the diagram of the airplane shown, determine the approximate exterior perimeter of each wing. Round to the nearest tenth meter.

10. After the hurricane, the small tree in Mrs. Gross' neighbor's yard was leaning as she was hunkered down in her house. To keep it from falling, she nailed a 6-foot strap into the ground 4 feet from the base of the tree. She attached the strap to the tree \(31 / 2\) feet above the ground. How far from vertical was the tree leaning?

\(\qquad\)
\(\qquad\) HR: \(\qquad\)

\section*{8.1-8.7 (After Break Intervention)}

Draw a picture if one is not provided. Find all values to the nearest tenth. Show all work to receive full credit!
1. A man flies a kite with a 100 foot string. The angle of elevation of the string is \(52^{\circ}\). How high off the ground is the kite?

2. An airplane takes off 200 yards in front of a 60 foot building. At what angle of elevation must the plane take off in order to avoid crashing into the building? Assume that the airplane flies in a straight line and the angle of elevation remains constant until the airplane flies over the building. (3 feet = 1 yard)

3. A person stands at the window of a building so that his eyes are 12.6 m above the level ground. An object is on the ground 58.5 m away from the building on a line directly beneath the person. Compute the angle of depression of the person's line of sight to the object on the ground.
4. From a plane flying due east at 265 m above sea level, the angles of depression of two ships sailing due east measure \(35^{\circ}\) and \(25^{\circ}\). How far apart are the ships?
5. Tom and Sam are on the opposite sides of a tower of 160 meters height. They measure the angle of elevation of the top of the tower as \(40^{\circ}\) and \(55^{\circ}\) respectively. Find the distance between Tom and Sam.

6. A man on the deck of a ship is 13 ft above water level. He observes that the angle of elevation of the top of a cliff is \(40^{\circ}\) and the angle of depression of the base is \(20^{\circ}\). Find the distance of the cliff from the ship and the height of the cliff if the base of the cliff is at sea level. (Find, x, y, and z)

7. Bert is building a kite using side lengths of 2 feet and 3 feet. The long vertical dowel to construct the kite is 4.12 ft . find the measure of the angle that the 2 foot edge makes with the 3 foot edge.

8. ARCHITECTURE An architect is designing a playground in the shape of a quadrilateral. Find the perimeter of the playground to the nearest tenth.

9. In a scalene triangle, one side is 12 feet and another side is 20 feet. The angle opposite 20 feet is 83 degrees. Find the measure of the angle opposite the side that is 12 feet.
10. A tree grew at a \(3^{\circ}\) slant from the vertical. At a point 50 feet from the tree, the angle of elevation to the top of the tree is \(17^{\circ}\). Find the length of the tree to the nearest tenth of a foot.

11. The angle of elevation from a point on the street to the top of a building is \(29^{\circ}\). The angle of elevation from another point on the street, 50 feet farther away from the building, to the top of the building is \(25^{\circ}\). To the nearest foot, how tall is the building?

\[
\begin{aligned}
& \mathrm{a}^{\mathrm{o}}=\square \\
& \mathrm{b}^{\mathrm{o}}= \\
& \mathrm{x}=\square \\
&
\end{aligned}
\]
height of building y : \(\qquad\)
12. Which set of measures could represent the sides of a right triangle?
a.
9, 40, 41
b. \(8,30,31\)
c. \(\quad 7,8,15\)
d. \(\sqrt{2}, \sqrt{3}, \sqrt{6}\)

Directions: For the following questions, find the variables as exact values.
13.


14.

16.

of the trapezoid.

18.

PAINTING A painter sets a ladder up to reach the bottom of a second-story window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint, a neighbor's dog bumps the ladder, which moves the base 2 feet farther away from the house. How far up the side of the house does the ladder reach?

19. LANDSCAPING Six congruent
square stones are arranged in an L-shaped walkway through a garden. If \(x=15\) inches, then find the area of the L-shaped walkway.


Directions: For the following questions round to the nearest tenth.
20.

22. If the length of one leg of a right triangle is 4 times the length of the other and the hypotenuse is 30 in , find the exact length of the shorter leg. This is not a special right triangle.
23. If \(x=16, y=30\), and \(z=34\) in the right triangles, find the 6 trigonometric ratios.
24. If \(\tan \theta=\frac{6}{16}\), find \(\sin \theta\) and \(\cos \theta\).```

