Name: _____ Date: _____ Hour: _____

TRIANGLE INEQUALITIES

Objectives: Students will investigate Triangle Inequality Conjecture, the Side-Angle Inequality Conjecture, and the Triangle Exterior Angle Conjecture. Once the conjectures have been established students will use them to answer questions about and solve problems involving triangles.

Can We Make a Triangle?

The Anglegs in each bag are measured and color coded based on the length. The lengths are:

Blue = 12.24 cm   Red = 14.14 cm   Yellow = 10 cm

Purple = 7.07 cm   Green = 8.66 cm   Orange = 5 cm

Take each of the ~~combinations~~ side lengths from above and try to form a triangle. Draw the possible triangles in the space provided below. Label all the sides and measure all of the angles (the drawings should be scaled so that they will all fit into the space below). List the side lengths of any groups that don't form a triangle in the space provided.

Triangles Formed

                                                                Lengths That Didn't
                                                                Form a Triangle

Look at the information above. Come up with a conclusion as to the relationship between the side lengths and whether or not they form triangles.

Conclusion: Sum of any 2 side lengths is greater than the length of the 3rd side

Does the conclusion hold true for all of the combinations?

Yes

Side Lengths and Angle Measures
Directions: Set up a triangle using three different colored anglegs and draw it below. They have lengths on them and label your lengths on your drawing. Measure all angles and label them in your drawing. ~~Make sure~~ ~~you color them the same color as your anglegs.~~

Now look at all of the triangles that were formed. Come up with a conclusion about the relationship between the angles the sides of the triangles.

Conclusion: _opposite the greatest side = greatest angle + vice versa_

Does the conclusion hold true for all of the triangles formed? _Yes_

Exterior Angles vs. Interior Angles, Is There a Relationship?

In the space provided below each person in the group should draw a different scalene triangle ABC. Make sure that each group has at least one acute and one obtuse triangle. Extend side AB past point B and label a point D as the endpoint of the segment. Drawing:
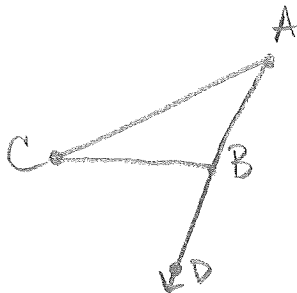
Use the patty paper and trace $\angle A$ and $\angle C$ to show their sum (trace them so they share a vertex and one side also meaning adjacent). Place the patty paper so that the angles you just traced are over the exterior angle $\angle CBD$. What do you notice? _They are the same_

Angles $\angle A$ and $\angle C$ are called the remote interior angles of the exterior angle $\angle CBD$.

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Forming the Conjectures: Triangle Inequalities

Triangle Inequality Conjecture
The sum of the lengths of any two sides of a triangle is _greater than_ the length of the third side.

Side-Angle Inequality Conjecture
In a triangle, if one side is longer than another side, then the angle opposite the longer side is _the greatest angle. The angle opposite the smallest side = smallest angle_

Triangle Exterior Angle Conjecture
The measure of an exterior angle of a triangle is _equal to the sum of the remote interior ∠'s_

Examples

1. Determine whether a triangle can be formed by the given set of sides. If not make a sketch demonstrating why it is not possible.

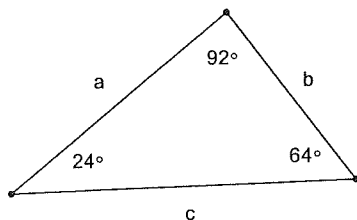   a. 2 in, 4in, 5in   b. 3 cm, 8 cm, 12 cm   c. 8 ft, 12 ft, 20 ft

    yes   no   no   20?

2. If two of the sides of a triangle are 12 and 30, what is the range of possible values for the third side?

Case 1: 12 + 30 are the 2 smaller sides
Case 2: 30 is the largest side

$$\underset{30-12}{\underline{18}} < X < \underset{30+12}{\underline{42}}$$

3. Arrange the unknown measures in order from **least to greatest.**

a.   b.   c.

Triangle a: angles 92°, 24°, 64°; sides a, b, c

Triangle b: sides 13, 9, 11; angles d, e, f

Triangle c: angle 48°, 46°, 86°; sides x, y, z

46 + 86 = 132
180 - 132 = 48

b, a, c   f, d, e   y, z, x
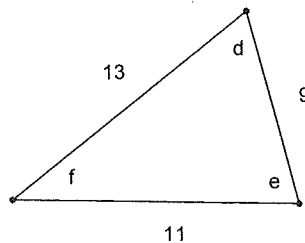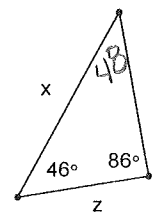
4. Solve for the missing variables in each of the following.

Triangle with sides: 4x-12, 3x-40, 5x+24
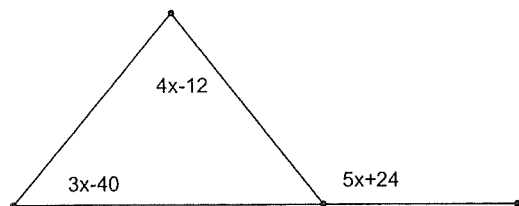
$4x-12 + 3x-40 = 5x+24$
$7x - 52 = 5x + 24$
$-5x + 52 \quad -5x + 52$
$$\frac{2x}{2} = \frac{76}{2} \qquad x = 38$$

Triangle Inequalities: Individual Practice

In Exercises 1 and 2, determine whether it is possible to draw a triangle with sides of the given measures. If it is possible, write yes. If it is not possible, write no and make a sketch demonstrating why it is not possible.
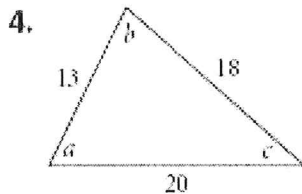
1. 16 cm, 30 cm, 45 cm
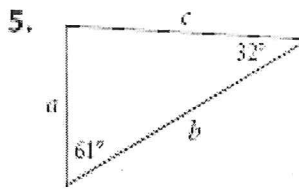
yes

2. 9 km, 17 km, 28 km

no

3. If 17 and 36 are the lengths of two sides of a triangle, what is the range of possible values for the length of the third side?
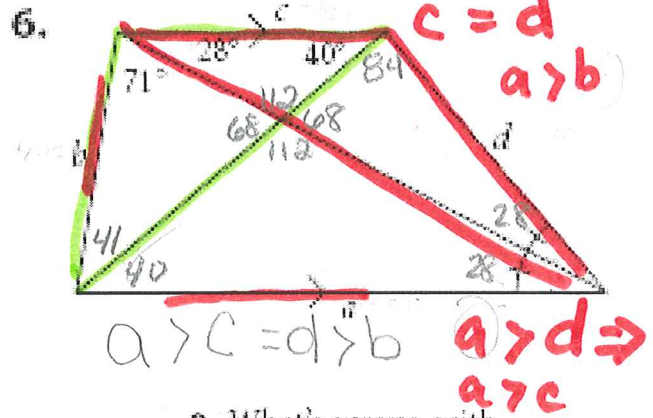
$19 < x < 53$

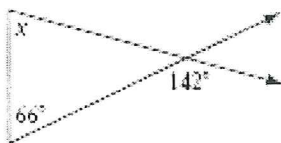In Exercises 4–6, arrange the unknown measures in order from <u>greatest</u> to <u>least</u>.

4.

13, 18, 20, b, a, c

b, a, c

5.

$32°$, $61°$, c, b, a

b, c, a

6.

$c > b$
$c = d$
$a > b$

28°, 40°, 71°, 84, 68, 112, 68, 112, 41, 40, 26, 28

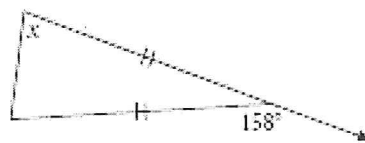$a > c = d > b$ $a > d \Rightarrow$
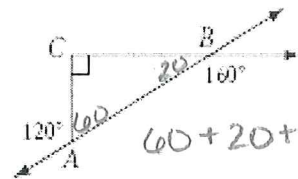$a > c$

7. $x =$ __76__
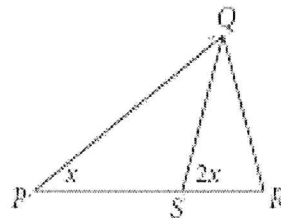
$142 = 66 + x$

8. $x =$ __79__

$2x = 158$

9. What's wrong with this picture?

$60 + 20 + 90 \neq 180$

10. Explain why $\triangle PQS$ is isosceles.

$\angle P = x$
$\angle PQS = x$ b/c Ext. $\angle$ thm
$\angle P + \angle PQS = \angle QSR$
$x + x = 2x$
$\therefore$ base $\angle$'s are $\cong$