Chapter 13

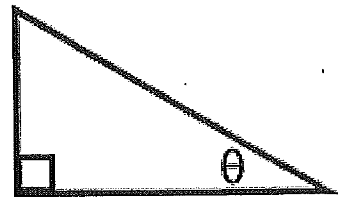
(Algebra 2 Book)

Trig Functions Unit

		textbook reference	date learned	end of unit: heard this before	end of unit: can do	end of unit: can teach
A	Solve problems involving side length of 30-60-90 and 45-45-90 degree triangles.	13-1				
B	Know the sine, cosine and tangent ratios. Use these ratios to solve problems.	13-1				
C	Understand that cosecant, secant and cotangent are the inverses of sine, cosine and tangent respectively.	13-1				
D	Identify sine and cosine values on the unit circle for multiples of 30 and 45 degrees.	various sections				
E	Identify co-terminal angles and use as a reference.	13-2				
F	Understand radian measure of an angle as the length of the arc on the unit circle.	13-2				
G	Translate between degree and radian measure.	13-2				
H	Extend the unit circle to solve trigonometry problems for all real numbers.	13-3				

13.1 Right Triangle Trigonometry Notes

Using these sides, you can define six **trigonometric functions: sine, cosine, tangent, cosecant, secant,** and **cotangent**. These functions are abbreviated sin, cos, tan, csc, sec, and cot, respectively.



KEY CONCEPT ***Trigonometric Functions***

If θ is the measure of an acute angle of a right triangle, *opp* is the measure of the leg opposite θ, *adj* is the measure of the leg adjacent to θ, and *hyp* is the measure of the hypotenuse, then the following are true.

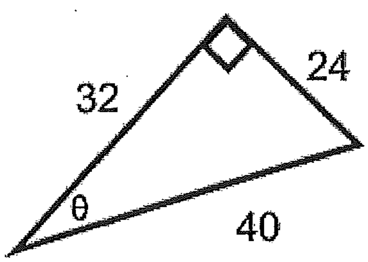
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

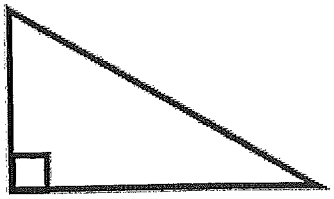
Notice that the sine, cosine, and tangent functions are reciprocals of the cosecant, secant, and cotangent functions, respectively. Thus, the following are also true.

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

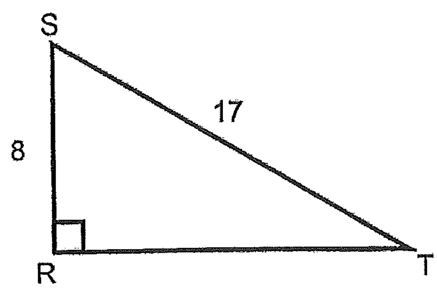
__Ex1__ Find the 6 trigonometric ratios.



Ex 2 If tan A = $\frac{\sqrt{2}}{5}$, then find csc A.

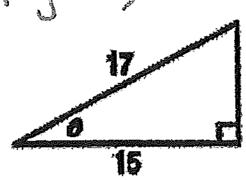


Ex 3 Solve ΔRST. Round measures of sides to nearest tenth and angle measures to nearest degree.



Show your work on a separate paper. (or next page ☺)

1. Find the values of the six trigonometric functions for angle θ.



2. Standardized Test Practice

If $\sin A = \frac{7}{10}$, find the value of $\cos A$.

A. $\frac{7\sqrt{149}}{149}$ **B.** $\frac{\sqrt{51}}{10}$ **C.** $\frac{10}{7}$ **D.** $\frac{\sqrt{51}}{7}$

3. Solve $\triangle ABC$ if $A = 20°$, $C = 90°$, and $b = 10$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

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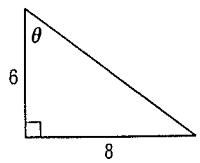
13-1 Skills Practice

Right Triangle Trigonometry

Find the values of the six trigonometric functions for angle θ.

1.



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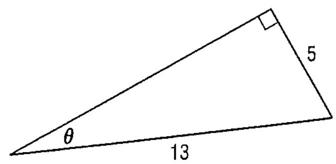

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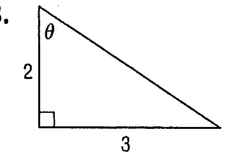
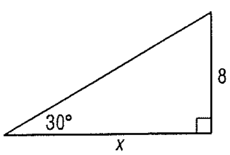


Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

No Laws for RT △s!

4.



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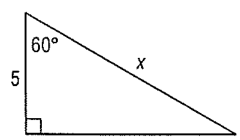

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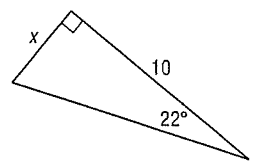

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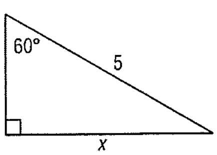

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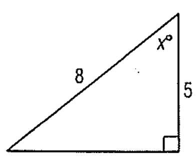

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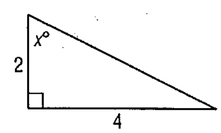
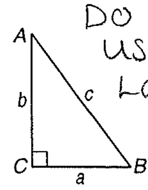


Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



Do Not use any Laws for RT △s

10. $A = 72°, c = 10$

11. $B = 20°, b = 15$

12. $A = 80°, a = 9$

13. $A = 58°, b = 12$

14. $b = 4, c = 9$

15. $a = 7, b = 5$

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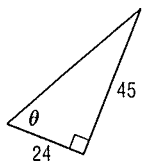
13-1 Practice

Right Triangle Trigonometry

Lesson 13-1

Find the values of the six trigonometric functions for angle θ.

1.



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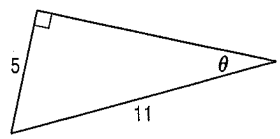

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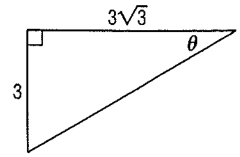
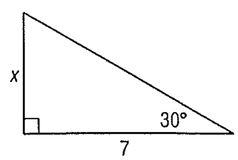


NO Laws for RT Δs

Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4.



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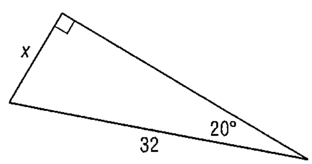

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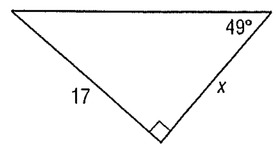

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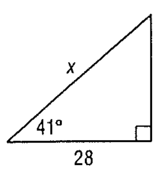

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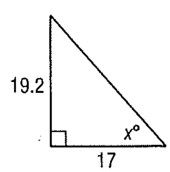

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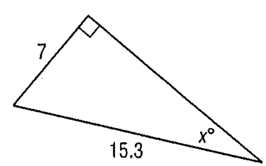
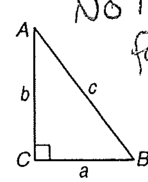


Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



No laws for RT Δs

10. $A = 35°, a = 12$

11. $B = 71°, b = 25$

12. $B = 36°, c = 8$

13. $a = 4, b = 7$

14. $A = 17°, c = 3.2$

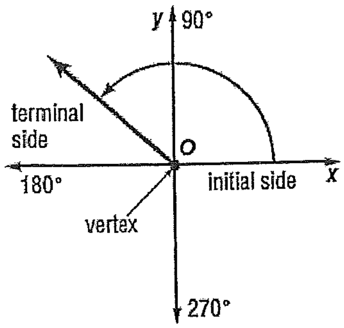
15. $b = 52, c = 95$

16. SURVEYING John stands 150 meters from a water tower and sights the top at an angle of elevation of 36°. How tall is the tower? Round to the nearest meter.

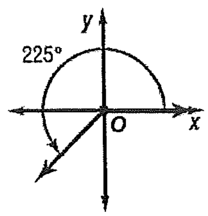
13-2/13-3 Angles, Reference Angles & Radians Notes

Angle of Rotation
In trigonometry, an angle is sometimes referred to as an *angle of rotation.*

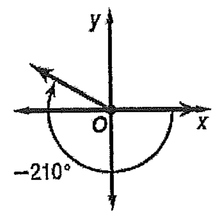
Remember: When sketching an angle, always start at the positive x-axis.
The positive x-axis represents ________° or ________°.

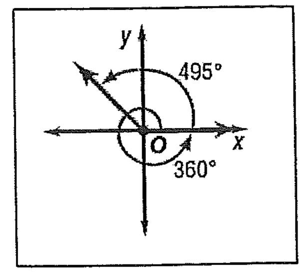


Positive Angle Measure
counterclockwise



Negative Angle Measure
clockwise

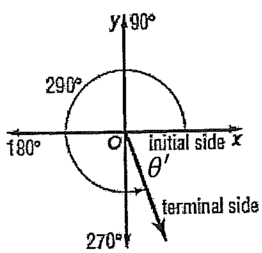




A **reference angle** is the acute angle formed by the terminal side and the x-axis. (denoted by θ')

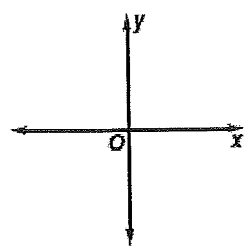
Examples: Sketch each angle. Then find its reference angle.

1. 290°

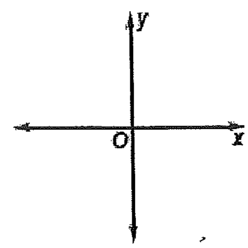


$\theta'=70°$

2. 135°

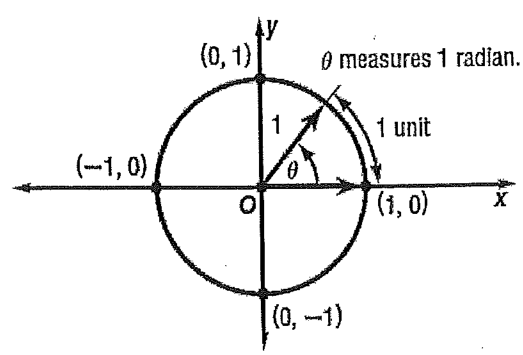


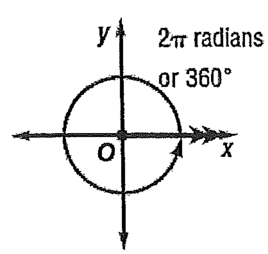
3. -40°



Radians

The definition of a radian is based on the unit circle, a circle of radius 1 which centers at the origin.
One radian is the measure of angle θ in standard position whose rays intercept and arc of length 1 unit.





Circle Review: The circumference of any circle is ________________, where r is the radius.
The circumference of the unit circle would be ______________, so 360° = _________ Radians 180° = _________Radians.

Conversions

Radian Measure The word *radian* is usually omitted when angles are expressed in radian measure. Thus, when no units are given for an angle measure, radian measure is implied.

Converting degrees to radians:

$$R=D\left(\frac{\pi}{180°}\right)$$

Converting radians to degrees:

$$D=R\left(\frac{180}{\pi}\right)$$

Examples:
Rewrite the degree measures in radians and the radian measure in degrees.

1. 60°

$$R = 60° \cdot \frac{\pi}{180°} = \frac{60\pi}{180} = \frac{\pi}{3}$$

2. 45°

3. $-\frac{7\pi}{4}$

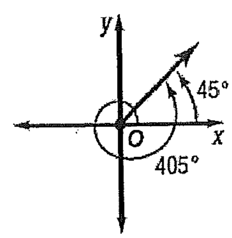
$$D = -\frac{7\pi}{4} \cdot \frac{180}{\pi} = -\frac{1260}{4} = -315°$$

4. $\frac{5\pi}{3}$

Coterminal Angles: The graph shows a 405° angle and a 45° angle. They both share the same terminal side. When two angles in standard position have the same terminal sides, they are called co terminal angles.

405°- 360°=45°

In degrees, you add/subtract 360
In radians, you would add/subtract 2π



Examples: *Find one angle with positive measure and one angle with negative measure coterminal with each angle.*

1. 240°

2. $\frac{9\pi}{4}$

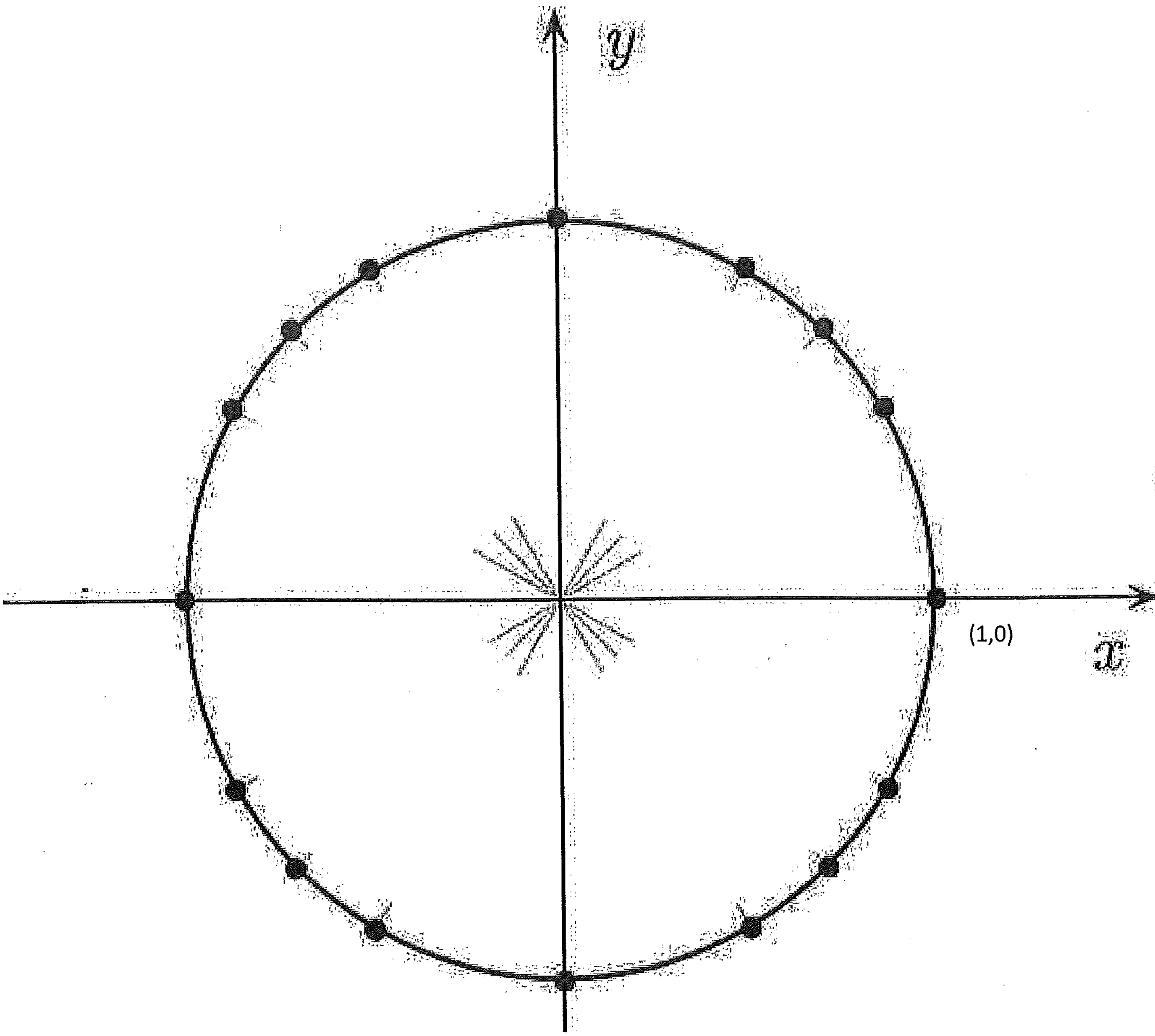
3. 15°

4. $-\frac{\pi}{4}$

Name: ___________________________________ Date: ____________

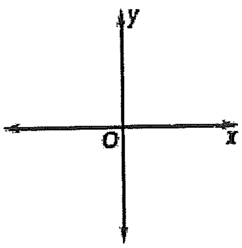
13-2/13-3 Angles, Reference Angles & Radians Hwk:

1. On the circle below, draw and label all of the following degrees:
0, 30, 45, 60, 90, 120, 135, 150, 180, 210, 225, 240, 270, 300, 315, 330, and 360.
Then find and label all radians of each degree.

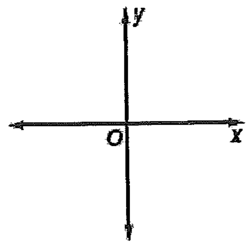


2. Without looking on the front, sketch each angle and find its reference angle.

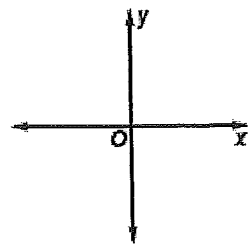
a). 315°



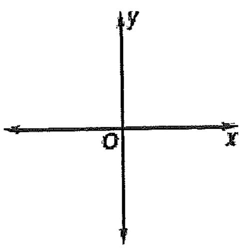
b). $\frac{7\pi}{4}$



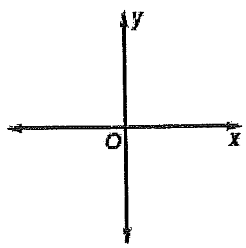
c). -240°



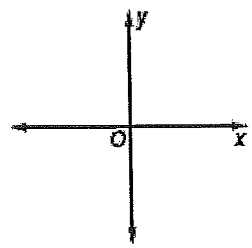
d). $-\frac{2\pi}{3}$



e). 750°



f). $\frac{5\pi}{6}$



Find one angle with positive measure and one angle with negative measure coterminal with each angle.

19. 45°

20. 60°

21. 370°

22. −90°

23. $\frac{2\pi}{3}$

24. $\frac{5\pi}{2}$

25. $\frac{\pi}{6}$

26. $-\frac{3\pi}{4}$

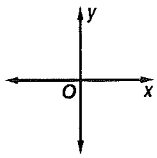
NAME ________________________________ DATE ____________ PERIOD _____

13-2 Practice

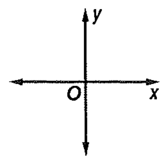
Angles and Angle Measure

Draw an angle with the given measure in standard position.

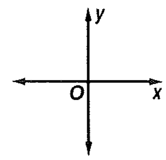
1. 210°



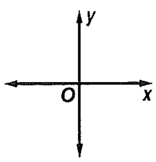
2. 305°



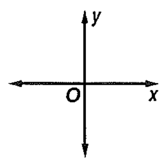
3. 580°



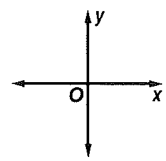
4. 135°



5. −450°



6. −560°



Rewrite each degree measure in radians and each radian measure in degrees.

7. 18° **8.** 6° **9.** 870° **10.** 347°

11. −72° **12.** −820° **13.** −250° **14.** −165°

15. 4π **16.** $\frac{5\pi}{2}$ **17.** $\frac{13\pi}{5}$ **18.** $\frac{13\pi}{30}$

19. $-\frac{9\pi}{2}$ **20.** $-\frac{7\pi}{12}$ **21.** $-\frac{3\pi}{8}$ **22.** $-\frac{3\pi}{16}$

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

23. 65° **24.** 80° **25.** 285°

26. 110° **27.** −37° **28.** −93°

29. $\frac{2\pi}{5}$ **30.** $\frac{5\pi}{6}$ **31.** $\frac{17\pi}{6}$

32. $-\frac{3\pi}{2}$ **33.** $-\frac{\pi}{4}$ **34.** $-\frac{5\pi}{12}$

35. TIME Find both the degree and radian measures of the angle through which the hour hand on a clock rotates from 5 A.M. to 10 A.M.

36. ROTATION A truck with 16-inch radius wheels is driven at 77 feet per second (52.5 miles per hour). Find the measure of the angle through which a point on the outside of the wheel travels each second. Round to the nearest degree and nearest radian.

13.3 Trig Functions of General Angles Notes Day 1

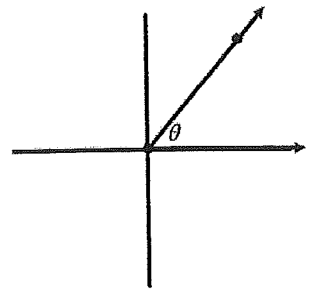
Example 1:

When point (x,y) is on the terminal side of angle θ the trig functions are defined as:

sin θ = 　　　　csc θ =

cos θ= 　　　　sec θ=

tan θ= 　　　　cot θ=



Example 2:

Find the exact values of the 6 trig functions of θ if the terminal side contains (8,-15).

sin θ = 　　　　csc θ =

cos θ= 　　　　sec θ=

tan θ= 　　　　cot θ=

Example 3:

Find the exact values of the 6 trig functions of θ if the terminal side contains (0,-2).

sin θ = 　　　　csc θ =

cos θ= 　　　　sec θ=

tan θ= 　　　　cot θ=

Example 4:

Suppose θ is an angle in Quadrant II and tan θ= $^{-2}/_{3}$. Find the exact values of the other five trig functions

sin θ =

csc θ =

cos θ=

sec θ=

tan θ=

cot θ=

Example 5:

Suppose θ is an angle in Quadrant II and sin θ= $^{-6}/_{11}$. Find the exact values of the other five trig functions

sin θ =

csc θ =

cos θ=

sec θ=

tan θ=

cot θ=

13.3 Trig Functions of General Angles HW Day 1

Find the exact values of the 6 trig functions of θ if the terminal side in standard position contains the given point.

1. (7,24)

2. (2,1)

3. (5,-8)

4. (4,-3)

5. (0,-6)

6. (-1,0)

7. $(\sqrt{2}, -\sqrt{2})$

8. $(-\sqrt{3}, -\sqrt{6})$

Suppose θ is an angle in standard position whose termianal side is in the given quadrant. For each function, find the exact values of the remaining five trig functions.

9. $\cos \theta = {}^{3}/_{5}$, Quadrant IV

10. $\tan \theta = {}^{-1}/_{5}$, Quadrant II

11. $\sin \theta = {}^{1}/_{3}$, Quadrant II

12. $\cot \theta = {}^{1}/_{2}$, Quadrant III

Review:

Sketch each angle. Then find its reference angle.

34. $315°$

35. $240°$

36. $\frac{5\pi}{4}$

37. $\frac{5\pi}{6}$

38. $-210°$

39. $-125°$

40. $\frac{13\pi}{7}$

41. $-\frac{2\pi}{3}$

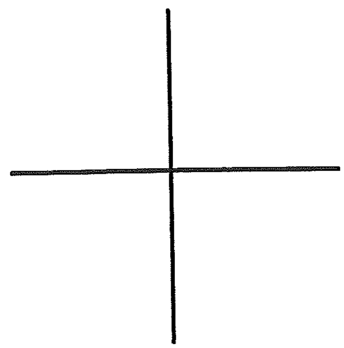
13.3 Trig Functions of General Angles Notes Day2

To find the EXACT trigonometric values Notes

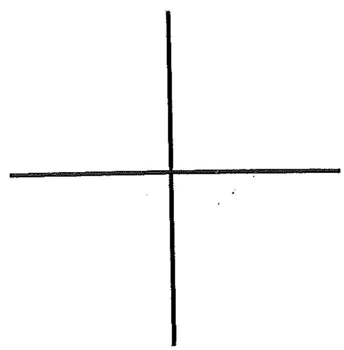
1.) Sketch the angle
2.) Label the reference angle
3.) Draw a triangle to the x-axis and label sides
4.) Find the trig values

Examples:

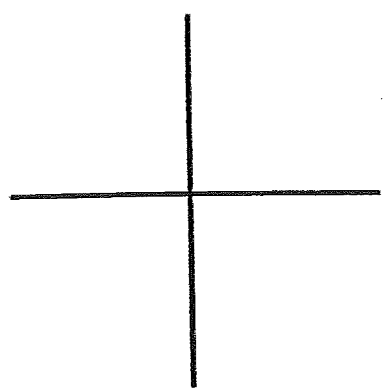
a.) Find the EXACT value for $\sin 120°$



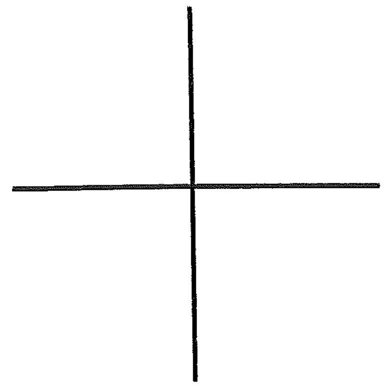
b.) Find the EXACT value for $\cos 300°$



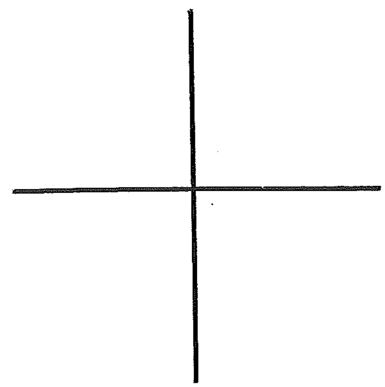
c.) Find the EXACT value for $\cos 180°$



d.) Find the EXACT value of $\cos \frac{5\pi}{4}$

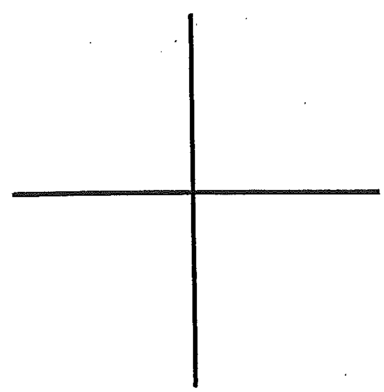


e.) Find the EXACT value for $\sec \frac{7\pi}{6}$

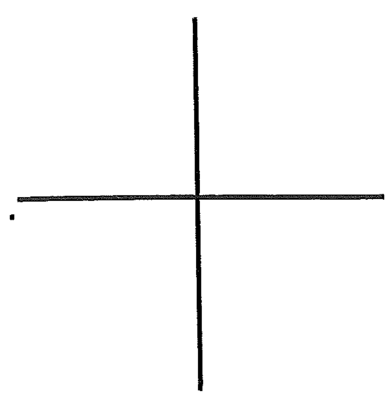


Show Me Problems:

1.) Find the EXACT value of csc -300°



2.) Find the EXACT value for $\tan \frac{5\pi}{3}$



NAME ______________________________ DATE ____________ PERIOD _____

13-3 Skills Practice

Trigonometric Functions of General Angles

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. (5, 12)

2. (3, 4)

3. (8, −15)

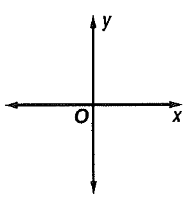
4. (−4, 3)

5. (−9, −40)

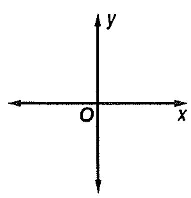
6. (1, 2)

Sketch each angle. Then find its reference angle.

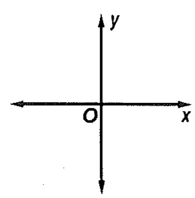
7. 135°



8. 200°



9. $\frac{5\pi}{3}$



Find the exact value of each trigonometric function.

10. sin 150°

11. cos 270°

12. cot 135°

13. tan (−30°)

14. $\tan \frac{\pi}{4}$

15. $\cos \frac{4\pi}{3}$

16. $\cot(-\pi)$

17. $\sin\left(-\frac{3\pi}{4}\right)$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

18. $\sin \theta = \frac{4}{5}$, Quadrant II

19. $\tan \theta = -\frac{12}{5}$, Quadrant IV

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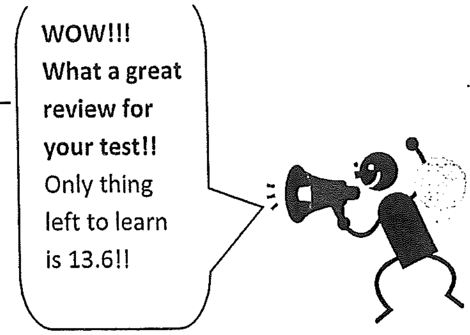
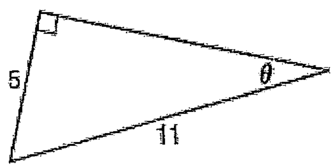


Accelerated Geometry 13.1-13.3

13.1 Right Triangle Trigonometry

Find the following 6 trigonometric values for the following triangle.



$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

$\csc\theta =$

$\sec\theta =$

$\cot\theta =$

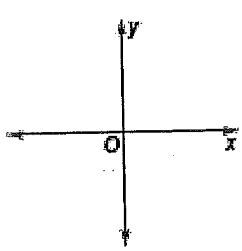
13.2 Angles and Angle Measures

1. Match each degree measure with the corresponding radian measure on the right.

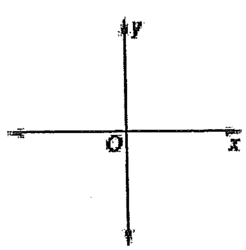
a. $30°$	i. $\frac{2\pi}{3}$
b. $90°$	ii. $\frac{\pi}{2}$
c. $120°$	iii. $\frac{7\pi}{6}$
d. $135°$	iv. π
e. $180°$	v. $\frac{\pi}{6}$
f. $210°$	vi. $\frac{3\pi}{4}$

(a.) Draw the angle with the given measure in standard position, (b) find θ′ as the reference angle and (c) find one positive and one negative conterminal angle with the given angles.

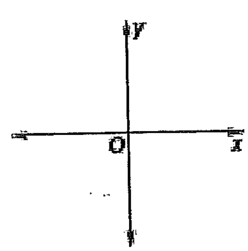
2. $140°$



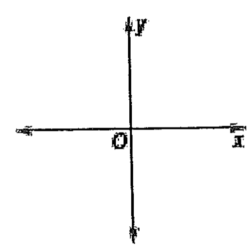
3. $-860°$



4. $-\frac{3\pi}{5}$



5. $\frac{11\pi}{3}$



13-3 Trigonometric Functions of General Angles
Show all work on separate paper.

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. $(6, 8)$
2. $(-20, 21)$
3. $(-2, -5)$

Find the reference angle for the angle with the given measure.

4. $236°$
5. $\frac{13\pi}{8}$
6. $-210°$
7. $-\frac{7\pi}{4}$

Find the exact value of each trigonometric function.

8. $\tan 135°$
9. $\cot 210°$
10. $\cot(-90°)$
11. $\cos 405°$
12. $\tan \frac{5\pi}{3}$
13. $\csc\left(-\frac{3\pi}{4}\right)$
14. $\cot 2\pi$
15. $\tan \frac{13\pi}{6}$

Find the exact value of each trigonometric function.

22. $\sin 240°$
23. $\sec 120°$
24. $\tan 300°$
25. $\cot 510°$
26. $\csc 5400°$
27. $\cos \frac{11\pi}{3}$
28. $\cot\left(-\frac{5\pi}{6}\right)$
29. $\sin \frac{3\pi}{4}$
30. $\sec \frac{3\pi}{2}$
31. $\csc \frac{17\pi}{6}$
32. $\cos(-30°)$
33. $\tan\left(-\frac{5\pi}{4}\right)$

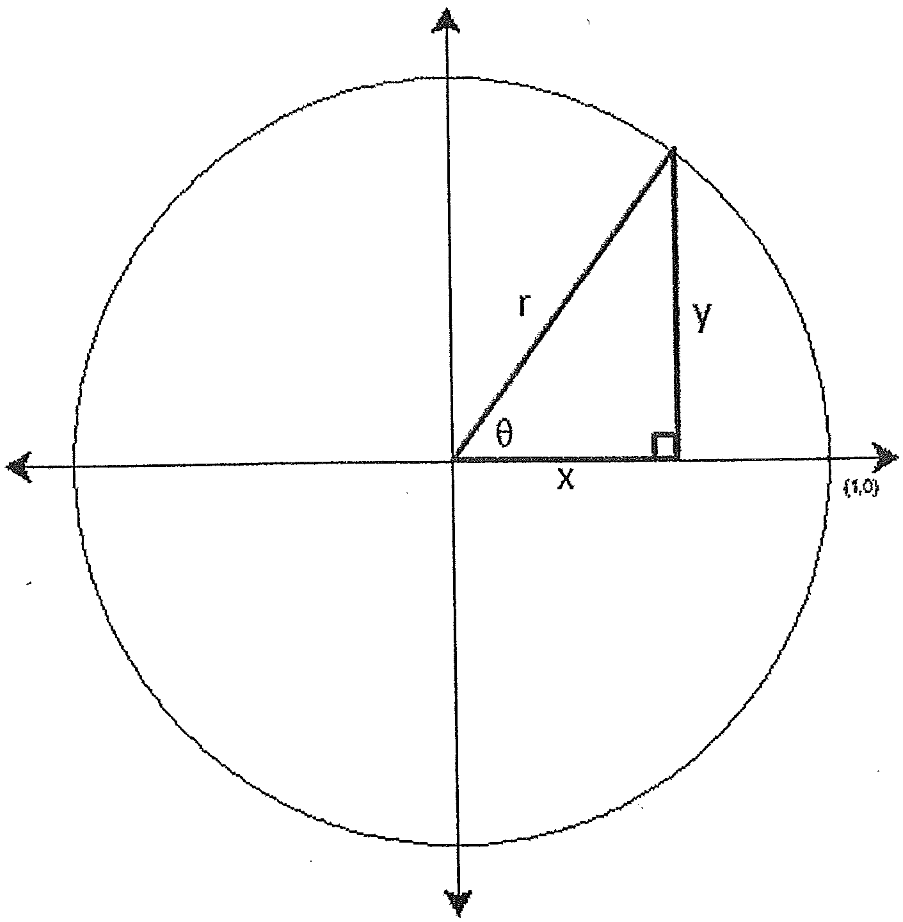
Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

16. $\tan\theta = -\frac{12}{5}$, Quadrant IV
17. $\sin\theta = \frac{2}{3}$, Quadrant III

13.3/13.6 Exact Values using the UNIT CIRCLE

ACC Geometry Notes

The circle below is called the ________________ ______________ because the value of the radius is _____________.



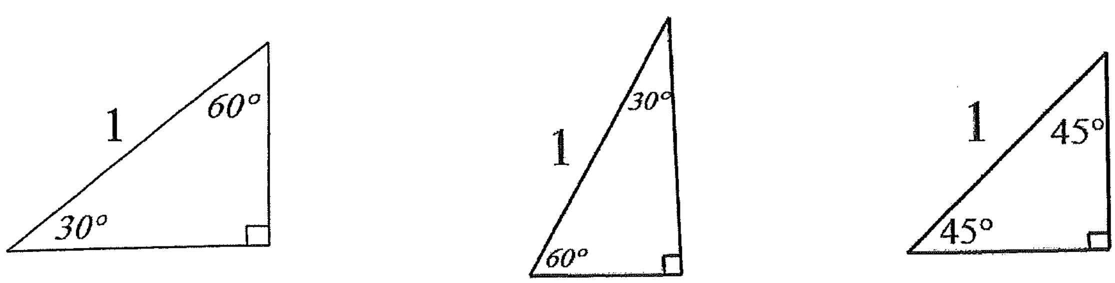
With radius= 1, find:

cosθ= ___________

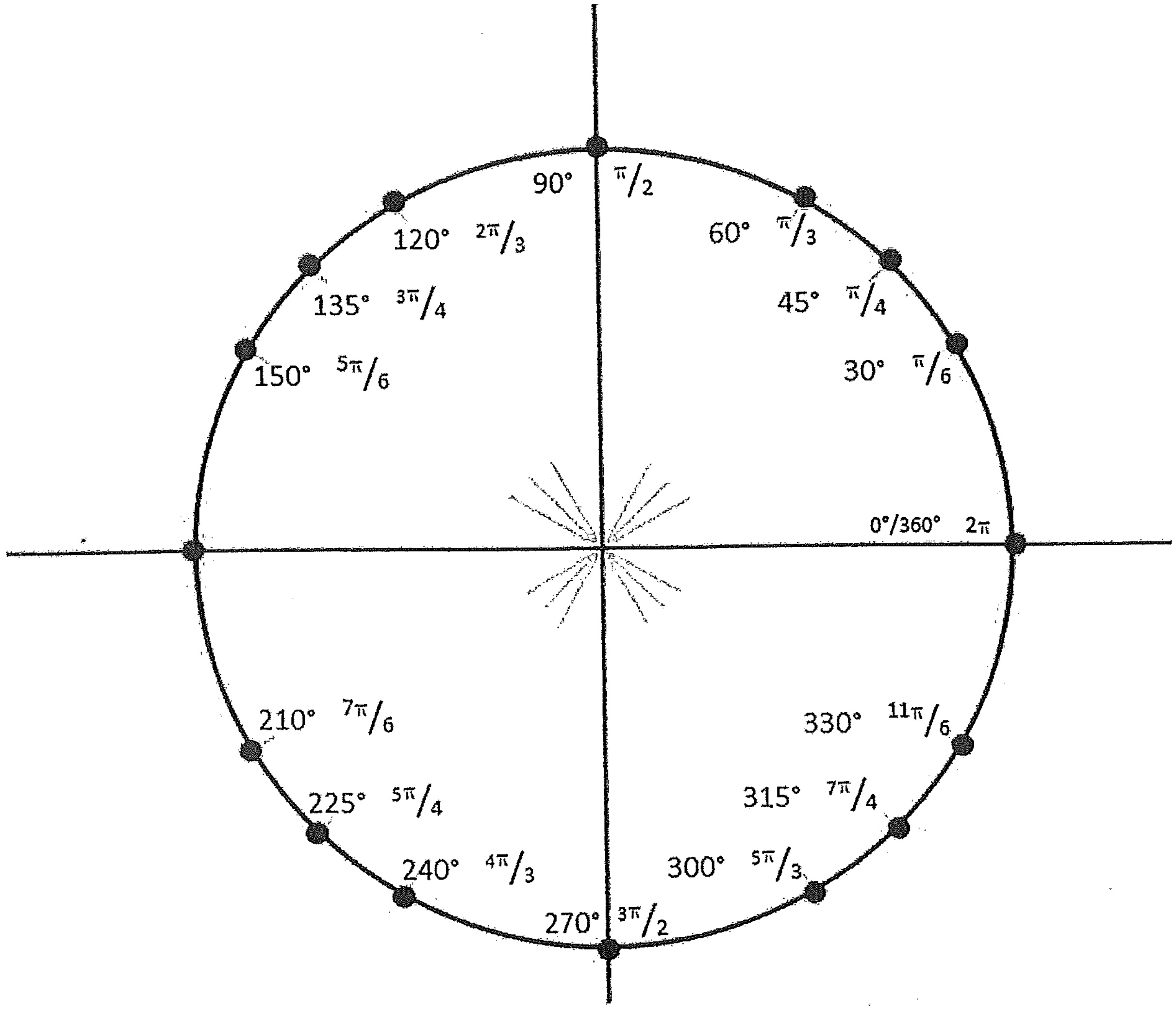
sinθ= ___________

tanθ= _________

Unit Circle



Find all points on the unit circle using special right triangles.



Find the exact value of each function by using the unit circle. Place the question # by the coordinates that correspond to the answer of the question.

1. cos (-240°)
2. tan $5\pi/4$
3. $\sin 5\pi$
4. $\csc(11\pi/4)$
5. $\sec(-3\pi/4)$
6. cot $7\pi/6$

7. Given an angle θ in standard position, if $P\left(\frac{\sqrt{6}}{5}, \frac{\sqrt{19}}{5}\right)$ lies on the terminal side and on the unit circle, find $\sin\theta$ and $\cos\theta$.

8. Find the exact value of the function. 2(sin45°)-6(cos135°).

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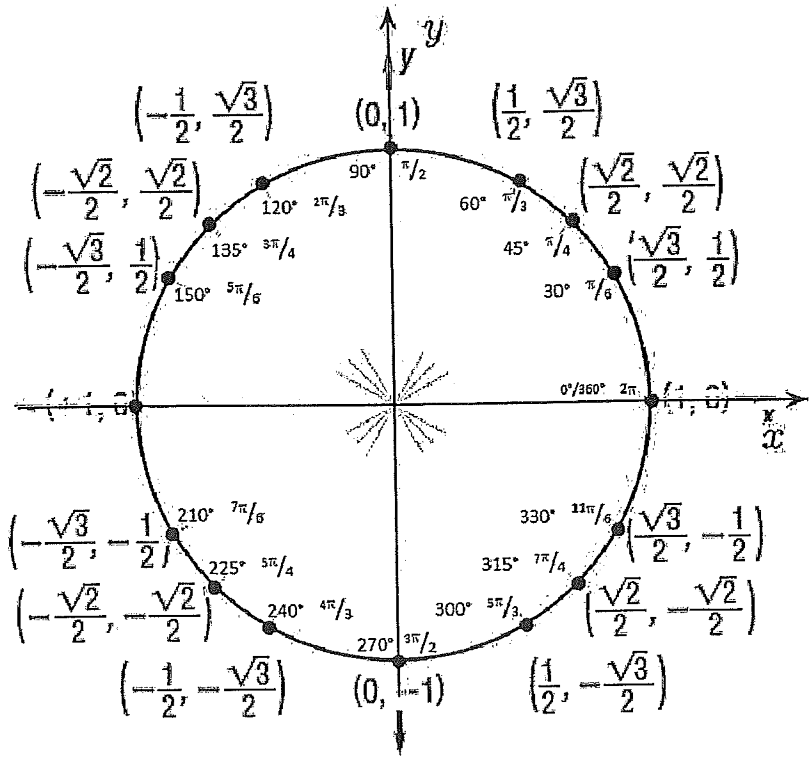
Unit Circle Homework

Find the exact value of each function by using the unit circle. Place the question # by the coordinates that correspond to the answer of the question.



1. $\sin 690°$

2. $\cos 750°$

3. sec 5π

4. tan $\left(\frac{14\pi}{6}\right)$

5. cot $\left(-\frac{3\pi}{2}\right)$

6. Csc (-225°)

Directions: *Find the exact value of each function... and I mean EXACT. No decimals!!!!!!!*

7. $\dfrac{cos60°+sin30°}{4}$

8. $3(sin60°)(cos30°)$

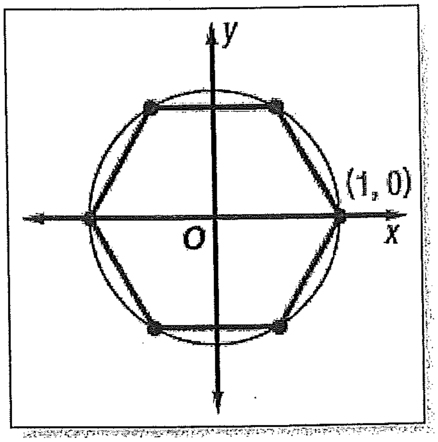
9. $\sin 30° - \sin 60°$

10. $\dfrac{4\cos 330°+2\sin 60°}{3}$

11. $12(\sin 150°)(\cos 150°)$

12. $(\sin 30°)^2 + (\cos 30°)^2$

13. A Regular hexagon is inscribed in a unit circle centered at the origin. If one vertex of the hexagon is at (1,0), find the exact coordinates of the remaining vertices. Use the picture to help!

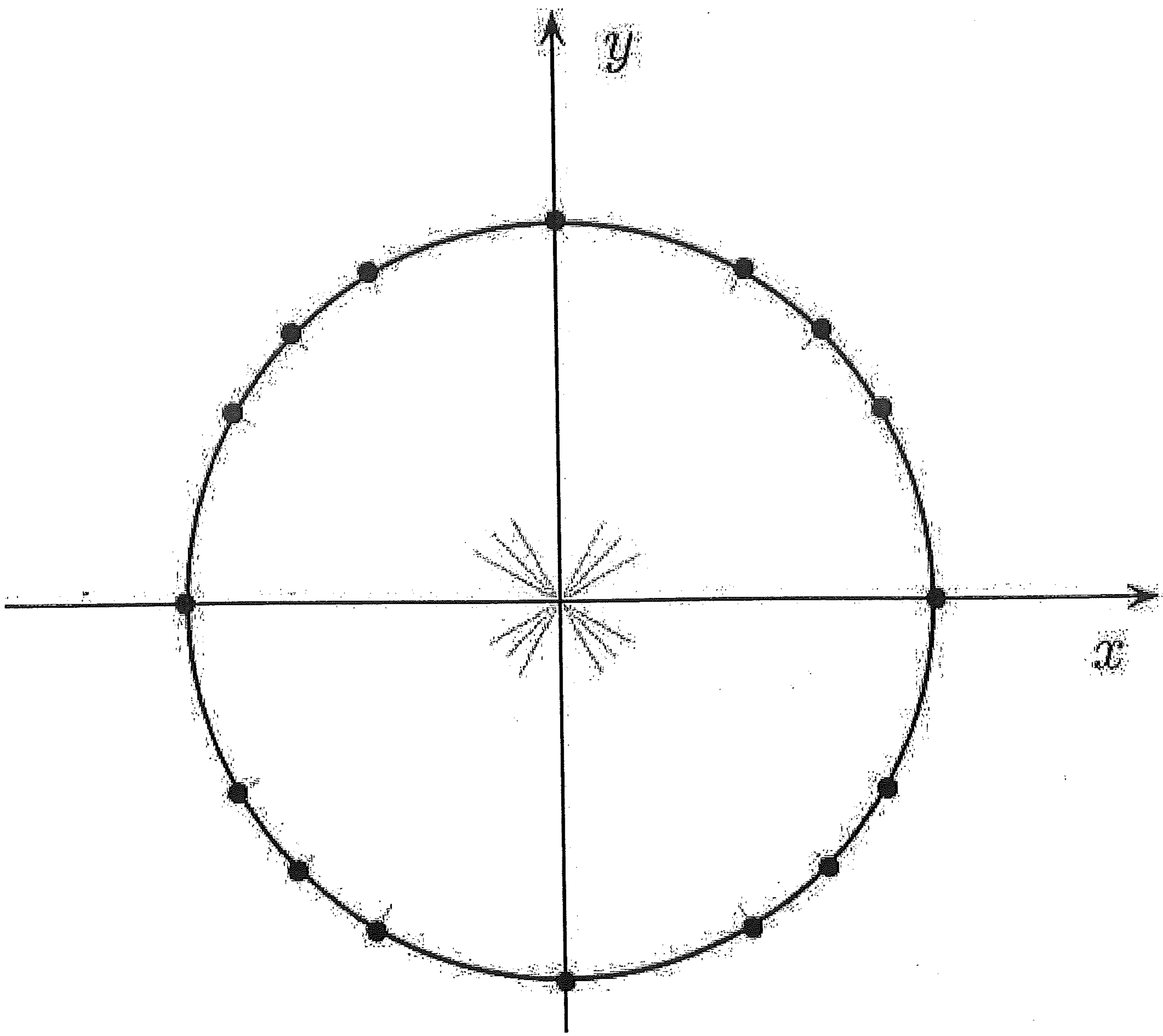


14. **WHICH ONE DOESN'T BELONG?** Identify the expression that does not belong with the other three. Explain your reasoning.

$\sin 90°$	$\tan \frac{\pi}{4}$	$\cos 180°$	$\csc \frac{\pi}{2}$

Acc Geo 13.3/13.6 Review Name__________________

Label all angles in degrees & radians. Label the coordinates for each point on the unit circle.



Find the exact value of each trigonometric function.

1. $\tan(-510°)$

2. $\csc \frac{11\pi}{4}$

3. $\cos 270°$

4. $\sin(-90°)$

5. $\cot 1665°$

6. $\cos \frac{4\pi}{3}$

7. $\cot 30°$

8. $\tan 315°$

10. $\cot(-\pi)$

11. $\csc \frac{\pi}{4}$

12. $\tan \frac{4\pi}{3}$

13. $\tan \frac{5\pi}{3}$

14. $\cos 45°$

15. $\sin 210°$

16. $\sin 330°$

17. $\cos 330°$

18. $\cos(-60°)$

19. $\sin(-390°)$

20. $\sin 5\pi$

21. $\cos 3\pi$

22. $\sin \frac{5\pi}{2}$

23. $\sin \frac{7\pi}{3}$

24. $\cos\left(-\frac{7\pi}{3}\right)$

25. $\cos\left(-\frac{5\pi}{6}\right)$

26. $\cos 30^o + \cos 60^o$

27. $5(\sin 45^o)(\cos 45^o)$

28. $\frac{\sin 210^o + \cos 240^o}{3}$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

29. $\sin\theta = \frac{4}{5}$, Quadrant II

30. $\tan\theta = -\frac{12}{5}$, Quadrant IV

13.4 Law of Sines-Ambiguous Case

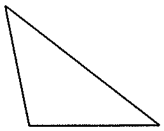
What are the maximum and minimum values that $\sin\theta$ can equal?

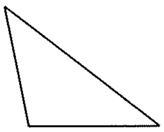
When you are solving a non-right triangle given ______ information and Law of Sines there can be:

1).
2).
3).

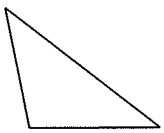
Determine whether each triangle has no solution, one solution, or two solutions. Then solve each triangle.

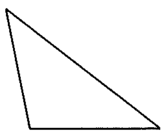
Ex1 **In ΔABC, B = 95, b = 10, c = 12.**



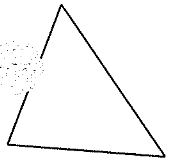


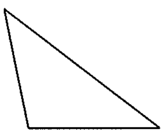
Ex2 **In ΔABC, B = 95, b = 19, c = 12.**





<u>Ex3</u> In ΔABC, A = 44, b = 19, a = 14.





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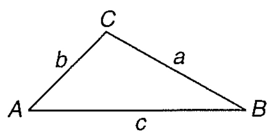
13-4 Study Guide and Intervention

Law of Sines

Law of Sines The area of any triangle is one half the product of the lengths of two sides and the sine of the included angle.

Area of a Triangle	area $= \frac{1}{2}bc\sin A$ area $= \frac{1}{2}ac\sin B$ area $= \frac{1}{2}ab\sin C$	

You can use the Law of Sines to solve any triangle if you know the measures of two angles and any side, or the measures of two sides and the angle opposite one of them.

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example 1 **Find the area of $\triangle ABC$ if $a = 10$, $b = 14$, and $C = 40°$.**

Area $= \frac{1}{2}ab\sin C$ — Area formula

$= \frac{1}{2}(10)(14)\sin 40°$ — Replace *a*, *b*, and *C*.

≈ 44.9951 — Use a calculator.

The area of the triangle is approximately 45 square units.

Example 2 **If $a = 12$, $b = 9$, and $A = 28°$, find B.**

$\frac{\sin A}{a} = \frac{\sin B}{b}$ — Law of Sines

$\frac{\sin 28°}{12} = \frac{\sin B}{9}$ — Replace *A*, *a*, and *b*.

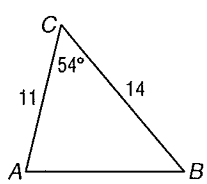
$\sin B = \frac{9\sin 28°}{12}$ — Solve for sin *B*.

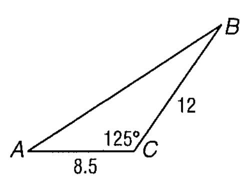
$\sin B \approx 0.3521$ — Use a calculator.

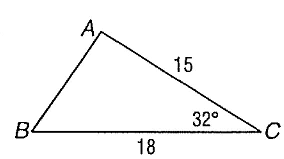
$B \approx 20.62°$ — Use the $\sin^{-1}$ function.

Exercises

Find the area of $\triangle ABC$ to the nearest tenth.

1. 

2. 

3. 

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4. $B = 42°, C = 68°, a = 10$
5. $A = 40°, B = 14°, a = 52$
6. $A = 15°, B = 50°, b = 36$

13-4 Study Guide and Intervention (continued)

Law of Sines

One, Two, or No Solutions

Possible Triangles Given Two Sides and One Opposite Angle	Suppose you are given a, b, and A for a triangle. If a is acute: $a < b \sin A \Rightarrow$ no solution $a = b \sin A \Rightarrow$ one solution $b > a > b \sin A \Rightarrow$ two solutions $a > b \Rightarrow$ one solution If A is right or obtuse: $a \leq b \Rightarrow$ no solution $a > b \Rightarrow$ one solution

Example **Determine whether $\triangle ABC$ has no solutions, one solution, or two solutions. Then solve $\triangle ABC$.**

a. $A = 48°$, $a = 11$, and $b = 16$

Since A is acute, find $b \sin A$ and compare it with a.
$b \sin A = 16 \sin 48° \approx 11.89$
Since $11 < 11.89$, there is no solution.

b. $A = 34°$, $a = 6$, $b = 8$

Since A is acute, find $b \sin A$ and compare it with a; $b \sin A = 8 \sin 34° \approx 4.47$. Since $8 > 6 > 4.47$, there are two solutions. Thus there are two possible triangles to solve.

Acute B

First use the Law of Sines to find B.

$$\frac{\sin B}{8} = \frac{\sin 34°}{6}$$
$$\sin B \approx 0.7456$$
$$B \approx 48°$$

The measure of angle C is about $180° - (34° + 48°)$ or about $98°$.

Use the Law of Sines again to find c.

$$\frac{\sin 98°}{c} \approx \frac{\sin 34°}{6}$$
$$c \approx \frac{6 \sin 98°}{\sin 34°}$$
$$c \approx 10.6$$

Obtuse B

To find B you need to find an obtuse angle whose sine is also 0.7456.

To do this, subtract the angle given by your calculator, 48°, from 180°. So B is approximately 132°.

The measure of angle C is about $180° - (34° + 132°)$ or about $14°$.

Use the Law of Sines to find c.

$$\frac{\sin 14°}{c} \approx \frac{\sin 34°}{6}$$
$$c \approx \frac{6 \sin 14°}{\sin 34°}$$
$$c \approx 2.6$$

Exercises

Determine whether each triangle has no solutions, one solution, or two solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. $A = 50°$, $a = 34$, $b = 40$ **2.** $A = 24°$, $a = 3$, $b = 8$ **3.** $A = 125°$, $a = 22$, $b = 15$

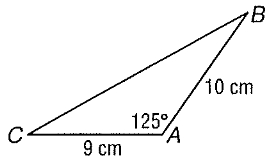
NAME ______________________ DATE ____________ PERIOD _____

13-4 Skills Practice

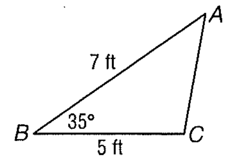
Law of Sines

Find the area of $\triangle ABC$ to the nearest tenth.

1.



2.



3. $A = 35°, b = 3$ ft, $c = 7$ ft

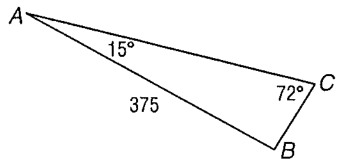
4. $C = 148°, a = 10$ cm, $b = 7$ cm

5. $C = 22°, a = 14$ m, $b = 8$ m

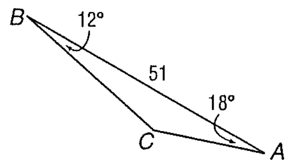
6. $B = 93°, c = 18$ mi, $a = 42$ mi

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

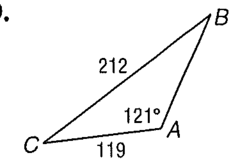
7.



8.



9.



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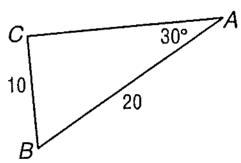

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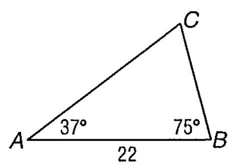

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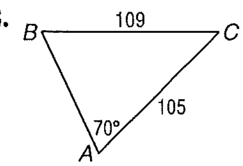


Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

13. $A = 30°, a = 1, b = 4$

14. $A = 30°, a = 2, b = 4$

15. $A = 30°, a = 3, b = 4$

16. $A = 38°, a = 10, b = 9$

17. $A = 78°, a = 8, b = 5$

18. $A = 133°, a = 9, b = 7$

19. $A = 127°, a = 2, b = 6$

20. $A = 109°, a = 24, b = 13$

Lesson 13-4

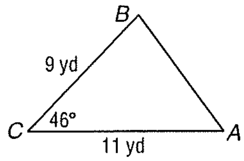
NAME ____________________ DATE __________ PERIOD _____

13-4 Practice

Law of Sines

Find the area of $\triangle ABC$ to the nearest tenth.

1.



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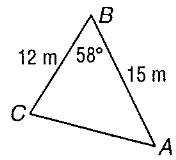
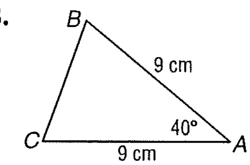


3.



4. $C = 32°, a = 12.6$ m, $b = 8.9$ m

5. $B = 27°, a = 14.9$ cm, $c = 18.6$ cm

6. $A = 17.4°, b = 12$ km, $c = 14$ km

7. $A = 34°, b = 19.4$ ft, $c = 8.6$ ft

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

8. $A = 50°, B = 30°, c = 9$

9. $A = 56°, B = 38°, a = 12$

10. $A = 80°, C = 14°, a = 40$

11. $B = 47°, C = 112°, b = 13$

12. $A = 72°, a = 8, c = 6$

13. $A = 25°, C = 107°, b = 12$

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

14. $A = 29°, a = 6, b = 13$

15. $A = 70°, a = 25, b = 20$

16. $A = 113°, a = 21, b = 25$

17. $A = 110°, a = 20, b = 8$

18. $A = 66°, a = 12, b = 7$

19. $A = 54°, a = 5, b = 8$

20. $A = 45°, a = 15, b = 18$

21. $A = 60°, a = 4\sqrt{3}, b = 8$

22. WILDLIFE Sarah Phillips, an officer for the Department of Fisheries and Wildlife, checks boaters on a lake to make sure they do not disturb two osprey nesting sites. She leaves a dock and heads due north in her boat to the first nesting site. From here, she turns 5° north of due west and travels an additional 2.14 miles to the second nesting site. She then travels 6.7 miles directly back to the dock. How far from the dock is the first osprey nesting site? Round to the nearest tenth.