

7-1 Study Guide and Intervention

Proportions

** On each page only do the circled questions + Read through the examples that say, read! ;)*

Write Ratios A **ratio** is a comparison of two quantities. The ratio a to b , where b is not zero, can be written as $\frac{a}{b}$ or $a:b$. The ratio of two quantities is sometimes called a **scale factor**. For a scale factor, the units for each quantity are the same.

Read **Example 1** In 2002, the Chicago Cubs baseball team won 67 games out of 162. Write a ratio for the number of games won to the total number of games played.

To find the ratio, divide the number of games won by the total number of games played. The result is $\frac{67}{162}$, which is about 0.41. The Chicago Cubs won about 41% of their games in 2002.

Read **Example 2** A doll house that is 15 inches tall is a scale model of a real house with a height of 20 feet. What is the ratio of the height of the doll house to the height of the real house?


To start, convert the height of the real house to inches.

$$20 \text{ feet} \times 12 \text{ inches per foot} = 240 \text{ inches}$$

To find the ratio or scale factor of the heights, divide the height of the doll house by the height of the real house. The ratio is 15 inches:240 inches or 1:16. The height of the doll house is $\frac{1}{16}$ the height of the real house.

Exercises

- In the 2005 Major League baseball season, Alex Rodriguez hit 48 home runs and was at bat 605 times. Find the ratio of home runs to the number of times he was at bat.
- There are 182 girls in the sophomore class of 305 students. Find the ratio of girls to total students.
- The length of a rectangle is 8 inches and its width is 5 inches. Find the ratio of length to width.
- The sides of a triangle are 3 inches, 4 inches, and 5 inches. Find the scale factor between the longest and the shortest sides.


- The length of a model train is 18 inches. It is a scale model of a train that is 48 feet long. Find the scale factor.

7-1 Study Guide and Intervention *(continued)*

Proportions

Use Properties of Proportions A statement that two ratios are equal is called a **proportion**. In the proportion $\frac{a}{b} = \frac{c}{d}$, where b and d are not zero, the values a and d are the **extremes** and the values b and c are the **means**. In a proportion, the product of the means is equal to the product of the extremes, so $ad = bc$.

$$\frac{a}{b} = \frac{c}{d}$$

$$a \cdot d = b \cdot c$$

\uparrow \uparrow
 extremes means

Example 1 Solve $\frac{9}{16} = \frac{27}{x}$.

$$\frac{9}{16} = \frac{27}{x}$$

$$9 \cdot x = 16 \cdot 27 \quad \text{Cross products}$$

$$9x = 432 \quad \text{Multiply.}$$

$$x = 48 \quad \text{Divide each side by 9.}$$

Example 2 A room is 49 centimeters by 28 centimeters on a scale drawing of a house. For the actual room, the larger dimension is 14 feet. Find the shorter dimension of the actual room.

If x is the room's shorter dimension, then

$$\frac{28}{49} = \frac{x}{14} \quad \begin{array}{l} \text{shorter dimension} \\ \text{longer dimension} \end{array}$$

$$49x = 392 \quad \text{Cross products}$$

$$x = 8 \quad \text{Divide each side by 49.}$$

The shorter side of the room is 8 feet.

Exercises

Solve each proportion.

1. $\frac{1}{2} = \frac{28}{x}$

2. $\frac{3}{8} = \frac{y}{24}$

3. $\frac{x + 22}{x + 2} = \frac{30}{10}$

4. $\frac{3}{18.2} = \frac{9}{y}$

5. $\frac{2x + 3}{8} = \frac{5}{4}$

6. $\frac{x + 1}{x - 1} = \frac{3}{4}$

Use a proportion to solve each problem.

7. If 3 cassettes cost \$44.85, find the cost of one cassette.
8. The ratio of the sides of a triangle are 8:15:17. If the perimeter of the triangle is 480 inches, find the length of each side of the triangle.
9. The scale on a map indicates that one inch equals 4 miles. If two towns are 3.5 inches apart on the map, what is the actual distance between the towns?

7-2 Study Guide and Intervention

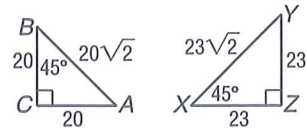
Similar Polygons

Identify Similar Figures

Read!

Example 1 Determine whether the triangles are similar.

Two polygons are similar if and only if their corresponding angles are congruent and their corresponding sides are proportional.



$\angle C \cong \angle Z$ because they are right angles, and $\angle B \cong \angle X$.
By the Third Angle Theorem, $\angle A \cong \angle Y$.

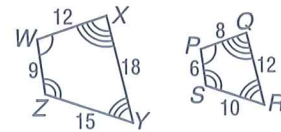
For the sides, $\frac{BC}{XZ} = \frac{20}{23}$, $\frac{BA}{XY} = \frac{20\sqrt{2}}{23\sqrt{2}} = \frac{20}{23}$, and $\frac{AC}{YZ} = \frac{20}{23}$.

The side lengths are proportional. So $\triangle BCA \sim \triangle XZY$.

Read!

Example 2 Is polygon $WXYZ \sim$ polygon $PQRS$?

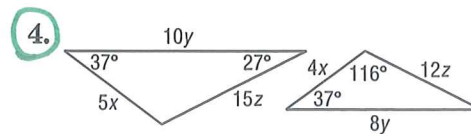
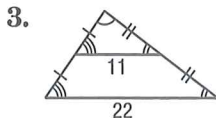
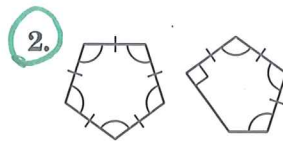
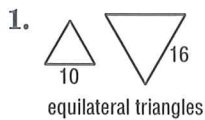
For the sides, $\frac{WX}{PQ} = \frac{12}{8} = \frac{3}{2}$, $\frac{XY}{QR} = \frac{18}{12} = \frac{3}{2}$, $\frac{YZ}{RS} = \frac{15}{10} = \frac{3}{2}$,
and $\frac{ZW}{SP} = \frac{9}{6} = \frac{3}{2}$. So corresponding sides are proportional.



Also, $\angle W \cong \angle P$, $\angle X \cong \angle Q$, $\angle Y \cong \angle R$, and $\angle Z \cong \angle S$, so corresponding angles are congruent. We can conclude that polygon $WXYZ \sim$ polygon $PQRS$.

Exercises

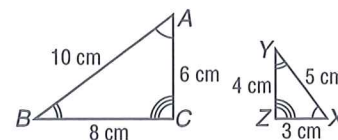
Determine whether each pair of figures is similar. If they are similar, give the ratio of corresponding sides.



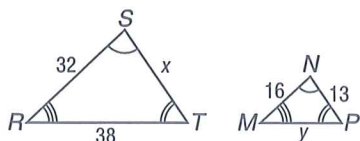
7-2 Study Guide and Intervention *(continued)*

Similar Polygons

Scale Factors When two polygons are similar, the ratio of the lengths of corresponding sides is called the **scale factor**. At the right, $\triangle ABC \sim \triangle XYZ$. The scale factor of $\triangle ABC$ to $\triangle XYZ$ is 2 and the scale factor of $\triangle XYZ$ to $\triangle ABC$ is $\frac{1}{2}$.



Example 1 The two polygons are similar. Find x and y .



Use the congruent angles to write the corresponding vertices in order.

$$\triangle RST \sim \triangle MNP$$

Write proportions to find x and y .

$$\frac{32}{16} = \frac{x}{13}$$

$$\frac{38}{y} = \frac{32}{16}$$

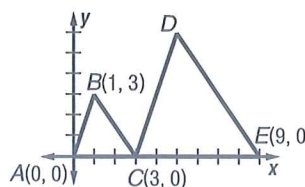
$$16x = 32(13)$$

$$32y = 38(16)$$

$$x = 26$$

$$y = 19$$

Example 2 $\triangle ABC \sim \triangle CDE$. Find the scale factor and find the lengths of \overline{CD} and \overline{DE} .



$AC = 3 - 0 = 3$ and $CE = 9 - 3 = 6$. The scale factor of $\triangle CDE$ to $\triangle ABC$ is $6:3$ or $2:1$.

Using the Distance Formula,

$$AB = \sqrt{1^2 + 9} = \sqrt{10} \text{ and}$$

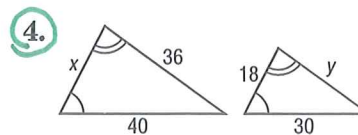
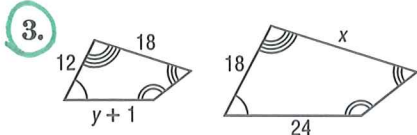
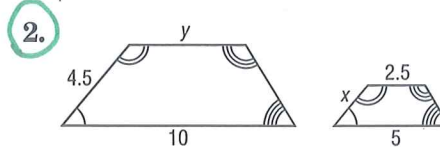
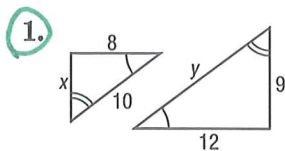
$BC = \sqrt{4^2 + 9} = \sqrt{13}$. The lengths of the sides of $\triangle CDE$ are twice those of $\triangle ABC$,

$$\text{so } DC = 2(BA) \text{ or } 2\sqrt{10} \text{ and}$$

$$DE = 2(BC) \text{ or } 2\sqrt{13}.$$

Exercises

Each pair of polygons is similar. Find x and y .



5. In Example 2 above, point D has coordinates $(5, 6)$. Use the Distance Formula to verify the lengths of \overline{CD} and \overline{DE} .

Try!

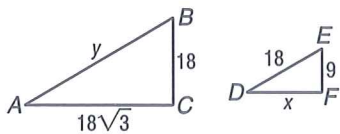
7-3 Study Guide and Intervention *(continued)*

Similar Triangles

Use Similar Triangles Similar triangles can be used to find measurements.

Example 1 $\triangle ABC \sim \triangle DEF$.

Find x and y .



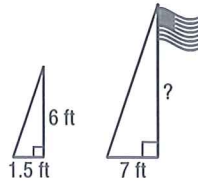
$$\frac{AC}{DF} = \frac{BC}{EF} \qquad \frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{18\sqrt{3}}{x} = \frac{18}{9} \qquad \frac{y}{18} = \frac{18}{9}$$

$$18x = 9(18\sqrt{3}) \qquad 9y = 324$$

$$x = 9\sqrt{3} \qquad y = 36$$

Example 2 A person 6 feet tall casts a 1.5-foot-long shadow at the same time that a flagpole casts a 7-foot-long shadow. How tall is the flagpole?

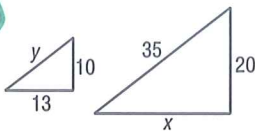


The sun's rays form similar triangles. Using x for the height of the pole, $\frac{6}{x} = \frac{1.5}{7}$, so $1.5x = 42$ and $x = 28$. The flagpole is 28 feet tall.

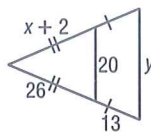
Exercises

Each pair of triangles is similar. Find x and y .

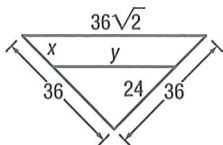
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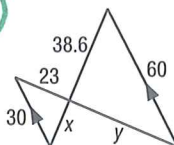
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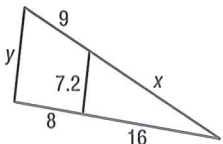
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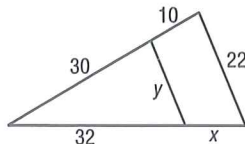
4.



5.



6.



7. The heights of two vertical posts are 2 meters and 0.45 meter. When the shorter post casts a shadow that is 0.85 meter long, what is the length of the longer post's shadow to the nearest hundredth?