Notes – Special Right Triangles & Trigonometry	Name:
Standard:	Hour:

Objective: I can find the exact value of trig ratios for 30°, 45°, 60° using special right triangles.

Warm-up.

1. $\dfrac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$$

2. $\dfrac{6\sqrt{3}}{2}$

$\boxed{3\sqrt{3}}$

3. $\dfrac{3}{3\sqrt{2}}$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

Find the missing side lengths of each of the special right triangles.

Special Right Triangles

(special right triangle diagram: 60°–30° triangle with sides $2x$, x, $x\sqrt{3}$; 45°–45° triangle with sides s, $s\sqrt{2}$, s)

4.

18   $18\sqrt{2}$ $45°$ $45°$ 18 $\cdot\sqrt{2}$

isosc. $\cong \| $

5. $\div 2$

SL 3   hyp 6 $\times\sqrt{3}$ $3\sqrt{3}$ 30°   LL

6. $\div 2$

SL 4   60°   8   Hyp $\sqrt{3}$ $4\sqrt{3}$ LL
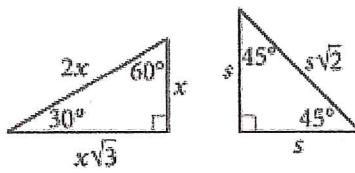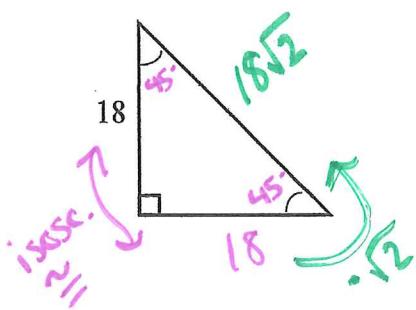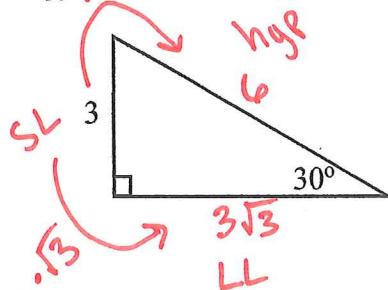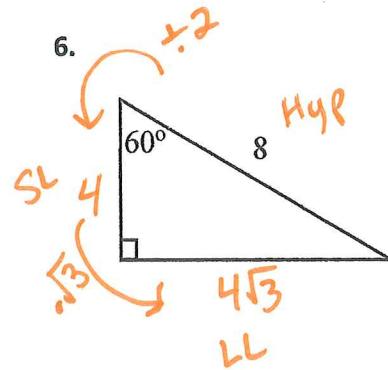
$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

**Explore**

Find the exact value each of the indicated trig ratios. Simplify your answers leaving no radicals in the denominator.

7.

Triangle A, C, B with right angle at C. A marked 60°, side 9 from A to C, hypotenuse 18 from A to B, side $9\sqrt{3}$ from C to B, angle 30° at B.
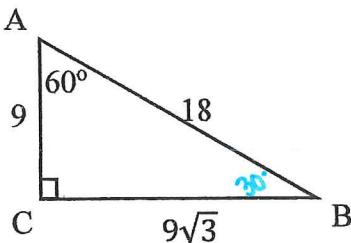
$\sin 30° = \dfrac{9}{18}$ $\dfrac{1}{2}$

$\cos 30° = \dfrac{9\sqrt{3}}{18}$ $\dfrac{\sqrt{3}}{2}$

$\tan 30° = \boxed{\dfrac{9}{9\sqrt{3}}}$ $\dfrac{\sqrt{3}}{3}$

$\dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} =$

$\sin 60° = \dfrac{9\sqrt{3}}{18} = \dfrac{\sqrt{3}}{2}$

$\cos 60° = \dfrac{9}{18} = \dfrac{1}{2}$

$\tan 60° = \dfrac{9\sqrt{3}}{9}$ $\sqrt{3}$

$1\sqrt{3}$

8.

Triangle K, F, J with right angle at F. K marked 60°, side 4 from K to F, hypotenuse 8 from K to J, side $4\sqrt{3}$ from F to J, angle 30° at J.
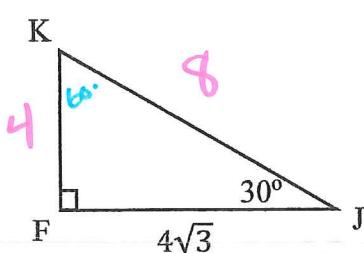
$\sin 30° = \dfrac{4}{8} = \dfrac{1}{2}$

$\cos 30° = \dfrac{4\sqrt{3}}{8} = \dfrac{\sqrt{3}}{2}$

$\tan 30° = \dfrac{4}{4\sqrt{3}}$ $= \dfrac{\sqrt{3}}{3}$

$\sin 60° = \dfrac{4\sqrt{3}}{8}$ $\dfrac{\sqrt{3}}{2}$

$\cos 60° = \dfrac{4}{8} = \dfrac{1}{2}$

$\tan 60° = \dfrac{4\sqrt{3}}{4} = \sqrt{3}$

9.

Triangle X, Y, Z with right angle at Z. Y marked 60°, side 2 from X to Y, side $1\sqrt{3}$ from X to Z, side 1 from Y to Z.
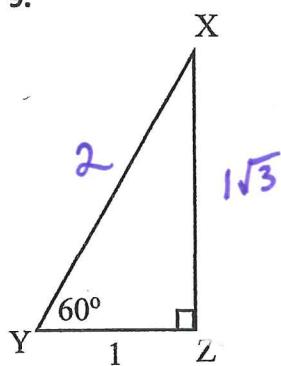
$\sin 30° = \dfrac{1}{2}$

$\cos 30° = \dfrac{\sqrt{3}}{2}$

$\tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

$\sin 60° = \dfrac{\sqrt{3}}{2}$

$\cos 60° = \dfrac{1}{2}$

$\tan 60° = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

10.
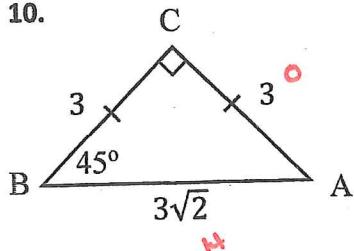
$\sin 45° = \dfrac{3}{3\sqrt{2}}$ $\dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\cos 45° = \dfrac{3}{3\sqrt{2}} \rightarrow$ $\dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$ $\dfrac{\sqrt{2}}{2}$

$\tan 45° = \dfrac{3}{3} = 1$

11.
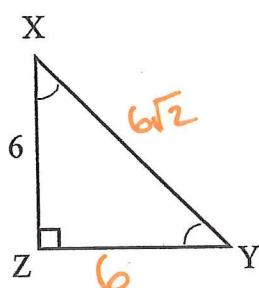

$\sin 45° = \dfrac{6}{6\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\cos 45° = \dfrac{6}{6\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\tan 45° = \dfrac{6}{6} = 1$

12.
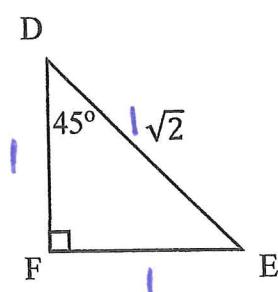
$\sin 45° = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\cos 45° = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\tan 45° = \dfrac{1}{1} = 1$

Summary – So regardless of the size of the special right triangle, the trig ratios all reduce down to the same value!

	30°	45°	60°
sin	$\dfrac{1}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$
cos	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$
tan	$\dfrac{\sqrt{3}}{3}$	1	$\sqrt{3}$