

NAME _____ DATE _____ PERIOD _____

13-4 Study Guide and Intervention *(continued)*

Law of Sines

One, Two, or No Solutions

<p>Possible Triangles Given Two Sides and One Opposite Angle</p> <p>Suppose you are given a, b, and A for a triangle. If a is acute:</p> <p>$a < b \sin A \Rightarrow$ no solution $a = b \sin A \Rightarrow$ one solution $b > a > b \sin A \Rightarrow$ two solutions $a > b \Rightarrow$ one solution</p> <p>If A is right or obtuse:</p> <p>$a \leq b \Rightarrow$ no solution $a > b \Rightarrow$ one solution</p>
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Example Determine whether $\triangle ABC$ has no solutions, one solution, or two solutions. Then solve $\triangle ABC$.

- a. $A = 48^\circ$, $a = 11$, and $b = 16$
 Since A is acute, find $b \sin A$ and compare it with a .
 $b \sin A = 16 \sin 48^\circ \approx 11.89$
 Since $11 < 11.89$, there is no solution.
- b. $A = 34^\circ$, $a = 6$, $b = 8$
 Since A is acute, find $b \sin A$ and compare it with a ; $b \sin A = 8 \sin 34^\circ \approx 4.47$. Since $8 > 6 > 4.47$, there are two solutions. Thus there are two possible triangles to solve.

Acute B

First use the Law of Sines to find B .

$$\frac{8}{\sin 34^\circ} = \frac{6}{\sin B}$$

$$\sin B \approx 0.7456$$

$$B \approx 48^\circ$$

The measure of angle C is about $180^\circ - (34^\circ + 48^\circ)$ or about 98° .

Use the Law of Sines again to find c .

$$\frac{\sin 98^\circ}{c} \approx \frac{\sin 34^\circ}{6}$$

$$c \approx \frac{6 \sin 98^\circ}{\sin 34^\circ}$$

$$c \approx 10.6$$

Exercises

Determine whether each triangle has no solutions, one solution, or two solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. $A = 50^\circ$, $a = 34$, $b = 40$ 2. $A = 24^\circ$, $a = 3$, $b = 8$
 two solutions; $B \approx 64^\circ$, $C \approx 66^\circ$ no solutions
 $B \approx 40.5^\circ$; $B \approx 116^\circ$; $C \approx 14^\circ$; $c \approx 10.7$ $c \approx 9.6$

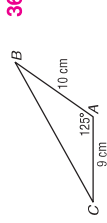
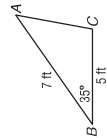
Answers (Lesson 13-4)

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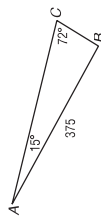
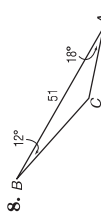
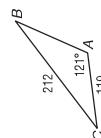
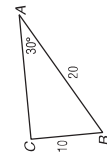
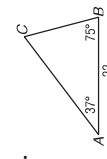
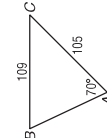
13-4 Skills Practice

Law of Sines

Find the area of $\triangle ABC$ to the nearest tenth.

1.  **36.9 cm²** 2.  **10.0 ft²**
3. $A = 35^\circ$, $b = 3$ ft, $c = 7$ ft, $a = 10$ cm, $b = 7$ cm **18.5 cm²**
 4. $C = 148^\circ$, $a = 10$ cm, $b = 7$ cm **18.5 cm²**
 5. $C = 22^\circ$, $a = 14$ m, $b = 8$ m **21.0 m²** 6. $B = 93^\circ$, $c = 18$ mi, $a = 42$ mi **377.5 mi²**

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

7.  **$B = 93^\circ$, $a \approx 102.1$, $b \approx 393.8$**
8.  **$C = 150^\circ$, $a \approx 31.5$, $b \approx 21.2$**
9.  **$B \approx 29^\circ$, $C \approx 30^\circ$, $c \approx 124.6$**
10.  **$B = 60^\circ$, $C = 90^\circ$, $b \approx 17.3$**
11.  **$C = 68^\circ$, $a \approx 14.3$, $b \approx 22.9$**
12.  **$B \approx 65^\circ$, $C \approx 45^\circ$, $c \approx 82.2$**

Determine whether each triangle has no solution, one solution, or two solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

13. $A = 30^\circ$, $a = 1$, $b = 4$ **no solution**
 14. $A = 30^\circ$, $a = 2$, $b = 4$ **one solution; $B = 90^\circ$, $C = 60^\circ$, $c \approx 3.5$**
15. $A = 30^\circ$, $a = 3$, $b = 4$ **two solutions; $B \approx 42^\circ$, $C \approx 108^\circ$, $c \approx 5.7$; $B \approx 138^\circ$, $C \approx 12^\circ$, $c \approx 1.2$**
16. $A = 38^\circ$, $a = 10$, $b = 9$ **one solution; $B \approx 34^\circ$, $C \approx 108^\circ$, $c \approx 15.4$**
17. $A = 78^\circ$, $a = 8$, $b = 5$ **one solution; $B \approx 38^\circ$, $C \approx 64^\circ$, $c \approx 7.4$**
18. $A = 133^\circ$, $a = 9$, $b = 7$ **one solution; $B \approx 35^\circ$, $C \approx 12^\circ$, $c \approx 2.6$**
19. $A = 127^\circ$, $a = 2$, $b = 6$ **no solution**
 20. $A = 109^\circ$, $a = 24$, $b = 13$ **one solution; $B \approx 31^\circ$, $C \approx 40^\circ$, $c \approx 16.4$**

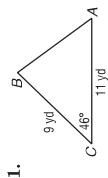
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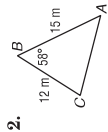
13-4 Practice

Law of Sines

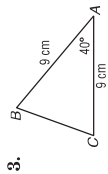
Find the area of $\triangle ABC$ to the nearest tenth.



35.6 yd²
29.7 m²



76.3 m²



26.0 cm²

4. $C = 32^\circ, a = 12.6 \text{ m}, b = 8.9 \text{ m}$

62.9 cm²

7. $A = 17.4^\circ, b = 12 \text{ km}, c = 14 \text{ km}$

25.1 km²

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

8. $A = 50^\circ, B = 30^\circ, c = 9$

$C = 100^\circ, a \approx 7.0, b \approx 4.6$

10. $A = 80^\circ, C = 14^\circ, a = 40$

$B = 86^\circ, b \approx 40.5, c \approx 9.8$

12. $A = 72^\circ, a = 8, c = 6$

$B = 62^\circ, c \approx 46^\circ, b \approx 7.5$

Determine whether each triangle has *no solution*, *one solution*, or *two solutions*. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

14. $A = 29^\circ, a = 6, b = 13$ **no solution**

16. $A = 113^\circ, a = 21, b = 25$ **no solution**

18. $A = 66^\circ, a = 12, b = 7$ **one solution;**

$B \approx 32^\circ, C \approx 82^\circ, c \approx 13.0$

20. $A = 45^\circ, a = 15, b = 18$ **two solutions;**

$B \approx 58^\circ, C \approx 77^\circ, c \approx 20.7;$
 $B \approx 122^\circ, C \approx 13^\circ, c \approx 4.8$

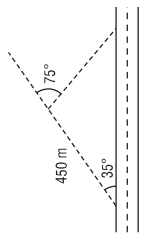
22. **WILDLIFE** Sarah Phillips, an officer for the Department of Fisheries and Wildlife, checks boaters on a lake to make sure they do not disturb two osprey nesting sites. She leaves a dock and heads due north in her boat to the first nesting site. From here, she turns 5° north of due west and travels an additional 2.14 miles to the second nesting site. She then travels 6.7 miles directly back to the dock. How far from the dock is the first osprey nesting site? Round to the nearest tenth. **6.2 mi**

13-4 Word Problem Practice

Law of Sines

WALKING For Exercises 1 and 2, use the following information.

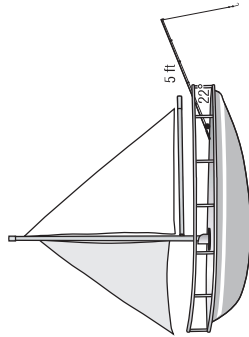
Alliya is taking a walk along a straight road. She decides to leave the road, so she walks on a path that makes an angle of 35° with the road. After walking for 450 meters, she turns 75° and heads back towards the road.



1. How far does Aliya need to walk on her current path to get back to the road?
402 m

2. When Aliya returns to the road, how far along the road is she from where she started?
676 m

4. **FISHING** A fishing pole is resting against the railing of a boat making an angle of 22° with the boat's deck. The fishing pole is 5 feet long, and the hook hangs 3 feet from the tip of the pole. The movement of the boat causes the hook to sway back and forth. Determine which angles the fishing line must make with the pole in order for the hook to be level with the boat's deck. **119.4° or 16.6°**



CAMERAS For Exercises 5 and 6, use the following information.

A security camera is located on top of a building at a certain distance from the sidewalk. The camera revolves counterclockwise at a steady rate of one revolution per minute. At one point in the revolution it directly faces a point on the sidewalk that is 20 meters from the camera. 4 seconds later, it directly faces a point 10 meters down the sidewalk.

5. How many degrees does the camera rotate in 4 seconds? **24°**

6. To the nearest tenth of a meter, how far is the security camera from the sidewalk? **19.6 m**

3. **ROCK CLIMBING** A rock climber is part of the way up a climb when he can see both the peak and the base of the Gray Mountain. When viewing the peak of the mountain, his angle of elevation is 42° . When viewing the base of the mountain, his angle of depression is 36° . If he knows the Gray Mountain is 2000 feet high and the base of the mountain is at sea level, then what is the elevation of the climber to the nearest foot? **893 ft**