

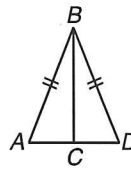
4-4 Study Guide and Intervention

Proving Congruence—SSS, SAS

SSS Postulate You know that two triangles are congruent if corresponding sides are congruent and corresponding angles are congruent. The Side-Side-Side (SSS) Postulate lets you show that two triangles are congruent if you know only that the sides of one triangle are congruent to the sides of the second triangle.

SSS Postulate	If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.
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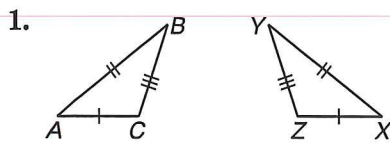
Example Write a two-column proof.
Given: $\overline{AB} \cong \overline{DB}$ and C is the midpoint of \overline{AD} .
Prove: $\triangle ABC \cong \triangle DBC$



Statements	Reasons
1. $\overline{AB} \cong \overline{DB}$	1. Given
2. C is the midpoint of \overline{AD} .	2. Given
3. $\overline{AC} \cong \overline{DC}$	3. Midpoint Theorem
4. $\overline{BC} \cong \overline{BC}$	4. Reflexive Property of \cong
5. $\triangle ABC \cong \triangle DBC$	5. SSS Postulate

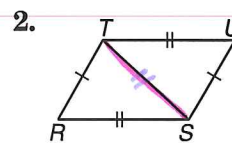
Exercises

Write a two-column proof.



Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$
Prove: $\triangle ABC \cong \triangle XYZ$

Statements	Reasons
1. $AB \cong XY$ $AC \cong XZ$ $BC \cong YZ$	1. given
2. $\triangle ABC \cong \triangle XYZ$	2. SSS



Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$
Prove: $\triangle RST \cong \triangle UTS$

Statements	Reasons
1. $RS \cong UT$ $RT \cong US$	1. given
2. $TS \cong TS$	2. Reflexive
3. $\triangle RST \cong \triangle UTS$	3. SSS

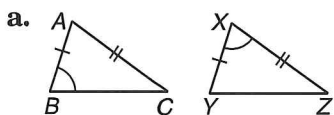
4-4 Study Guide and Intervention *(continued)*

Proving Congruence—SSS, SAS

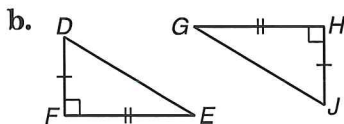
SAS Postulate Another way to show that two triangles are congruent is to use the Side-Angle-Side (SAS) Postulate.

SAS Postulate	If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.
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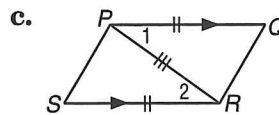
Example For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.



In $\triangle ABC$, the angle is not "included" by the sides \overline{AB} and \overline{AC} . So the triangles cannot be proved congruent by the SAS Postulate.



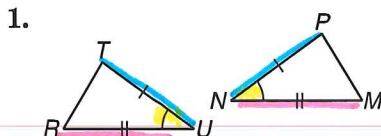
The right angles are congruent and they are the included angles for the congruent sides. $\triangle DEF \cong \triangle JGH$ by the SAS Postulate.



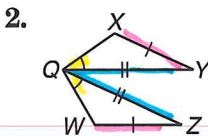
The included angles, $\angle 1$ and $\angle 2$, are congruent because they are alternate interior angles for two parallel lines. $\triangle PSR \cong \triangle RQP$ by the SAS Postulate.

Exercises

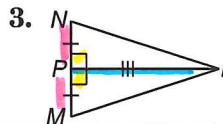
For each figure, determine which pairs of triangles can be proved congruent by the SAS Postulate.



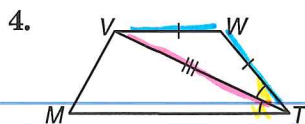
$\triangle TUR \cong \triangle PNM$
by SAS



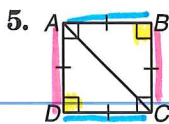
$\triangle XYQ \cong$ none
by _____



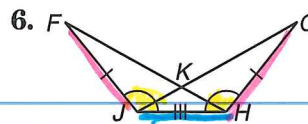
$\triangle NPL \cong \triangle MPL$
by SAS



$\triangle VWT \cong$ _____
by _____



$\triangle ABC \cong \triangle CDA$
by SAS (HL)
(many more shortcuts are optional!)



$\triangle FJH \cong \triangle GKH$
by SAS

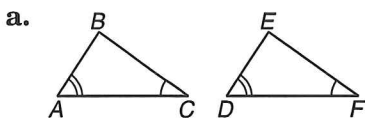
4-5 Study Guide and Intervention

Proving Congruence—ASA, AAS

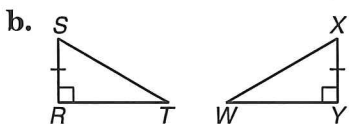
ASA Postulate The Angle-Side-Angle (ASA) Postulate lets you show that two triangles are congruent.

ASA Postulate	If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
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Example Find the missing congruent parts so that the triangles can be proved congruent by the ASA Postulate. Then write the triangle congruence.



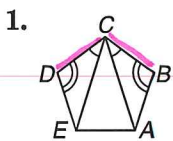
Two pairs of corresponding angles are congruent, $\angle A \cong \angle D$ and $\angle C \cong \angle F$. If the included sides \overline{AC} and \overline{DF} are congruent, then $\triangle ABC \cong \triangle DEF$ by the ASA Postulate.



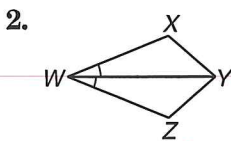
$\angle R \cong \angle Y$ and $\overline{SR} \cong \overline{XY}$. If $\angle S \cong \angle X$, then $\triangle RST \cong \triangle YXW$ by the ASA Postulate.

Exercises

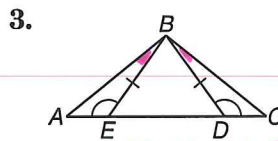
What corresponding parts must be congruent in order to prove that the triangles are congruent by the **ASA** Postulate? Write the triangle congruence statement.



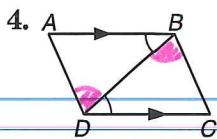
$CD \cong CB$
 $\triangle EDC \cong \triangle ABC$



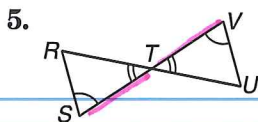
$\angle XYW \cong \angle ZYW$
 $\triangle WXY \cong \triangle WZY$



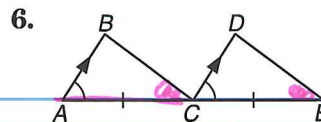
$\angle ABE \cong \angle CBD$
 $\triangle AEB \cong \triangle CDB$



$\angle ADB \cong \angle CBD$
 $\triangle ABD \cong \triangle CDB$



$ST \cong TV$
 $\triangle RST \cong \triangle VUT$



$\angle BCA \cong \angle DEC$
 $\triangle ABC \cong$

4-5 Study Guide and Intervention (continued)

Proving Congruence—ASA, AAS

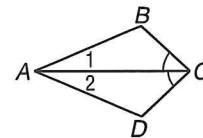
AAS Theorem Another way to show that two triangles are congruent is the Angle-Angle-Side (AAS) Theorem.

AAS Theorem	If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.
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You now have five ways to show that two triangles are congruent.

- definition of triangle congruence
- SSS Postulate
- SAS Postulate
- ASA Postulate
- AAS Theorem

Example In the diagram, $\angle BCA \cong \angle DCA$. Which sides are congruent? Which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Postulate?

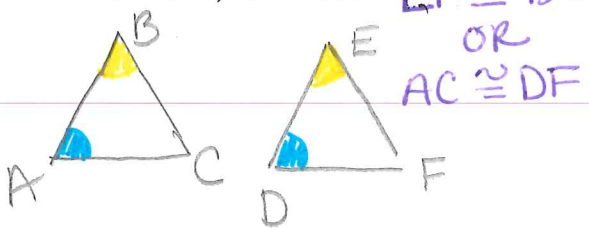


$\overline{AC} \cong \overline{AC}$ by the Reflexive Property of congruence. The congruent angles cannot be $\angle 1$ and $\angle 2$, because \overline{AC} would be the included side. If $\angle B \cong \angle D$, then $\triangle ABC \cong \triangle ADC$ by the AAS Theorem.

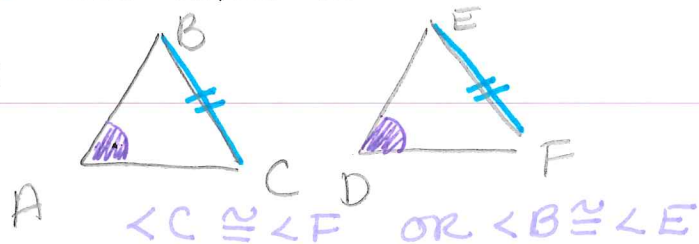
Exercises

In Exercises 1 and 2, draw and label $\triangle ABC$ and $\triangle DEF$. Indicate which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Theorem.

1. $\angle A \cong \angle D$; $\angle B \cong \angle E$



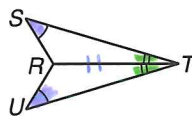
2. $BC \cong EF$; $\angle A \cong \angle D$



3. Write a flow proof.

Given: $\angle S \cong \angle U$; \overline{TR} bisects $\angle STU$.

Prove: $\angle SRT \cong \angle URT$



1. $\angle S \cong \angle U$
 \overline{TR} bisects $\angle STU$
2. $\angle STR \cong \angle UTR$
3. $\overline{RT} \cong \overline{RT}$
4. $\triangle STR \cong \triangle UTR$
5. $\angle SRT \cong \angle URT$

1. given
2. def of \angle bisector
3. Reflexive
4. AAS
5. CPCTC

4-4 Study Guide and Intervention

Proving Congruence—SSS, SAS

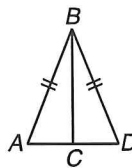
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Example Write a two-column proof.

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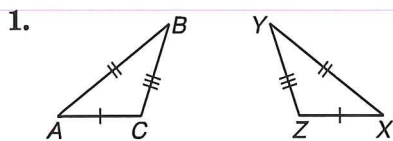
Prove: $\triangle ABC \cong \triangle DBC$



Statements	Reasons
1. $\overline{AB} \cong \overline{DB}$	1. _____
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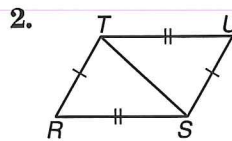
Exercises

Write a two-column proof.



Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$
Prove: $\triangle ABC \cong \triangle XYZ$

Statements	Reasons



Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$
Prove: $\triangle RST \cong \triangle UTS$

Statements	Reasons

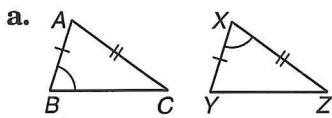
4-4 Study Guide and Intervention *(continued)*

Proving Congruence—SSS, SAS

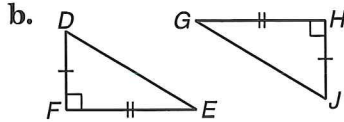
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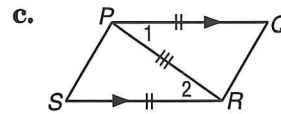
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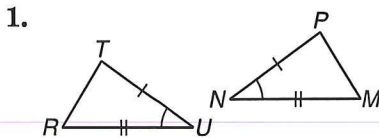
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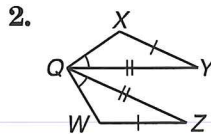
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Exercises

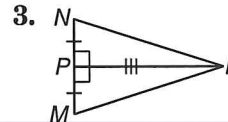
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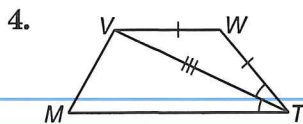
$\triangle TUR \cong \triangle NPM$
by _____



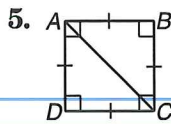
$\triangle XYQ \cong \triangle XWZ$
by _____



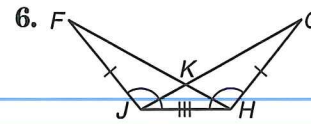
$\triangle NPL \cong \triangle MPL$
by _____



$\triangle VWT \cong \triangle VUT$
by _____



$\triangle ABC \cong \triangle DCB$
by _____



$\triangle FJK \cong \triangle GHK$
by _____

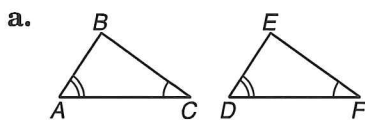
4-5 Study Guide and Intervention

Proving Congruence—ASA, AAS

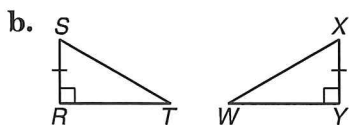
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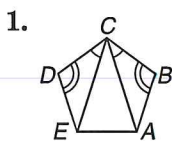
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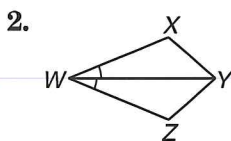
$\angle R \cong \angle Y$ and $\overline{SR} \cong \overline{XY}$. If $\angle S \cong \angle X$, then $\triangle RST \cong \triangle YXW$ by the ASA Postulate.

Exercises

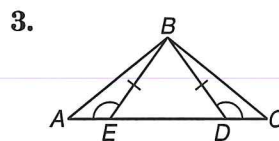
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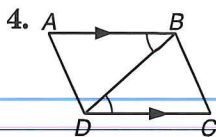
$\triangle EDC \cong \underline{\hspace{2cm}}$



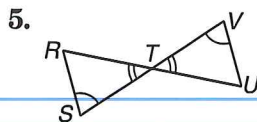
$\triangle WXY \cong \underline{\hspace{2cm}}$



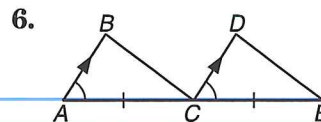
$\triangle AEB \cong \underline{\hspace{2cm}}$



$\triangle ABD \cong \underline{\hspace{2cm}}$



$\triangle RTS \cong \underline{\hspace{2cm}}$



$\triangle ABC \cong \underline{\hspace{2cm}}$

4-5 Study Guide and Intervention *(continued)*

Proving Congruence—ASA, AAS

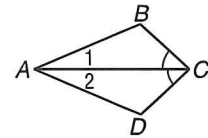
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Exercises

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1. $\angle A \cong \angle D$; $\angle B \cong \angle E$

2. $BC \cong EF$; $\angle A \cong \angle D$

3. Write a flow proof.

Given: $\angle S \cong \angle U$; \overline{TR} bisects $\angle STU$.

Prove: $\angle SRT \cong \angle URT$

