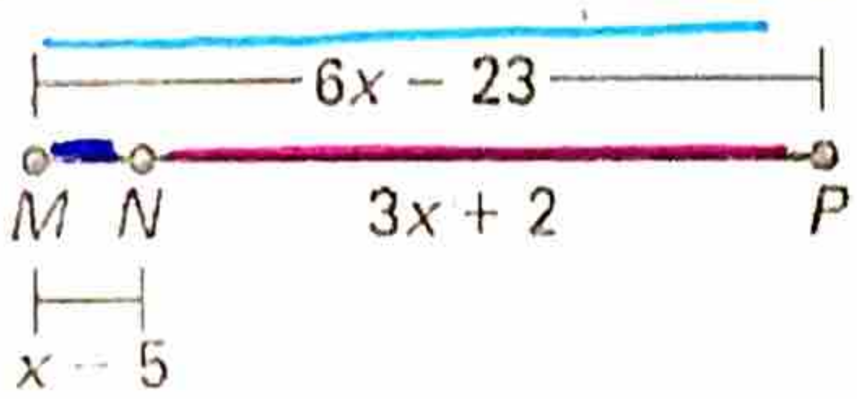
Segment Relationships Review Notes Key

ACC Geometry

Warm Up:

Segment Addition with Algebra: Find the variables and indicated lengths.

A.



Geometry:

$MN + NP = MP$

$x - 5 + 3x + 2 = 6x - 23$

$4x - 3 = 6x - 23$

$20 = 2x$

$\boxed{10 = x}$

Justification:

Segment addition

$MN = (10) - 5 = 5$ ✓

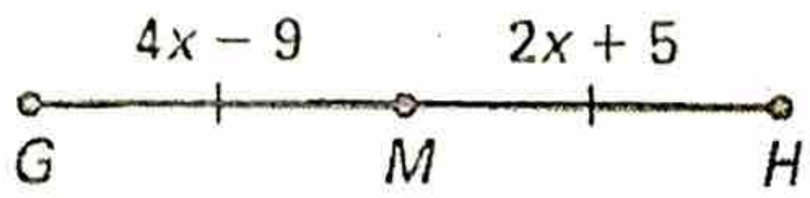
$NP = 3(10) + 2 = 32$ ✓

$MP = 6(10) - 23 = 37$ ✓

$5 + 32 = 37$ yes!!

X = 10 MN = 5 NP = 32 MP = 37

B. Midpoints with Algebra: In each diagram, M is the midpoint of the segment. Find the indicated length.



Geometry:

$GM \cong MH$

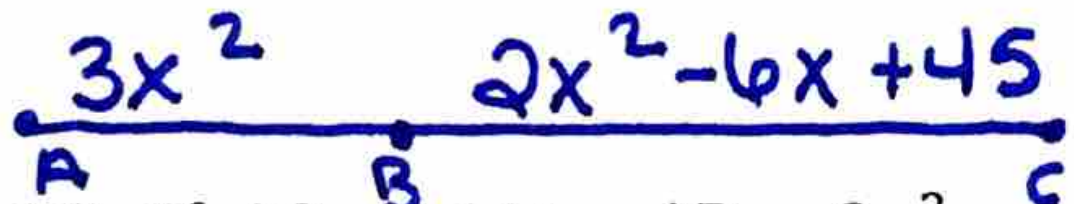
$4x - 9 = 2x + 5$

$x = 7$

Justification:

def of midpt

X = 7 GM = 19 MH = 19 GH = 38



$AC = 44in$

1. B lies between A and C on $\overline{AC}$. If $AC = 44in$, $AB = 3x^2$, and $BC = 2x^2 - 6x + 45$, Find the value of x and the lengths of each segment.

Geometry:

$AB + BC = AC$

$3x^2 + 2x^2 - 6x + 45 = 44$

$5x^2 - 6x + 1 = 0$

$(x - \frac{5}{5})(x - \frac{1}{5}) = 0$

$(x - 1)(5x - 1) = 0$

$x - 1 = 0 \quad 5x - 1 = 0$

$x = 1$ or $x = \frac{1}{5}$

Justification:

Segment addition

$5 \cdot 1 = 5$

Check Work:

check $x = 1$

$AB = 3(1)^2 = \boxed{3in}$

$BC = 2(1)^2 - 6(1) + 45 \boxed{= 41in}$

Geo: $3 + 41 = 44$ ✓ yes!

$\therefore$ $x = 1$ is a solution

check $x = \frac{1}{5}$

$AB = 3(\frac{1}{5})^2 = 0.12in$

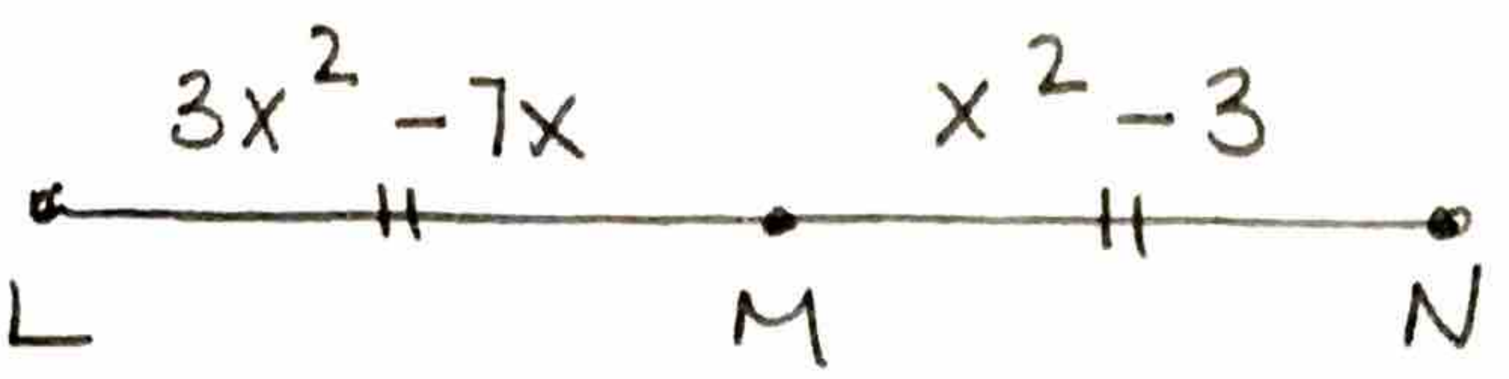
$BC = 2(\frac{1}{5})^2 - 6(\frac{1}{5}) + 45 = 43.85in$

Geo: $0.12 + 43.85 = 44in$ yes!

$\therefore$ $x = \frac{1}{5}$ is a solution

X = 1 and $\frac{1}{5}$ AB = 3in and 0.12in BC = 41in and 43.85in AC = 44in ~~and 44in~~ ☺

2. M is the midpoint of LN. If $LM = 3x^2 - 7x$, and $MN = x^2 - 3$, Find the value of x and the lengths of each segment.



Check Work:

check $x = 3$

$LM \cong MN$

$3(3)^2 - 7(3) \stackrel{?}{=} (3)^2 - 3$

$6 = 6$ ✓ yes

$LN = 6 + 6 = 12$

Geometry:

$LM \cong MN$

$3x^2 - 7x = x^2 - 3$

$2x^2 - 7x + 3 = 0$

$(x - \frac{1}{2})(x - \frac{6}{2}) = 0$

$(2x - 1)(x - 3) = 0$

$2x - 1 = 0$ $\quad x - 3 = 0$ $\quad x = 3$ OR $\cancel{x = \frac{1}{2}}$

must check!

Justification:

def of midpt

$2 \cdot 3 = 6$

$-1 \wedge -6$

check $x = \frac{1}{2}$

$LM \cong MN$

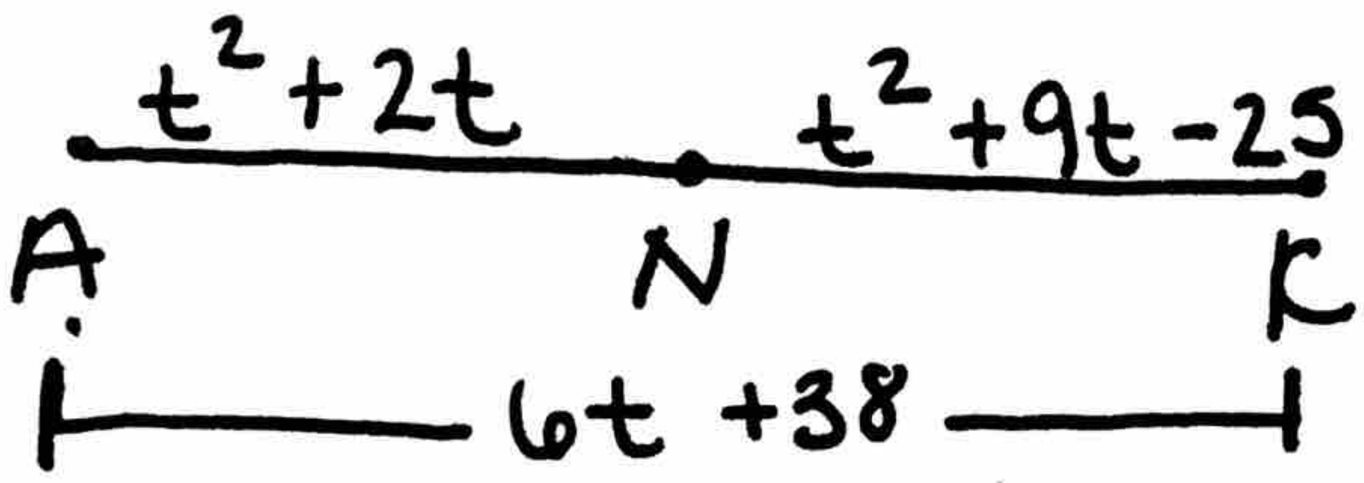
$\underbrace{3(\frac{1}{2})^2 - 7(\frac{1}{2})}_{} \stackrel{?}{=} \underbrace{(\frac{1}{2})^2 - 3}_{}$

$-2.75 = -2.75$

They are equal But distance can't be negative. ∴ $x \neq \frac{1}{2}$

X = 3 LM = 6 MN = 6 LN = 12

3. N lies between A and K on AK. If $AN = t^2 + 2t$, $NK = t^2 + 9t - 25$, and BC AK $= 6t + 38$, Find the value of t and the lengths of each segment.



Check Work:

check $t = \frac{9}{2}$

$AN = (\frac{9}{2})^2 + 2(\frac{9}{2}) = 29.25$

$NK = (\frac{9}{2})^2 + 9(\frac{9}{2}) - 25 = 35.75$

$AK = 6(\frac{9}{2}) + 38 = 65$

$29.25 + 35.75 = 65$ ✓ yes!

∴ $t = \frac{9}{2}$ is a solution

Geometry:

$AN + NK = AK$

$t^2 + 2t + t^2 + 9t - 25 = 6t + 38$

$2t^2 + 11t - 25 = 6t + 38$

$2t^2 + 5t - 63 = 0$

$(t - \frac{9}{2})(t + \frac{14}{2}) = 0$

$(2t - 9)(t + 7) = 0$

$2t - 9 = 0$ $\quad t + 7 = 0$

$t = \frac{9}{2}$ or $\cancel{t = -7}$

Justification:

Segment addition

$a \cdot c = -126$

$-9 \wedge 14$

check $t = -7$

$AN = (-7)^2 + 2(-7) = 35$

$NK = (-7)^2 + 9(-7) - 25 = -39$

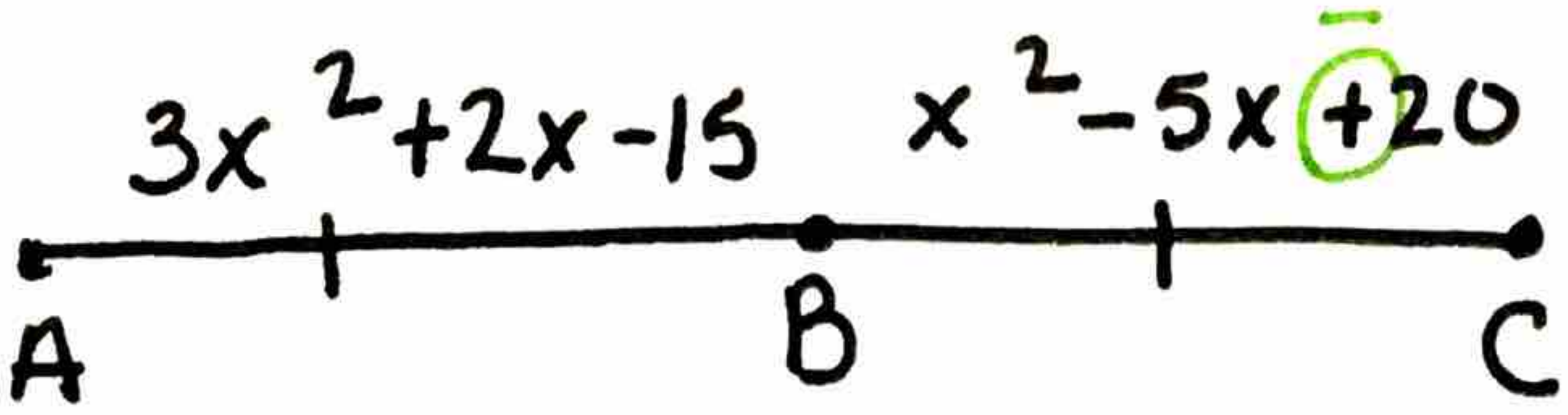
$AK = 6(-7) + 38 = -4$

can't have Neg dist.

∴ $x = -7$ is NOT a solution

t = $\frac{9}{2}$ AN = 29.25 NK = 35.75 AK = 65

4. B is the midpoint of $\overline{AC}$. If AB = $3x^2 + 2x - 15$, BC = $x^2 - 5x + 20$, find the value of the variable and the lengths of each segment.



Check Work:

check $x=-1$

$AB = 3(-1)^2+2(-1)-15=-14$

$BC = (-1)^2-5(-1)-20=-14$

Distance can't be negative So $x=-1$ is not a Solution.

Geometry: Justification:

$AB \cong BC$ def of midpoint

$3x^2+2x-15=x^2-5x-20$

$2x^2+7x+5=0$

$2\cdot 5 = 10$

$+2 \quad 5$

$(x+\frac{2}{2})(x+\frac{5}{2})=0$

$(x+1)(2x+5)=0$

$x+1=0 \quad 2x+5=0$

$\cancel{x=-1}$ or $x=\frac{-5}{2}$

check $x=\frac{-5}{2}$

$AB = 3(-\frac{5}{2})^2+2(-\frac{5}{2})-15=-1.25$

$BC = (-\frac{5}{2})^2-5(-\frac{5}{2})-20=-1.25$

Distance cannot be neg. $x=-\frac{5}{2}$ is not a Solution

x = No solution AN = ____________ NK = ____________ AK = ____________

5. If QR = $2x^2 - 9x - 12$ and ST = $-2x^2 - 18x - 3$, find all possible value(s) for x.

Geometry:

$QR \cong ST$

$2x^2-9x-12=-2x^2-18x-3$

$4x^2+9x-9=0$

$a\cdot c=-36$

$-3 \quad 12$

$(x-\frac{3}{4})(x+\frac{12}{4})=0$

$(4x-3)(x+3)=0$

$4x-3=0 \quad x+3=0$

$\cancel{x=\frac{3}{4}} \quad x=-3$

Check Work:

check $x=\frac{3}{4}$

$QR=2(\frac{3}{4})^2-9(\frac{3}{4})-12=-17.625$

No neg dist.

check $x=-3$

$QR=2(-3)^2-9(-3)-12=33$ ✓

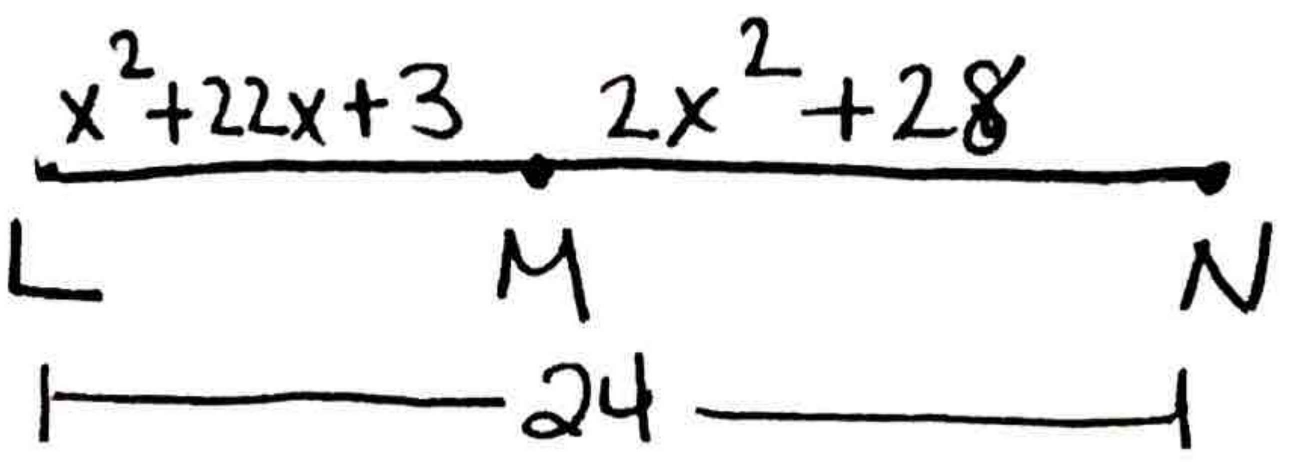
$ST=-2(-3)^2-18(-3)-3=33$ ✓

Yay!!!

X = -3

6. M is between L and N. If LM = $x^2 + 22x + 3$, LN = 24, and MN = $2x^2 + 28$, find the value of x and the lengths of each segment.

Check Work:



Check $x=-7$

$LM=(-7)^2+22(-7)+3=\boxed{-102}$

Distance can't be negative so $x=-7$ is not a solution for this geometry question.

Geometry: Justification:

$LM+MN=LN$ Segment addition

$x^2+22x+3+2x^2+28=24$

$3x^2+22x+31=24$

$3x^2+22x+7=0$

$a \cdot c = 21$

21 1

$(x+\frac{21}{3})(x+\frac{1}{3})=0$

$(x+7)(3x+1)=0$

$x+7=0$ $3x+1=0$

$x=-7$ $x=\frac{-1}{3}$

check $x=-\frac{1}{3}$

$LM=(-\frac{1}{3})^2+22(-\frac{1}{3})+3$

$LM=-4.\overline{2}$

Distance can't be negative $\therefore$

X = No Solution AB = ________ BC = ________ AC = ________