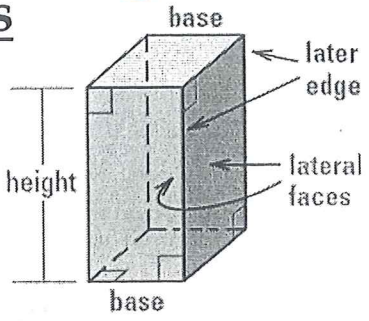
Key

Acc Geometry: Surface Area Notes

Prisms

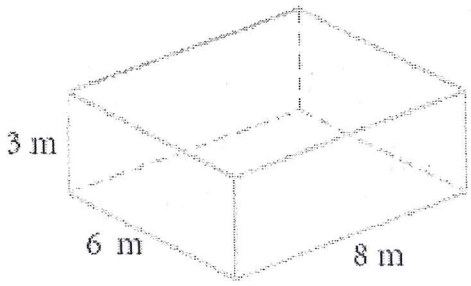
A **prism** is a polyhedron with two congruent faces, called **bases**, that lie in parallel planes. The other faces, called **lateral faces**, are parallelograms formed by connecting the corresponding vertices of the bases. The segments connecting these vertices are *lateral edges*.

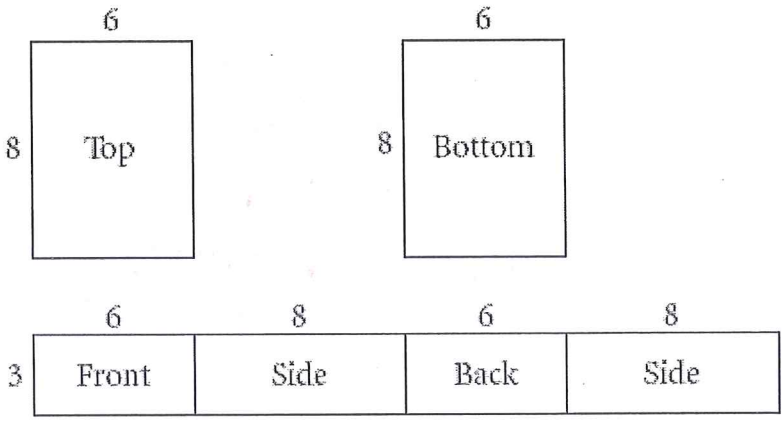


Right rectangular prism

Surface area of prisms: The surface area of a prism is the sum of the areas of all the faces including the bases.

Example1:
Find the surface area of the prism.



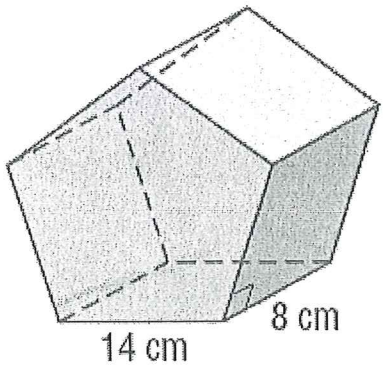


$SA = 3\cdot6 + 3\cdot8 + 3\cdot6 + 3\cdot8 + 2(8\cdot6)$

$SA = 180m^2$

Find the lateral area and the surface area of the following solids. Round your answers to the nearest thousandth.

Example 2:

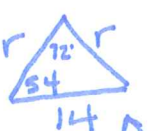


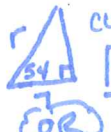
Base: Pentagon

$A_B = 5\frac{1}{2}rr\sin\theta$

$A_B = 5\frac{1}{2}(11.909)^2\sin(72)$ $\quad \theta = \frac{360}{5} = 72$

$A_B \approx 337.207 cm^2$





$\cos 54 = \frac{7}{r}$

$r = 11.909$

OR

Law of Sines

$\frac{\sin(72)}{14} = \frac{\sin(54)}{r}$

$r = 11.909$

Lateral Area

$LA = 5$ (14×8 rectangles)

$LA = 560\ cm^2$

Area of base: $337.207 cm^2$

Lateral Area: $560 cm^2$

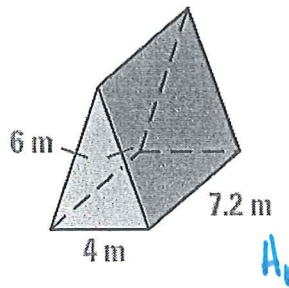
Surface Area: $1234.415 cm^2$

Surface Area

2 Bases + 5 faces

$2\left(5\cdot\frac{1}{2}(11.909)^2\sin(72)\right) + 5(14\cdot8)$ →

Example 3: Find the surface area of the following solids. Round your answers to the nearest



Area of Bases

Find h

$h^2+2^2=6^2$

$h=\sqrt{32}$

$h=4\sqrt{2}$m

$A_B = 2\Delta s = 2(\frac{1}{2}b\cdot h)$

$A_B = 2\frac{1}{2}4\cdot 4\sqrt{2}$

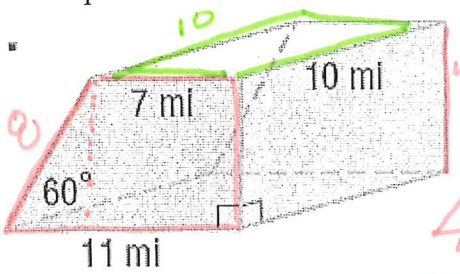
$A_B = 16\sqrt{2}\ m^2$

$LA = 2(6\times 7.2) + 4\times 7.2$

$LA = 115.2\ m^2$

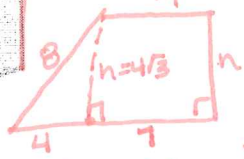
$SA \approx 137.827\ m^2$

Example 4: Find the exact surface area of the following solids.



Area of Bases

Trapezoid x2



$A_B = 2(\frac{1}{2}4\sqrt{3}(11+7))$

$A_B = 72\sqrt{3}\ mi^2$

Lateral Area

$10\times 7 + 10\cdot 4\sqrt{3}$

$10\times 11 + 10\cdot 8$

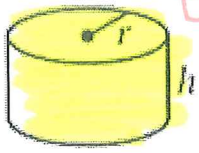
$LA = 260 + 40\sqrt{3}\ mi^2$

SA = Lateral area + Area of Both Bases

$SA = 260 + 40\sqrt{3} + 72\sqrt{3}$

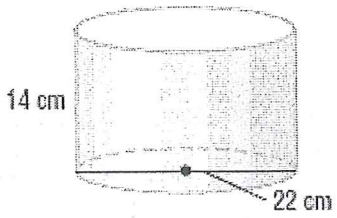
$SA = 260 + 112\sqrt{3}\ mi^2$ exact value

Cylinders:



$SA = 2\pi r^2 + 2\pi rh$

Example 5: Find the surface area of the figure. Leave in terms of pi.



$SA = 2\pi r^2 + 2\pi rh$

SA = 2 Base + lateral area

$SA = 2\pi 11^2 + 2\pi 11\cdot 14$

$SA = 242\pi + 308\pi$

$SA = 550\pi\ cm^2$

use this instead

300π

Example 6: The surface area of a cylinder is ~~301.6~~ square centimeters, and its height is 8 cm. Find its radius.

$SA = ~~301.6~~\ 300\pi$

$H = 8$ cm

$SA = 2\pi r^2 + 2\pi rh$

$300\pi = 2\pi r^2 + 2\pi r 8$

$0 = 2\pi r^2 + 16\pi r - 300\pi$

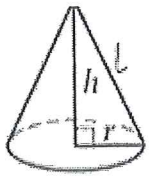
$0 = 2\pi(r^2 + 8r - 150)$

$r = \frac{-8\pm\sqrt{8^2-4\cdot 1\cdot -150}}{2\cdot 1}$

$r = \frac{-8\pm\sqrt{664}}{2} = \frac{-8\pm 2\sqrt{166}}{2}$

$r = -4+\sqrt{166} \approx 8.884$ cm

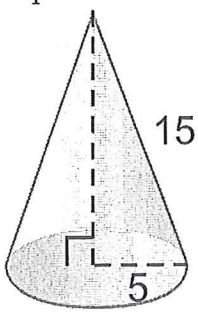
Cones:



$SA = \pi r^2 + \pi rl$

Example 7: Find the surface area of the figure. Leave in terms of pi.

A)

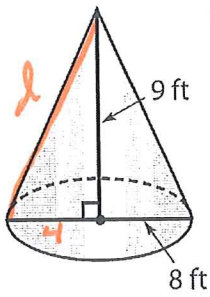


$SA = \pi 5^2 + \pi 5 \cdot 15$

$SA = 25\pi + 75\pi$

$SA = 100\pi \text{ units}^2$

B)



Find L 1st

$4^2 + 9^2 = l^2$

$l = \sqrt{97}$

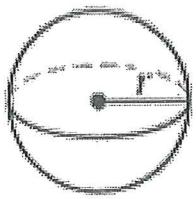
$SA = \pi 4^2 + \pi 4 \cdot \sqrt{97}$

Exact $= 16\pi + 4\pi\sqrt{97}$

Rounded ≈ 174.0298797

$SA \approx 174.030 \text{ ft}^2$

Spheres:



$SA = 4\pi r^2$

Example 8: Find the surface area if the circumference of the sphere is 24π.

$24\pi = C$

$24\pi = d\pi$

$d = 24$

$r = 12$

$SA = 4\pi r^2$

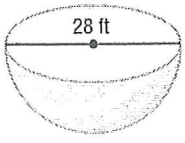
$SA = 4\pi \cdot 12^2$

$SA = 576\pi \text{ units}^2$

Example 9: Find the surface area of the hemisphere.

$d = 28$

$r = 14 \text{ ft}$



$SA = \frac{1}{2}(\text{sphere}) + \text{top circle}$

$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$

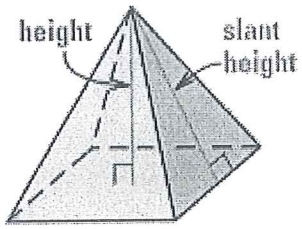
$SA = \frac{1}{2} \cdot 4\pi 14^2 + \pi 14^2$

$SA = 392\pi + 196\pi$

$SA = 588\pi \text{ ft}^2$

Pyramids:

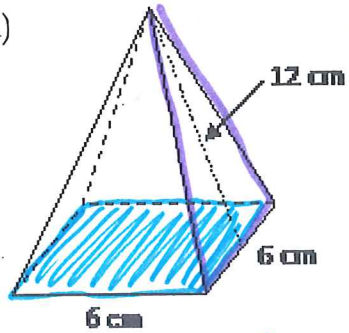
The surface area of a pyramid is the sum of the areas of all the faces including the base.



Directions: Find the surface area of the pyramid, round to the nearest thousandth if needed.

Example 10:

A)



LA = 4

$LA = 4 \cdot \frac{1}{2} \cdot 6 \cdot 12$

$LA = 144\ cm^2$

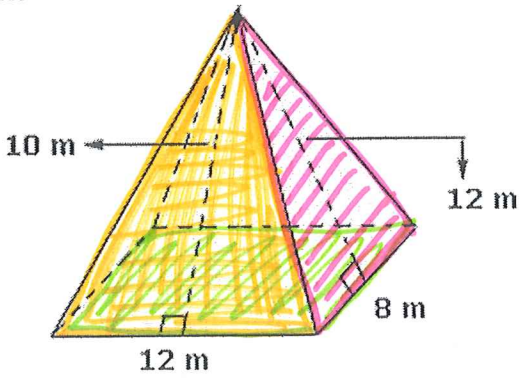
Square Base

$A_B = 6 \cdot 6$ $\quad A_B = 36\ cm^2$

$SA = 4 \cdot \frac{1}{2} \cdot 6 \cdot 12 + 6 \cdot 6$

$SA = 180\ cm^2$

B)



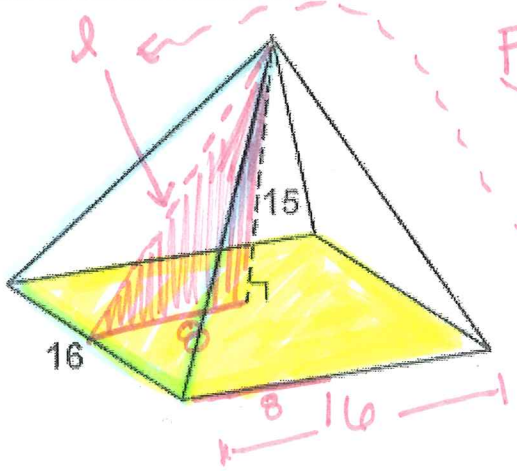
Base: 12×8

$2\Delta: 2 \cdot \frac{1}{2} \cdot 8 \cdot 12$

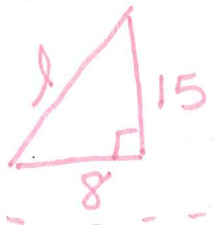
$2\Delta: 2 \cdot \frac{1}{2} \cdot 12 \cdot 10$

$SA = 312\ m^2$

Example 11: Find the missing parts then find the surface area of the pyramid, round to the nearest thousandth if needed.



Find slant height



$15^2 + 8^2 = l^2$

$\sqrt{289} = l$

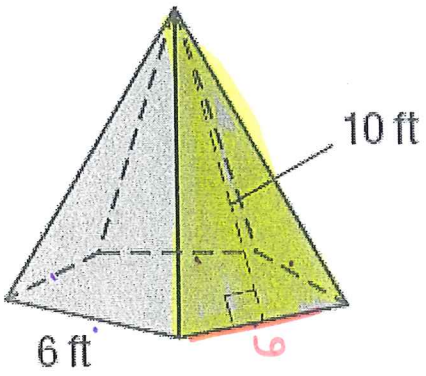
$17 = l$

Base: 16×16

$4\Delta: 4 \cdot \frac{1}{2} \cdot 16 \cdot 17$

$SA = 800\ units^2$

Example 12: Find the surface area of the regular, right pyramid, round to the nearest thousandth if needed.



Area of Base
Pentagon

$A_B = 5\frac{1}{2} r \cdot r \cdot \sin 72$

$A_B = 5\frac{1}{2}(5.102)^2 \sin(72)$

Find r

$\frac{\sin(72)}{} = \frac{\sin(54)}{}$

$r = 5.104$

$LA = 5\,\Delta s = 5\frac{1}{2} b \cdot k$

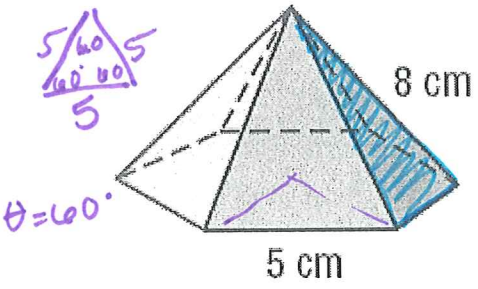
$LA = 5\,\frac{1}{2} 6 \cdot 10$

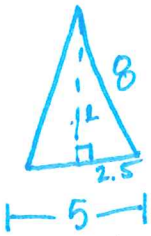
$SA = 5 \cdot \frac{1}{2}(5.102)^2 \sin(72) + 5 \cdot \frac{1}{2} \cdot 6 \cdot 10$

$SA \approx 211.891 ft^2$

Example 13: Find the missing slant height then find the surface area of the regular, right pyramid, round to the nearest thousandth if needed.

We are given an edge. need to find slant height.





$2.5^2 + l^2 = 8^2$

$l = \sqrt{57.75}$

$LA = 6\,\Delta s = 6\,\frac{1}{2} b \cdot h$

$LA = 6 \cdot \frac{1}{2} 5 \cdot \sqrt{57.75}$

Area of hexagon Base

$A_B = 6\,\frac{1}{2} r \cdot r \sin\theta$

$A_B = 6\,\frac{1}{2} 5 \cdot 5 \sin(60)$

Base + $6 \cdot \frac{1}{2} \cdot 5 \cdot 5 \sin(60)$

$SA \approx 178.942 cm^2$

Parts (Base + apothem)

Example 14: Find the missing slant height then find the surface area of the regular, right pyramid, round to the nearest thousandth if needed.

Find apothem

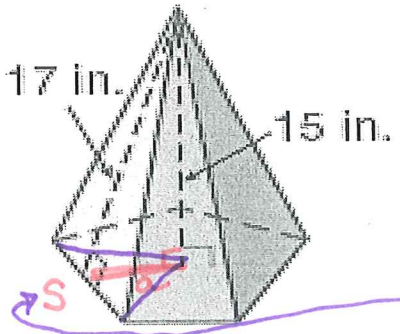
$a^2 + 15^2 = 17^2$

$a = 8$

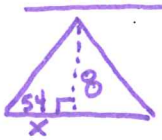
Base: $5\,\frac{1}{2} 11.625 \cdot 8$

$LA = 5\,\frac{1}{2} 11.625 \cdot 17$

$SA \approx 726.563 in^2$



Find S



$\tan(54) = \frac{8}{x}$

$x = 5.812340 22$

$S = 11.625$