

Study Guide 4.6 Isosceles & Equilateral Δ 's

2. Determine whether each statement is *always*, *sometimes*, or *never* true.
- If a triangle has three congruent sides, then it has three congruent angles.
 - If a triangle is isosceles, then it is equilateral.
 - If a right triangle is isosceles, then it is equilateral.
 - The largest angle of an isosceles triangle is obtuse.
 - If a right triangle has a 45° angle, then it is isosceles.
 - If an isosceles triangle has three acute angles, then it is equilateral.
 - The vertex angle of an isosceles triangle is the largest angle of the triangle.
3. Give the measures of the three angles of each triangle.
- an equilateral triangle
 - an isosceles right triangle
 - an isosceles triangle in which the measure of the vertex angle is 70
 - an isosceles triangle in which the measure of a base angle is 70
 - an isosceles triangle in which the measure of the vertex angle is twice the measure of one of the base angles

Properties of Isosceles Triangles An isosceles triangle has two congruent sides. The angle formed by these sides is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

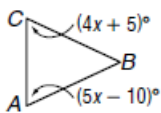
- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (**Isosceles Triangle Theorem**)
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.

If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$.

Example 1 Find x , given $\overline{BC} \cong \overline{BA}$.



$$BC = BA, \text{ so}$$

$$m\angle A = m\angle C. \quad \text{Isos. Triangle Theorem}$$

$$5x - 10 = 4x + 5 \quad \text{Substitution}$$

$$x - 10 = 5 \quad \text{Subtract } 4x \text{ from each side.}$$

$$x = 15 \quad \text{Add 10 to each side.}$$

Example 2 Find x .



$$m\angle S = m\angle T, \text{ so}$$

$$SR = TR. \quad \text{Converse of Isos. } \Delta \text{ Thm.}$$

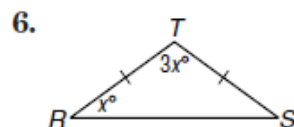
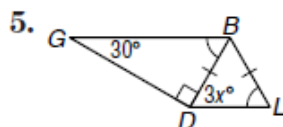
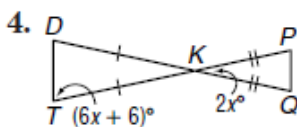
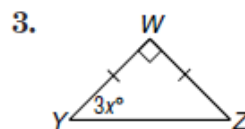
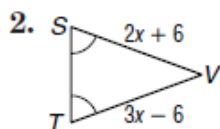
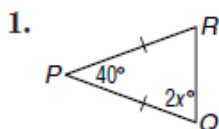
$$3x - 13 = 2x \quad \text{Substitution}$$

$$3x = 2x + 13 \quad \text{Add 13 to each side.}$$

$$x = 13 \quad \text{Subtract } 2x \text{ from each side.}$$

Exercises

Find x .

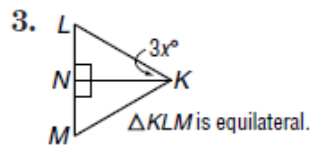
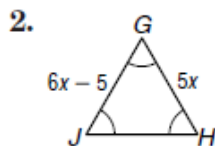
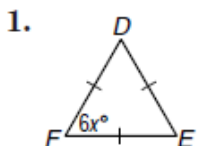


Properties of Equilateral Triangles An equilateral triangle has three congruent sides. The Isosceles Triangle Theorem can be used to prove two properties of equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral triangle measures 60° .

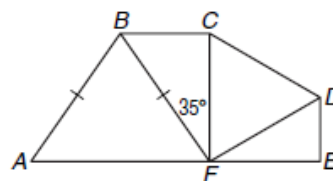
Exercises

Find x .



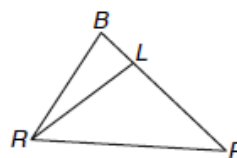
$\triangle ABF$ is isosceles, $\triangle CDF$ is equilateral, and $m\angle AFD = 150$. Find each measure.

5. $m\angle CFD$
6. $m\angle AFB$
7. $m\angle ABF$
8. $m\angle A$



In the figure, $\overline{PL} \cong \overline{RL}$ and $\overline{LR} \cong \overline{BR}$.

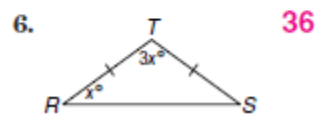
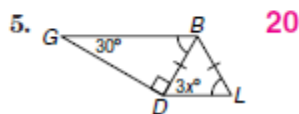
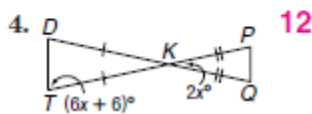
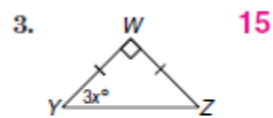
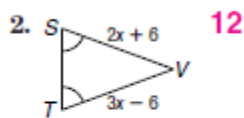
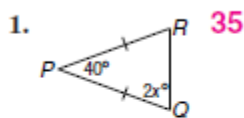
9. If $m\angle RLP = 100$, find $m\angle BRL$.
10. If $m\angle LPR = 34$, find $m\angle B$.



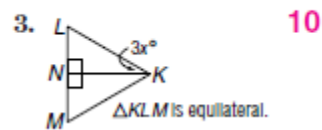
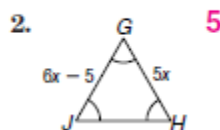
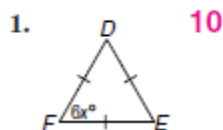
Solutions:

2. Determine whether each statement is *always*, *sometimes*, or *never* true.
- If a triangle has three congruent sides, then it has three congruent angles. **always**
 - If a triangle is isosceles, then it is equilateral. **sometimes**
 - If a right triangle is isosceles, then it is equilateral. **never**
 - The largest angle of an isosceles triangle is obtuse. **sometimes**
 - If a right triangle has a 45° angle, then it is isosceles. **always**
 - If an isosceles triangle has three acute angles, then it is equilateral. **sometimes**
 - The vertex angle of an isosceles triangle is the largest angle of the triangle. **sometimes**
3. Give the measures of the three angles of each triangle.
- an equilateral triangle **60, 60, 60**
 - an isosceles right triangle **45, 45, 90**
 - an isosceles triangle in which the measure of the vertex angle is 70 **70, 55, 55**
 - an isosceles triangle in which the measure of a base angle is 70 **70, 70, 40**
 - an isosceles triangle in which the measure of the vertex angle is twice the measure of one of the base angles **90, 45, 45**

Find x .

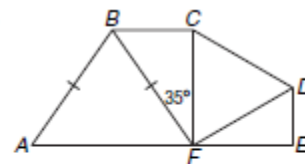


Find x .



$\triangle ABF$ is isosceles, $\triangle CDF$ is equilateral, and $m\angle AFD = 150$. Find each measure.

- $m\angle CFD$ **60**
- $m\angle AFB$ **55**
- $m\angle ABF$ **70**
- $m\angle A$ **55**



In the figure, $\overline{PL} \cong \overline{RL}$ and $\overline{LR} \cong \overline{BR}$.

- If $m\angle RLP = 100$, find $m\angle BRL$. **20**
- If $m\angle LPR = 34$, find $m\angle B$. **68**

