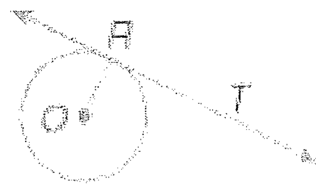
10-5 Tangent Notes

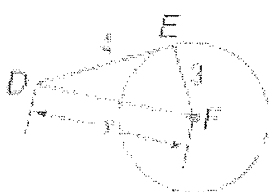
A line, line segment, or ray that intersects a circle in exactly one point is the tangent. The point that the line, line segment, or ray intersects with the circle is called the point of tangency.

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

Example: If $\overleftrightarrow{RT}$ is a tangent, $\overline{OR} \perp \overleftrightarrow{RT}$.



Example 1: $\overline{ED}$ is tangent to Circle F at point E. Find x.



$(EF)^2 + (DE)^2 = (DF)^2$

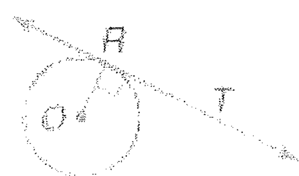
$3^2 + 4^2 = x^2$

$\sqrt{25} = \sqrt{x^2}$

$\boxed{x = 5}$

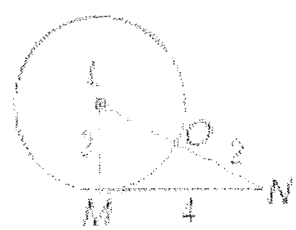
If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

Example: If $\overline{OR} \perp \overleftrightarrow{RT}$, $\overleftrightarrow{RT}$ is a tangent.



Example 2:

a) Determine whether $\overline{MN}$ is tangent to Circle L. Justify your reasoning.

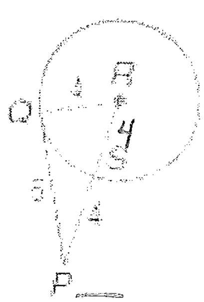


$(LM)^2 + (MN)^2 \overset{?}{=} (LN)^2$

$3^2 + 4^2 \overset{?}{=} 5^2$

$25 = 25$ ✓

$\therefore \overline{MN}$ is tangent



$\overline{RQ} = \overline{RS}$

$\therefore \overline{RP} = 8$

b) Determine whether $\overline{PQ}$ is tangent to Circle R. Justify your reasoning.

$(RQ)^2 + (PQ)^2 \overset{?}{=} (RP)^2$

$4^2 + 5^2 \overset{?}{=} 8^2$

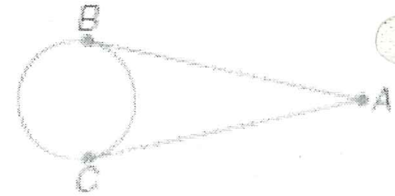
$41 \neq 64$

$\therefore \overline{PQ}$ is not tangent

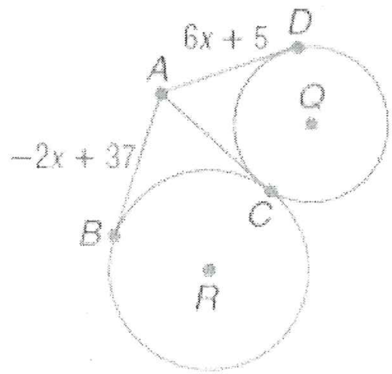
Congruent Tangents

If two segments from the same exterior point are tangent to a circle, then they are congruent.

Example: $\overline{AB} \cong \overline{AC}$



Example 3: Find x. Assume that segments that appear tangent to circles are tangent.



$\overline{AD} \cong \overline{AC} \rightarrow$ from same exterior point and are tangent to $\odot Q$

$\overline{AC} \cong \overline{AB} \rightarrow$ from same exterior point and tangent to $\odot R$

$\therefore \overline{AD} \cong \overline{AB}$

$AD = AB$

$6x+5 = -2x+37$

$8x+5 = 37$

$8x = 32$

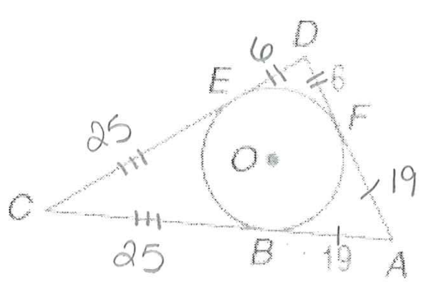
$\boxed{x = 4}$

Circumscribed Polygons

Polygons can also be <u>circumscribed</u> about a circle, or the circle is <u>inscribed</u> in the polygon.

The vertices of the polygon <u>DO NOT</u> lie on the circle, but every side of the polygon is <u>tangent</u> to the circle.

Example 4: Triangle ADC is circumscribed about Circle O. Find the perimeter of Triangle ADC if $\overline{EC} = \overline{DE} + \overline{AF}$.



$\overline{EC} = \overline{DE} + \overline{AF}$

$\overline{EC} = 6 + 19$

$\overline{EC} = 25$

$P = 6 + 25 + 25 + 19 + 19 + 6$

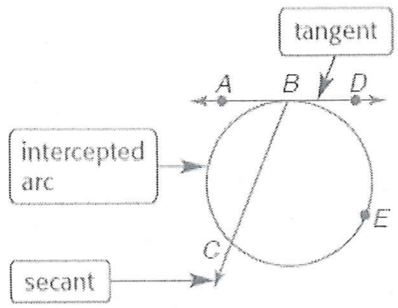
$\boxed{P = 100 \text{ units}}$

THEOREM 10.13

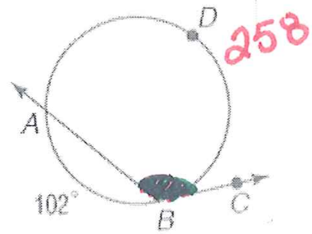
If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

Examples: $m\angle ABC = \frac{1}{2}m\overset{\frown}{BC}$

$m\angle DBC = \frac{1}{2}m\overset{\frown}{BEC}$



Example 1. Find $m\angle ABC$ if $m\overset{\frown}{AB} = 102$.



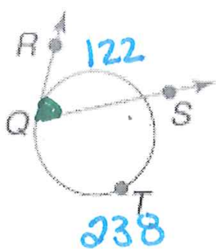
$m\overset{\frown}{ADC} = 360 - m\overset{\frown}{AB}$

$= 360 - 102 = 258$

$m\angle ABC = \frac{1}{2}\, m\overset{\frown}{ADC}$

$= \frac{1}{2}(258) = \boxed{129°}$

Example 2:



Find $m\angle RQS$ if $m\overset{\frown}{QTS} = 238$.

$m\overset{\frown}{QS} = 360 - 238$

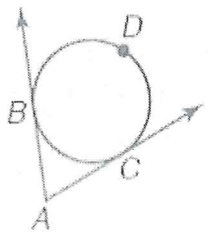
$= 122°$

$m\angle RQS = \frac{1}{2}(122)$

$= \boxed{61°}$

Name: ____

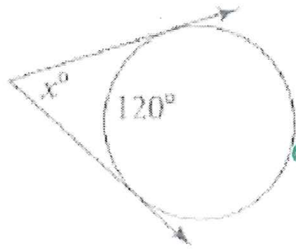
Two Tangents



$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

Example 3:

$360 - 120 = 240$



240

$x = \frac{1}{2}(\text{outside} - \text{inside})$

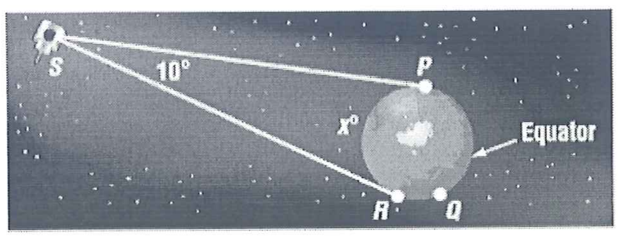
$x = \frac{1}{2}(240 - 120)$

$x = \frac{1}{2}(120)$

$\boxed{x = 60°}$

Example 4:

SATELLITES Suppose a satellite S orbits above Earth rotating so that it appears to hover directly over the equator. Use the figure to determine the arc measure on the equator visible to this satellite.



$10 = \frac{1}{2}(\text{outside} - \text{inside})$

outside $= 360 - x$

inside $= x$

$2 \cdot 10 = \cancel{\frac{1}{2}}(360 - x - x) \cdot \cancel{2}$

$20 = 360 - 2x$

$-340 = -2x$

$\boxed{170° = x}$