7-1 Proportions

Main Ideas

- Write ratios.
- Use properties of proportions.

BUILD YOUR VOCABULARY (pages 168–169)

A **ratio** is a comparison of two quantities. The ratio of a to b can be expressed $\frac{a}{b}$, where b is not zero.

An equation stating that two ratios are **equal** is called a **proportion**.

EXAMPLE Write a Ratio

1. **The total number of students who participate in sports programs at Central High School is 520. The total number of students in the school is 1850. Find the athlete-to-student ratio to the nearest tenth.**

To find this ratio, divide the number of athletes by the total number of students.

$$\frac{\text{number of athletes}}{\text{total number of students}} = \frac{520}{1850} \text{ or about } 0.3$$

The athlete-to-student ratio is 0.3 athletes for each student in the school.

Check Your Progress The country with the longest school year is China with 251 days. Find the ratio of school days to total days in a year for China to the nearest tenth. (Use 365 as the number of days in a year.)

0.7

EXAMPLE Solve Proportions by Using Cross Products **(parallels Example 3 in text)**

2. **Solve each proportion.**

a. $\frac{6}{18.2} = \frac{9}{y}$.

$\frac{6}{18.2} = \frac{9}{y}$ Original proportion

$6y = 163.8$ Cross products

$y = 27.3$ Divide each side by 6.

KEY CONCEPT

Property of Proportions
For any numbers a and c and any nonzero numbers b and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

b. $\frac{4x-5}{3} = \frac{-26}{6}$

$\frac{4x-5}{3} = \frac{-26}{6}$	Original proportion
$6(4x-5) = 3(-26)$	Cross multiply.
$24x - 30 = -78$	Multiply.
$24x = -48$	Add 30 to each side.
$x =$ −2	Divide.

Check Your Progress **Solve each proportion.**

a. $\frac{13.5}{42} = \frac{b}{14}$ 4.5

b. $\frac{7n-1}{8} = \frac{15.5}{2}$ 9

FOLDABLES

ORGANIZE IT

As you skim the lesson, write down questions that you have on the section for Lesson 7-1. Then write the answers next to each question.

○ 7-1	7-2
○ 7-3	7-4
○ 7-5	Vocabulary

EXAMPLE Solve Problems Using Proportions (parallels Example 4 in text)

3 TRAINS **A boxcar on a train has a length of 40 feet and a width of 9 feet. A scale model is made with a length of 16 inches. Find the width of the model.**

Write and solve a proportion.

$$\frac{\text{boxcar's length (ft)}}{\text{model's length (in.)}} = \frac{\text{boxcar's width (ft)}}{\text{model's width (in.)}}$$

$\frac{40}{16} = \frac{9}{x}$	Substitution
$40x = 16(9)$	Cross products
$40x = 144$	Multiply.
$x =$ 3.6	Divide each side by 40.

The width of the model is 3.6 inches.

Check Your Progress Two large cylindrical containers are in proportion. The height of the larger container is 25 meters with a diameter of 8 meters. The height of the smaller container is 7 meters. Find the diameter of the smaller container.

2.24 m

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7-2 Similar Polygons

Main Ideas

- Identify similar figures.
- Solve problems involving scale factors.

BUILD YOUR VOCABULARY (page 169)

When polygons have the same shape but may be different in **size**, they are called **similar polygons**.

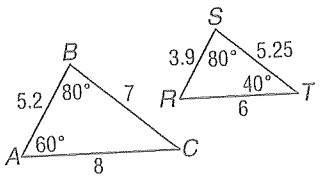
When you compare the lengths of **corresponding** sides of similar figures, you usually get a numerical ratio. This ratio is called the **scale factor** for the two figures.

KEY CONCEPT

Similar polygons Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.

EXAMPLE Similar Polygons

1 **Determine whether the pair of figures is similar. Justify your answer.**



Since $m\angle B = m\angle S$, **$\angle B \cong \angle S$**.

The $m\angle C = 40$ and $m\angle R = 60$.

So, $\angle C \cong \angle T$ and **$\angle A \cong \angle R$**.

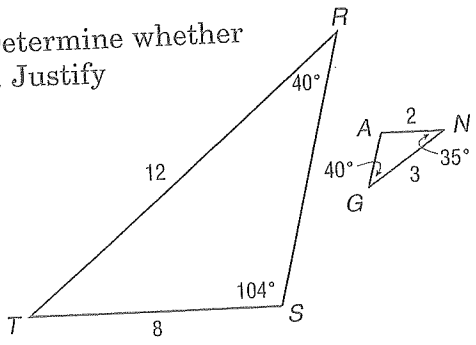
Thus, all the corresponding angles are congruent.

Now determine whether corresponding sides are proportional.

$\frac{AC}{RT} = \frac{8}{6}$ or $1.\overline{3}$ $\quad \frac{AB}{RS} = \frac{\textbf{5.2}}{\textbf{3.9}}$ or $1.\overline{3}$ $\quad \frac{BC}{ST} = \frac{\textbf{7}}{\textbf{5.25}}$ or $1.\overline{3}$

The ratio of the measures of the corresponding sides are equal and the corresponding angles are **congruent**, so $\triangle ABC \sim \triangle RST$.

Check Your Progress Determine whether the pair of figures is similar. Justify your answer.



Only one pair of angles are congruent, so the triangles are not similar.

EXAMPLE Scale Factor

2 ARCHITECTURE **An architect prepared a 12-inch model of a skyscraper to look like an actual 1100-foot building. What is the scale factor of the model compared to the actual building?**

Before finding the scale factor you must make sure that both measurements use the same unit of measure.

1100(12) = 13,200 inches

$$\frac{\text{height of model}}{\text{height of actual building}} = \frac{\boxed{12 \text{ inches}}}{\boxed{13{,}200 \text{ inches}}}$$

$$= \boxed{\frac{1}{1100}}$$

The ratio comparing the two heights is $\boxed{\frac{1}{1100}}$ or $\boxed{1} : \boxed{1100}$. The scale factor is $\boxed{\frac{1}{1100}}$, which means that the model is $\boxed{\frac{1}{1100}}$ the height of the actual skyscraper.

WRITE IT

Explain why two congruent polygons must also be similar.

Check Your Progress A space shuttle is about 122 feet in length. The Science Club plans to make a model of the space shuttle with a length of 24 inches. What is the scale factor of the model compared to the real space shuttle?

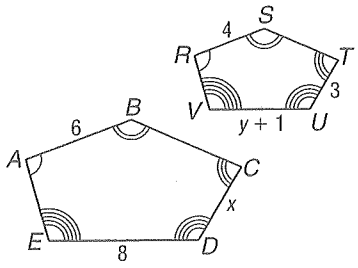
$\boxed{\frac{1}{61}}$

EXAMPLE Proportional Parts and Scale Factor

3 **The two polygons are similar.**

a. Write a similarity statement. Then find x, y, and UV.

Use the congruent angles to write the corresponding vertices in order.



polygon $\boxed{ABCDE}$ ~ polygon $\boxed{RSTUV}$

Now write propotions to find x and y.

To find x:

$\frac{AB}{RS} = \frac{CD}{TU}$ Similarity proportion

$\frac{\boxed{6}}{4} = \frac{\boxed{x}}{3}$ $AB = 6$, $RS = 4$, $CD = x$, $TU = 3$

$18 = 4x$ Cross products

$\boxed{\frac{9}{2}} = x$ Divide each side by 4.

To find y:

$\frac{AB}{RS} = \frac{DE}{UV}$ Similarity proportion

$\frac{6}{\boxed{4}} = \frac{8}{\boxed{y + 1}}$ $AB = 6$, $RS = 4$, $DE = 8$, $UV = y + 1$

$6y + 6 = 32$ Cross products

$6y = 26$ Subtract 6 from each side.

$y = \boxed{\frac{13}{3}}$ Divide each side by 6 and simplify.

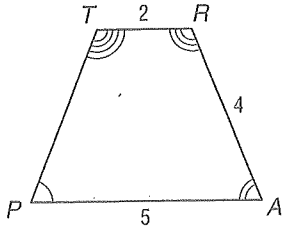
$UV = y + 1$, so $UV = \boxed{\frac{13}{3}} + 1$ or $\boxed{\frac{16}{3}}$.

b. Find the scale factor of polygon *ABCDE* to polygon *RSTUV*.

The scale factor is the ratio of the lengths of any two corresponding sides.

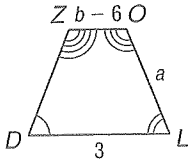
$\frac{AB}{RS} = \frac{\boxed{6}}{4}$ or $\frac{3}{2}$

Check Your Progress **The two polygons are similar.**



a. Write a similarity statement. Then find a, b, and ZO.

TRAP ~ *ZOLD*, $a = 2.4$; $b = 7.2$; $ZO = 1.2$



b. Find the scale factor of polygon *TRAP* to polygon *ZOLD*.

$\frac{5}{3}$

FOLDABLES

ORGANIZE IT

Write a description of the information you would include in a diagram of two polygons to enable a friend to decide that the polygons are similar. Record your description on the section for Lesson 7-2.

○ 7-1	7-2
○ 7-3	7-4
○ 7-5	Vocabulary

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7-3 Similar Triangles

Main Ideas

- Identify similar triangles.
- Use similar triangles to solve problems.

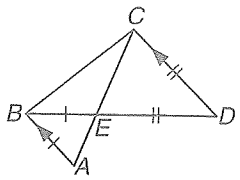
Postulate 7.1 Angle-Angle (AA) Similarity
If the two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Theorem 7.1 Side-Side-Side (SSS) Similarity
If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

Theorem 7.2 Side-Angle-Side (SAS) Similarity
If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

EXAMPLE Determine Whether Triangles are Similar

1 **In the figure, $\overline{AB} \parallel \overline{DC}$, $BE = 27$, $DE = 45$, $AE = 21$, and $CE = 35$. Determine which triangles in the figure are similar.**



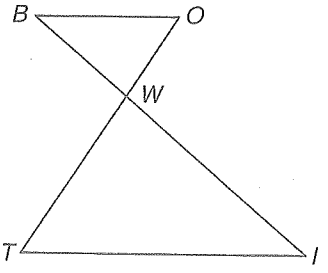
Since $\overline{AB} \parallel \overline{DC}$, $\angle BAC \cong$ $\angle DCE$ by the Alternate Interior Angles Theorem.

Vertical angles are congruent, so $\angle BAE \cong \angle DEC$.

Therefore, by the AA Similarity Theorem, $\triangle ABE \sim \triangle CDE$.

Check Your Progress In the figure, $OW = 7$, $BW = 9$, $WT = 17.5$, and $WI = 22.5$. Determine which triangles in the figure are similar.

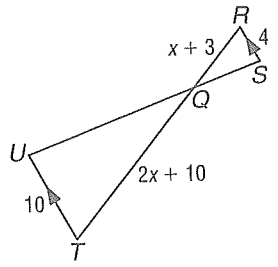
$\triangle BOW \sim \triangle ITW$



Theorem 7.3
Similarity of triangles is reflexive, symmeteric, and transitive.

EXAMPLE Parts of Similar Triangles

1 ALGEBRA Given $\overline{RS} \parallel \overline{UT}$, $RS = 4$, $RQ = x + 3$, $QT = 2x + 10$, $UT = 10$, find RQ and QT.



Since $\overline{RS} \parallel \overline{UT}$, $\angle SRQ \cong$ $\angle UTQ$ and $\angle RSQ$ $\cong TUQ$ because they are alternate interior angles. By AA Similarity, $\triangle RSQ$ $\cong$ $\triangle TUQ$. Using the definition of similar polygons, $\frac{RS}{TU} = \frac{RQ}{TQ}$.

$\frac{4}{10} = \frac{x + 3}{2x + 10}$	Substitution
$4(2x + 10) = 10(x + 3)$	Cross products
$8x + 40 = 10x + 30$	Distributive Property
$5 = x$	Simplify.

Now find RQ and QT.

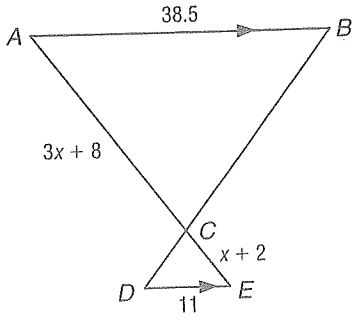
$RQ = x + 3$

$= 5 + 3$ or 8

$QT = 2x + 10$

$= 2(5) + 10$ or 20

Check Your Progress Given $\overline{AB} \parallel \overline{DE}$, $AB = 38.5$, $DE = 11$, $AC = 3x + 8$, and $CE = x + 2$, find AC and CE.



$AC = 14$; $CE = 4$

FOLDABLES

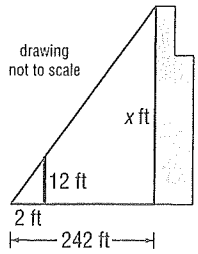
ORGANIZE IT

Write a short paragraph to describe how you could apply the postulate and theorems in this lesson to help you construct similar triangles. Include your paragraph on the section for Lesson 7-3.

○ 7-1	7-2
○ 7-3	7-4
○ 7-5	Vocabulary

EXAMPLE Find a Measurement

5 INDIRECT MEASUREMENT **Josh wanted to measure the height of the Sears Tower in Chicago. He used a 12-foot light pole and measured its shadow at 1 P.M. The length of the shadow was 2 feet. Then he measured the length of the Sears Tower's shadow and it was 242 feet at the time. What is the height of the Sears Tower?**



Assuming that the sun's rays form similar triangles, the following proportion can be written.

$$\frac{\text{height of Sears Tower (ft)}}{\text{Sears Tower shadow length (ft)}} = \frac{\text{height of the light pole (ft)}}{\text{light pole shadow length (ft)}}$$

Now substitute the known values and let x be the height of the Sears Tower.

$$\frac{x}{242} = \frac{12}{2}$$ Substitution

$$x \cdot 2 = 242(12)$$ Cross products

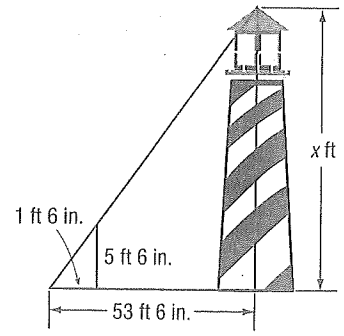
$$x = 1452$$ Simplify and divide each side by 2.

The Sears Tower is 1452 feet tall.

REMEMBER IT

Shadows and similar triangles are commonly used for indirectly measuring the heights of objects that are otherwise too tall to measure.

Check Your Progress On her trip along the East coast, Jennie stops to look at the tallest lighthouse in the U.S. located at Cape Hatteras, North Carolina. Jennie measures her shadow to be 1 feet 6 inches in length and the length of the shadow of the lighthouse to be 53 feet 6 inches. Jennie's height is 5 feet 6 inches. What is the height of the Cape Hatteras lighthouse to the nearest foot?



196 ft

HOMEWORK ASSIGNMENT

Page(s):

Exercises: